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MGT 3510 COURSE OUTLINE AND ASSIGNMENT SCHEDULE SPRING 2015 Assignments listed below are firm only for one week in advance. The complete list of tentative assignments is given to help you plan ahead.

1. POSITIONING STRATEGIES IN MANUFACTURING AND SERVICES

Competitive Priorities (Marketplace Drivers) Types of Technologies and Workforce Skills (Resource Capabilities) Positioning Strategies (Integration of Above)

Assignment due Tues. Jan 13

1.0 Cheryl Gaimon, Karen Napoleon, "Information Technology Worker Systems in Structured and Unstructured Environments," <u>New Service Development Creating Memorable Experiences</u> James A Fitzsimmons and Mona Fitzsimmons, Eds., Sage Publications, Thousand Oaks CA, 2000, pp. 183-192, 212-214.

Reading due Thurs. Jan 15

Excerpts from Operations and Supply Chain Management, Jacobs and Chase, 14th Edition, 2014.

II. IMPROVING FIRM PERFORMANCE

A. New Product Development: Management and Engineering Interface

Assignment due Tues. Jan 20

Case 1: Campbell Soup (Candidate for group write-up. See questions on syllabus) HBS Case 9-690-051, Aug. 1990.

Assignment due Thurs. Jan 22

1.25 Teradyne Corp : The Jaguar Project (not a group case) HBS Case 9-606-042, May 2006.

Assignment due Tues. Jan 27

1.0 Behnam Tabrizi, Rick Walleigh, "Defining Next-Generation Products: An Inside Look," Harvard Business Review, Nov. Dec. 1997.

B. Manufacturing Flexibility

Assignment due Thurs. Jan 29

Case 2: Eli Lilly Company: The Flexible Facility Decision (All groups must do this case and/or Universal Luxary) HBS Case 9-694-074, April 1994.

Assignment due Tues. Feb 3

Featuring Access to

1.25 Gaimon, Cheryl, Alysse Morton, "Investment in Changeover Flexibility for Early Entry in High Tech Markets," <u>Production and Operations Management</u>, Vol. 14, No. 2, 2005, 159-

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Sammy Sample 9-17-10 Hour 1 Book Report First Quarter

Title: The Adventures of Tom Sawyer Author: Mark Twain Genre: Realistic Fiction

Protagonist:

Tom Sawyer is the protagonist of the novel. He is about 12 years old and lives in the small town of St. Petersburg, Missouri. He wants to be an outlaw and likes to play hooky from school and church. He drives his Aunt Polly crazy and hates his half-brother Sid.

Antagonist

Injun Joe is the antagonist of this story. Joe is a half breed Indian and a robber and murderer. He kills Dr. Robinson and in addition plans a brutal attack on a wealthy widow. He is greedy and bitter because of his treatment by the townspeople.

Conflict:

This is a person vs. person conflict. Tom wants to get the \$12,000 that Joe has stolen and Joe wants to kill the Widow Douglas and then get away. Both characters cannot get what they want, so there is a conflict.

Setting:

The story takes place in a small town in Missouri during the 1830's. The setting is very important to the story because the town represents the place where Mark Twain grew up. Many of the characters are based on people he actually knew. The language, superstitions, and culture of the village are all important in this story.

Plot:

Tom Sawyer is a mischievous and hilarious boy growing up in the 1930's by the Mississippi River. He is constantly in trouble with Aunt Polly, who loves him and suffers trying to raise him. Tom is a very clever boy. He tricks his friends into whitewashing the fence, and wins a bible in church by cheating his friends out of their prize tickets.

Tom is not a good student and often plays hooky. He likes girls, and leaves Amy Lawrence for Becky Thatcher. His best friends are Joe Harper and Huck Finn. Huck is the son of the town drunk and disliked by all the adults. The kids respect him because he is the only free boy in town.

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If ϕ is the angle between (x, y) and B(x, y) then by (2) and what we just proved, $\cos \phi = (x, y) \cdot B(x, y)$ (x2 + y 2) $\cos \theta = \cos \theta$. 4 b) E = {(x, y) : $0 \le y \le 1$, $y \le x \le 1$, $0 \le y \le 1$, $y \le x \le 1$, $0 \le y \le x$, hence by Fubini's Theorem, Z 1 Z 0 Z 1 1 sin(x2) dx = 0 1 x sin(x2) dx = 0 1 - cos(1) . 10.5.4. a) If E is connected in R then E is an interval, hence E o is either empty or an interval, hence connected by definition or Theorem 10.56. Notice that $x_1 = 1 = y_1$. Fix $j \ge n$. By the Heine-Borel Theorem, xk is bounded, hence (by Bolzano-Weierstrass) has a convergent subsequence. Thus the trace looks like a sine wave traced on the parabolic cylinder $y = x_2$. $[0, \pi] \cup [2\pi, 3\pi] \cup$ c) $\lim x \to 0 \log(x/\sin x)/x^2 = \lim x \to 0 (\sin x - x \cos x)/(2x^2 \sin x) = \lim x \to 0 (\sin x - x \cos x)/(2x^2 \sin x) = \lim x \to 0 (\sin x - x \cos x)/(2x^2 \cos x + 4 \sin x) = \lim x \to 0 (\sin x - x \cos x)/(2x^2 \sin x) = \lim x \to 0 (\sin x - x \cos x)/(2x \cos x + 4 \sin x) = \lim x \to 0 (\sin x - x \cos x)/(2x \cos x + 4 \sin x) = \lim x \to 0 (\sin x - x \cos x)/(2x \cos x + 4 \sin x) = \lim x \to 0 (\sin x - x \cos x)/(2x \cos x + 4 \sin x) = \lim x \to 0 (\sin x - x \cos x)/(2x \cos x + 4 \sin x) = \lim x \to 0 (\sin x - x \cos x)/(2x \cos x + 4 \sin x) = \lim x \to 0 (\sin x - x \cos x)/(2x \cos x + 4 \sin x) = \lim x \to 0 (\sin x - x \cos x)/(2x \cos x + 4 \sin x) = \lim x \to 0 (\sin x - x \cos x)/(2x \cos x + 4 \sin x) = \lim x \to 0 (\sin x - x \cos x)/(2x \cos x + 4 \sin x) = \lim x \to 0 (\sin x - x \cos x)/(2x \cos x + 4 \sin x) = \lim x \to 0 (\sin x - x \cos x)/(2x \cos x + 4 \sin x) = \lim x \to 0 (\sin x - x \cos x)/(2x \cos x + 4 \sin x) = \lim x \to 0 (\sin x - x \cos x)/(2x \cos x + 4 \sin x) = \lim x \to 0 (\sin x - x \cos x)/(2x \cos x + 4 \sin x)$ for each $n \in N$. But by the Fundamental Theorem of Calculus and the fact that f(a) = 0, we have $Z \ge f(0) = f(x) = f(x) = f(x) = 1$ (sin((k - 1/2)x) - sin((k + 1/2)x) = sin(x/2) + sinthat $\partial(y, z) \partial(z, x) + (yu ut + yv vt) \partial(u, v) \partial(x, y) + (zu ut + zv vt) \partial(u, v) [] [] xu yu zu xv yv zv = ut det [xu yu zu] + vt de$ $0 (c1) + |h| |f 0 (c2)| \le 2M |h|$ for some cj 's and all $h \in \mathbb{R}$. Hence $P = \cos y$, Q = 0, R = xy, and $(P, Q, R) \circ \varphi = (\cos(\cos t/2), 0, \cos 2t/2)$. 3.4.0 a) False. Thus bk (f) = 4/(k\pi) when k is odd and 0 when k is even. b) Repeat the proof of Theorem 3.6, replacing the absolute value by the norm sign. 10.5.7. Suppose H is compact. Then U = E and $V = X \setminus E$ are nonempty open sets, $U \cap V = \emptyset$, and $X = U \cup V$. Let $\varepsilon > 0$ and set $\delta = \varepsilon/M$. Hence, $2|ak| \le |ak|$ for large k, and it follows from the Comparison Theorem that k=1 ak converges absolutely. And, if y = -1 then f (x, y) = x3 - 3x + 1 has critical points $x = \pm 1$ which correspond to extreme points of H. -2.40 Copyright © 2010 Pearson Education, Inc. c) The closure is R, the interior is R, the boundary is Ø. $\partial u \, \partial v \, \partial v \, \partial u \, E \, 2 \, G2 - F \, 2 = 142$ Copyright © 2010 Pearson Education, Inc. Thus f is absolutely integrable on (a, b) by the Comparison Theorem. 13.5.9. a) By hypothesis, F = (fx, fy). g \circ \phi(t) k \phi (t)k dt = \tau (Jk0) \cup N \tau (Jk0) k = 1 Since τ is 1-1, $\tau (Jk) \setminus \tau (Jk0)$ consists of two points, so this last integral is unchanged if Jk0 is replaced by Jk. b) If the Riemann sums converge to I(f), then there is a partition P such that $|S(f, P, t_j)| < |I(f)| + 1$ for all choices of $t_j \in [x_j-1, x_j]$. 3xn-1 + 6xn-1 + 4 Using an initial guess of $x_0 = 0$, we obtain $x_1 = -.25$, $x_2 = -.313953$. \sqrt{c} The inequality is equivalent to $n_2 + 1 > 500$. By definition and Exercise 5.3.7d, L(w) = x + y = L(s) + L(t) = L(st). Then x < x - 1 so 0 < -1, i.e., this case is empty. b) By definition, a and b are closed, $A \cap B = \emptyset$, but dist (A, B) = 0 because $1/x \rightarrow 0$ as $x \rightarrow \infty$. f 0 (0) π (g -1)0 (2) = 1 1 = . b) By vector algebra and the Cauchy-Schwarz inequality, $|x \cdot y - x \cdot z| = |x \cdot (y - z)| \le kxk ky - zk < 2 \cdot (3+4) = 14$. Conversely, suppose $xn \in E \setminus \{a\}$ and $xn \to a$ as $n \to \infty$. By Theorem 9.26, f - 1 (U) $\cap E$ is relatively open in E. Therefore, the series converges by the Alternating Series Test. Thus such a vector has the form (a, (20 - 8a)/7, (8 + a)/7), a = 0. $\neg \infty R \infty 5.4.2$. a) If p = 1 then 1 $dx/xp = x1 - p/(1 - p)^{-1}$. Thus f (x) $\leq \leq M1 + \cdots + MN =: M$ for all $x \in E * \supset E$. 9.1.8. a) If $E \cap Br$ (a) (a) is nonempty. d) We cannot multiply by the denominator x - 1 unless we consider its sign. b) This is the set of points on or inside the ellipse $x^2 + 4y^2 = 1$. Since $x \in [1, 3]$ implies $|3 - x^{36}| \leq 1$ 3 + 336 and $x3 + nx66 \ge 0 + n = n$, it follows that $3 + 336 3 + 336 3 + 336 3 + 336 3 + 336 3 + 336 3 = -x = \le 1$ implies $e4/N \to 1$ as $N \to \infty$, given $\varepsilon > 0$, we can choose $N \in N$ so that $2 \ 0 < e4/N - 1 < \varepsilon$. 14.1.6. a) Since $f(x) \cos kx$ is odd, ak (f) = 0 for k = 0, 1, ... By hypothesis, 0 < x1 < 1. SN $f(x) \ge 0 + \cdots + fxj(x) + \cdots + 0 = 1$ for all $x \in 10^{-1}$ for x = 0. E. Hence we can use (-a, -b, c) for a normal at the point (a, b, c). Therefore, this series converges by the Alternating Series Test. Moreover, by the Approximation Property, there exist $x_j \in E$ such that $x_j \to a$. 10.2.6. Modify the proofs of Theorems 3.21 and 3.22, replacing the absolute value signs with the metric ρ . Thus $Rx\sqrt{=}y + fx \sqrt{and}$ we can set f = 0, P = 0, and g = 0.22 Case 2. Fix x, $y \in [xk-1, xk]$. If f is differentiable, then f is continuous on [a, b]. If $\alpha > 0$ then $|x|\alpha \rightarrow 0$ as $x \rightarrow 0$. 11.3.4. a) If (x, y) = (0, 0), then by the Chain Rule, a normal to K at (x, y, z) is given by (x/z, y/z, -1). Therefore, x is not a cluster point of [a, b). By (8) in 1.1, $xn < 2\sqrt{10}$ for $n \ge N$, i.e., $xn \to 0$ as $n \to \infty$. Hence (ex - 1)/x = k=0 xk/(k + 1)! for $x \in R$. Thus given x1 < x2, f (x1) = lim gn (x2) = f (x2). Conversely, suppose every relatively open covering of H has a finite subcover. If f (c) < 0, then c - a > 0 and c - b < 0 imply that f 0 (x1) < 0 < f 0 (x2). Then y = 1 $2 2 \partial F \partial x \partial y \partial F \partial x \partial z \partial 2 F \partial y \partial z + 2 + 2 + 2$. Thus the series converges absolutely on (-1/e, 1/e). \sqrt{d} Suppose 3 < a < 5. b) The ratio of successive terms of this series is 2k + 1 4 2 = 1 - = 1 - x, i.e., $x^2 - x = 0$. Since each $V\alpha$ is nonempty, choose a point $x\alpha \in V\alpha$ for each $\alpha \in A$. 2 2 2 Thus $\{xn\}$ is decreasing and bounded below. 2.3.8. a) This follows immediately from Exercise 1.2.6. $p \sqrt{b}$ By a), xn+1 = (xn + yn)/2 < 2xn/2 = xn. The trace looks like a gull in flight (called a cubical parabola) traced in the z = x plane. If $n \ge N := max\{N1, N2\}$ and $x \in E$ then $|(fg)(x) - (fngn)(x)| \le |f(x) - fn(x)| |g(x) - gn(x)| 0$ choose $\delta > 0$ so small that $x, y \in E$ and |x| = (xn + yn)/2 < 2xn/2 = xn. $-y < \delta$ imply $|fN(x) - fN(y)| < ^2/3$. 91 Copyright © 2010 Pearson Education, Inc. 2 (a + b) sin 2 ϕ + 0 13.5.4. a) By Gauss' Theorem, ZZ Z 2 Z 4 Z Z 1 ω = 2 (yz + 2y + 1) dz dy dx = 2 S x2 - 2 0 (24 - x2 - 5x4/4) dx = 224/3. Then by Exercise 1.2.5b, 0 < xn < xn+1. Therefore, kf kH is finite and attained by the Extreme Value Theorem. Chapter 4 4.1 The Derivative. It follows from the Squeeze Theorem that $f \times g$ is differentiable and its total derivative is T. Then $\mu k \psi u k 2 = \partial x \partial v \P 2 \mu + \partial y \partial v \P 2 \mu + \partial z \partial v \Pi 2 \mu + \partial z \partial v H 2 \mu + \partial z \partial v$ $s s 0 0 R \infty$ for s > 0. Hence by Theorem 11.15, g is differentiable on V. Also, $0 \le s2n+1 - s2n = 1/(2n + 1) \rightarrow 0$ as $n \rightarrow \infty$. 2k)/(1 · 3 . f) By Cauchy-Schwarz and Remark 8.10, $|x \cdot (y \times z)| \le kxk kyk kzk < 1 \cdot 2 \cdot 3 = 6$. (σ , J) runs counterclockwise. Therefore, f is not differentiable at (0, 0). Thus by induction, 0 < xn < xn+1 for all $n \in N$. Conversely, suppose $x \in f - 1$ (f (A)). Then $y = x^2$ and $z = \sin x$. Let $x \in E$ and khk $< \delta$. dx j = 1 n X (-1)j - 1 Vol (E). Let $h = \alpha h^0$, α 6=0, where khk < δ and let $c \in L(a; a + h)$. Indeed, $0 \le f 0$ (x0) = 0, i.e., f 0 (x0) = 0, a) Suppose E is convex is convex by a characterise for the found only if f = 0 (see Exercise 5.1.4b). 2 Next, notice by part a) that $f 0 (x) = (2/x^3)e^{-1/x}$ for x = 0 and f 0 (0) = 0. a) Suppose E is convex is convex is convex is convex in the found on the found of t but not connected. $4x^2 - 1 < 0$. Hence $D(f + g)(x, y) = [2x + 1 2y - 1] b) Df(x, y) = [3x^2 - 2xy + y^2 2xy - x^2 - 3y^2]$. Let $^2 > 0$ and choose a grid $G = P \times Q$ on R, where $P = \{a0, . Since differentiability implies continuity, it follows from the Inverse Function Theorem that <math>f - 1$ is differentiable on f(I), and (f - 1)0(x) = 1/f 0. (f - 1 (x)). 8.4.2. a) This is the set of points on or inside the ellipse x2 + 4y 2 = 1. Then there is a $\delta 0 > 0$ such that $0 < c - x < \delta 0$ implies F (c) - F (x) > $\epsilon 0 / 2$. It follows that fxj (0) = 0 for j = 1, . Then f satisfies the condition for $\alpha = 1$ but fx (0, 0) = limh $\rightarrow 0$ |h|/h does not exist. Hence it follows from Parseval's Identity that $4 \propto X 1 \pi (a^2k (f) + b^2k (f))$ sin 2 kh = k = 1 Z π |f (x + h) - f (x - h)|2 dx - π for each h \in R. Then 2 e - 1/x 1/xj+1 (j + 1)/xj+2 (j + 1)/xj+1 = lim = and $\varphi(t_2) - \varphi(t_0)$. 12.5.3. If $f \in Cc_\infty$ (R) then f = 0 on some interval (a, b). It converges to the continuous function 0 as $k \to \infty$, so by Dini's Theorem, $(\log k + x)/(k + x) \to 0$ uniformly on [0, 1] as $k \to \infty$. Case 2. If it holds for n then $n+1 \ge 1 - n + n + 1 = 1 - n + n + 1 = 1 - n + 1 = 1$ $1/k \ge 1$ so the terms of this series are all ≥ 1 . So, let $x \in E$ and choose $xk \in D$ such that $xk \to x$. Moreover, $xn-1+1-1 \le 1-1 = 0$. d) Repeat the proof of Theorem 2.29. By the Triangle Inequality, $n \ge N$ implies $\rho(xn, yn) \le \rho(xn, a) + \rho(yn, a) < \varepsilon$. By definition, $f(A \setminus B) \supseteq f(A) \setminus f(B)$ holds whether f is 1-1 or not. Therefore, $\{sn\}$ is bounded. 10.3.9 Suppose $f: R \to R$ is continuous and I = (a, b). By definition and Exercise 5.3.7c, L(y) = xq = qL(t) = L(tq). Thus $(f-1)0(xn) = 1/f 0 (f-1(xn)) \to 1/f$ $n \rightarrow \infty \leq -$ lim inf $(-xn - yn) = \lim \sup(xn + yn)$. Let w = F(x, y, f(x, y)). c) By repeating the proof of Theorem 2.8, we can show that every Cauchy sequence is bounded. xp+1 By Remark 5.46 and Exercise 5.4.2a, this last integral is absolutely integrable since p + 1 > 1. If $w \in B^2(y)$ then $\rho(w, a) \leq \rho(w, y) + \rho(y, a) = r$ and $\rho(w, a) = r$ and $\rho(w, a) = r$ and $\rho(w, a) = r$. $3 + n \in Q$ then $+ 3 + n \ge Q$ then $+ 3 + n \ge Q$ and $x \in E$ imply |fk(x) - f(x)| < 2/2. By Definition 13.6 and the Fundamental Theorem of Calculus, $ds/dt = k\varphi 0$ (t)k, hence by the Inverse Function Theorem, $dt/ds = (-1)0(s) = 1/k\phi0(t)k$, where t = -1(s). b) Following Example 2.13, $n^3 + n - 21 + (1/n^2) - (2/n^3) 2$ as $n \to \infty$. Therefore, $1/(2x^2 + x - 3 \ge M \cdot d)$ The graph of $x^2 - 2x + 2$ is a parabola whose minimum is 1 at x = 1. By Remark 10.44, A and B are closed sets, so by Theorem 10.31, $A \cap B$ is a closed subset of the compact set A. On the other hand, since w + 1 is not the supremum of E, w + 1 > 10n+1 y. and $D(f \cdot g)(x, y) = [xy \cos x + 2xy \sin x - x \cos y]$. The limit is continuous because $\varphi 0$ is continuous. Let $x \in [c, d]$. This point is outside H so can be disregarded. However, the product of any irrational with 0 is a rational. In particular, it follows from Exercise 1.6.5c that f is 1-1 if and only if f is onto. Finally, by the Chain Rule, $(x\alpha) = E(\alpha L(x)) \cdot \alpha L 0$ (x) = $x\alpha \alpha = \alpha x\alpha - 1$. Since xyz = 16, neither x nor y is zero. Thus f (x/m) = f (x/m) f (x/m) = f (x/m) + Chain Rule, $(x\alpha) = x\alpha \alpha = \alpha x\alpha - 1$. Intermediate Value Theorem, there is an x (between -1 and 0) such that f (x) = 0. Hence x \in BX \cap Y \subset V, i.e., V is open in Y. 0 0 Hence by the Fundamental Theorem of Calculus, Z y fx = Pv (x, v) dv + h0 (x) = P (x, y) - P(x, 0) + h0 (x). 1.3 The Completeness Axiom. 11.7.6. Suppose D(2) f (a)(h0) < 0 for some $h0 \in R2$. Hence if a 6= 0, then it follows from the Root Test that this series converges absolutely when a|x| < 1, i.e., |x| < 1/a. 10.2.4. a) Surely a set which has infinitely many points is nonempty. Since $qn \rightarrow q$ and q is bounded by M, choose N2 so large that $|qn(x)| \le 2M$ for all $n \ge N^2$ and $x \in E$. 108 Copyright © 2010 Pearson Education, Inc. C1 : $x^2 + z^2 = 8$, y = 1, oriented in the clockwise direction when viewed from far out the y axis, and C2 : $x^2 + z^2 = 8$, y = 0, oriented in the counterclockwise direction when viewed from far out the y axis. Let $\varphi(x) = x^2 + z^2 = 8$, y = 0, oriented in the counterclockwise direction when viewed from far out the y axis. Let $\varphi(x) = x^2 + z^2 = 8$, y = 0, oriented in the counterclockwise direction when viewed from far out the y axis. Let $\varphi(x) = x^2 + z^2 = 8$, y = 0, oriented in the counterclockwise direction when viewed from far out the y axis. Let $\varphi(x) = x^2 + z^2 = 8$, y = 0, oriented in the counterclockwise direction when viewed from far out the y axis. Let $\varphi(x) = x^2 + z^2 = 8$, y = 0, oriented in the counterclockwise direction when viewed from far out the y axis. Let $\varphi(x) = x^2 + z^2 = 8$, y = 0, oriented in the counterclockwise direction when viewed from far out the y axis. Let $\varphi(x) = x^2 + z^2 = 8$, y = 0, oriented in the counterclockwise direction when viewed from far out the y axis. Let $\varphi(x) = x^2 + z^2 = 8$, y = 0, oriented in the counterclockwise direction when viewed from far out the y axis. Let $\varphi(x) = x^2 + z^2 = 8$, y = 0, oriented in the counterclockwise direction when viewed from far out the y axis. Let $\varphi(x) = x^2 + z^2 = 8$, y = 0, oriented in the counterclockwise direction when viewed from far out the y axis. Let $\varphi(x) = x^2 + z^2 = 8$, y = 0, $\cos(t^2) - 1)$ dt = 1 1 - $\sin(t)$. Thus choose $|x - y| < \delta$ and $x, y \in E0$ imply |f(x) - f(y)| 0, choose an integer n > 1 so large that $1/n < \frac{2}{2}$ and choose $\delta > 0$ so small that $2(n - 1)\delta < \frac{2}{2}$. Thus $|f(x, y)| \leq M |x - 1|$, and it follows from the Squeeze Theorem that $f(x, y) \rightarrow 0 = :L$ as $(x, y) \rightarrow (1, b)$. By the Approximation Property, choose $x \in E$ such that $xk \rightarrow 0$ sup E. Since Faa = 2 k=1 x2k > 0, this critical point is either a minimum or a saddle point. Let $\delta > 0$ be so small that B $\delta(x_j) \subset R_j0$ and define A as above. 1.3.2. Since a - 1/n < a + 1/n, i.e., |a - rn| < 1/n. b) Let $\varepsilon > 0$ and set $\delta := \varepsilon$. This is sometimes called the Reciprocal Rule. Let f(x) = x and g(x) = 1if x < 2 and 2 if $x \ge 2$. c) By b), $(1 + 1/n)n \ge 1n + n1n - 1$ (1/n) = 2. Otherwise, given any M > 0 there is an $N \in N$ such that xN > M. Hence its arc length is 2 2 2 4 + 4 + 4 = 4 3. x] and $Dg(x, y) = [x \cos x + \sin x \sin y]$. Hence by Theorem 10.34, Ao \subseteq Bo. If $x \in A$, then $x \le \sup A$. Since each of the numbers x0, y0, u0, v0, s0, to is nonzero, this determinant is nonzero. As $t \to \infty$, $(x, y) \to (0, 0)$ and $dy/dx = (2t - t4)/(1 - 2t3) \to \infty$. \sqrt{b} Let $f(x) = x - \log x - 0.6$. Since $x \ge 4$, $\sqrt{\sqrt{x-2}} x \sqrt{f} 0$ ($x) = 1/(2x) - 1/x = \ge 0$. Hence sn converges by Theorem 2.29. Moreover, by Exercise 12.5.1, products of functions with compact support. Since $(2/3)k \to 0$ as $k \to \infty$, it follows that E is of measure zero. By Gauss' Theorem, ZZ ZZ ZZ Z Z I Z Z 1 y |z| dV = 2 yz dy d(x, z) + 2 yz dy d(x, zx0 = y0. Since $f 2 \ge 0$, we conclude that f 2, hence f, is identically zero on [a, b]. Therefore, Vol $(x + E) := \inf G R |x + R | = \inf G$ $x \in 1/g(x)$ is positive. R Conversely, by part b) the function f (x, y) := C(x,y) F \cdot T ds is well-defined. 4.5 Inverse Functions. Therefore, $0 < a < 2 + a - \sqrt{2} = b$. Thus $y \in Br$ (a) \cap Bs (b). Then f is $C \infty$ on R. (Indeed, in either case, q < p so this expression is eventually bigger P than q.) ∞ The inequality implies |ak+1/ak| < 1 - q/k for k large. By Theorem

9.6, it follows that fk \rightarrow some function f pointwise on H. Hence n-1 U (f, P) = 2 X 2 + 2\delta = + 2(n - 1)\delta < 2/2 + 2/2 = 2. f (x) > |2M|/2 = M . 14.5.5. Suppose F is not convex. Since E is bounded above (by a), it follows from the Completeness Axiom and Theorem 1.15 that n0 = sup E exists and belongs to E. Thus x = - a , 2cD y = - b , 2cE and z = 1 4c2 µ $a^2 b^2 + D \in \P$.) onto the number with decimal expansion $0.x1 x^2 \cdots$. But this follows immediately from the Squeeze Theorem since $\varphi(N)$ is bounded as $N \to \infty$ and $e^{-(s-a)N \to 0}$ as $n \to \infty$. In particular, all the functions which appear in this problem are products of integrable and/or continuous functions, hence integrable. In particular, X contains more than two clopen sets. b) Suppose f is increasing and continuous on (a, b). kxk kyk n(b - a) 2 76 Copyright © 2010 Pearson Education, Inc. Telescoping, we obtain Z F · T ds = C N X f $\circ \phi j$ (bj) - f $\circ \phi j$ (a) = f $\circ \phi N$ (bN) - f $\circ \phi j$ (a) = f $\circ \phi N$ (b) here integrable. In particular, X contains more than two clopen sets. b) Suppose f is increasing and continuous on (a, b). kxk kyk n(b - a) 2 76 Copyright © 2010 Pearson Education, Inc. Telescoping, we obtain Z F · T ds = C N X f $\circ \phi j$ (b) - f $\circ \phi j$ (c) = f $\circ \phi N$ (b) - f $\circ \phi j$ (c) = f $\circ \phi N$ (b) - f $\circ \phi j$ (c) = f $\circ \phi N$ (c) = f \circ \phi N (c) = f $\circ \phi N$ (c) = f \circ \phi N (c) = f $\circ \phi N$ (c) = f \circ \phi N (c) = f $\circ \phi N$ (c) = f \circ \phi N (c) = f $\circ \phi N$ (c) = f \circ \phi N (c) = f $\log(1/|ak|) \log(\log k \log \log k) (\log \log k) 2 \log 2 u = = =$. Suppose it holds for some n. Finally, letting r0 \downarrow r, we \sqrt{k} conclude that lim supk $\rightarrow \infty$ ak \leq r, as required. d) Let $\eta > 0$ and let C² be the constants in part c). Thus choose M > 0 such that |xn| and |yn| are both $\leq M$ for all $n \in N$. Consequently, if j > J then (since k + j > k) $\sqrt{k} \propto \sqrt{k}$ are $X \ge 0$. $|ak|^k \leq 1 < 2$. Thus g is well defined on all of X. 11.5 The Mean Value Theorem and Taylor's Formula. Therefore, sk 1 > y and x + br1 $\leq s \leq y + bk1$ for all $k \leq 1 < y + bk1$ for all $k \leq 1 < y + bk1$ for all $n \in N$. But by part a), Ac $(\partial B)c \subseteq \partial A$ and $B \subset (\partial A)c \subseteq \partial B$. Hence by hypothesis, there is an $n \in N$ and a 1-1 function φ from E onto {1, 2, .13.1.5. a) This curve evidently lies on the cone x2 + y 2 + z 2)-3/2, y(x2 + y 2 + z 2)-3/2, z(x2 + y 2 + z 2)-3/2, z(x2 + y 2 + z 2)-3/2 , z(x2 + y 2 + z 2)-3/2) and E = B1 (0, 0, 0). 1.2.8. a) Since (1 - n)/(1 - n2) = 1/(1 + n), the inequality is equivalent to 1/(n + 1) < .01 = 1/100. Then $x \in B\delta x_j(x_j)$ for some $1 \le j \le N$, so $f(x) = f(x_j)$. It is closed and connected. Therefore, by Dini's Theorem and Theorem 7.10, Z 1 lim $k \rightarrow \infty \mu x_2 f(0) dx = 0 f(0)$. If $y = 0, 1 \le x \le 3$, then $f(x, y) = x_2 f(0) dx = 0 f(0)$. If $y = 0, 1 \le x \le 3$, then $f(x, y) = x_2 f(0) dx = 0 f(0)$. If $y = 0, 1 \le x \le 3$, then $f(x, y) = x_2 f(0) dx = 0 f(0)$. > 39/2, i.e., n ≥ 20. Moreover, since fxk is continuous on E * := k=1 [xk - rk, xk + rk], the Extreme Value Theorem implies that there are constants Mk that |fxk | ≤ Mk on E * for all k. gx (a, b, c) gz (a, b, c) has three pieces: C1 which runs from (1, 0, 0) to (0, 0, 1), C2 which runs from (0, 0, 1) to (0, 1, 0), and C3 which runs from (0, 1, 0) to (1, 0, 0). Define g on E by g = f. Hence by d), $\{y\} = f(\{a\}) \cap f(\{b\}) = \emptyset$, a contradiction. On the other hand, if $x \in E$ then since $E \circ = \emptyset$, Br (x) is not contained in E for any r > 0, i.e., Br $(x) \cap E \circ G = \emptyset$ for all r > 0. Thus $n \ge N$ implies $|1 + 2xn - 3| \equiv 2 |xn - 1| < \epsilon$. It converges to the continuous function sin x 1/2 as $k \to \infty$. Hence differentiating term by term, we obtain $-\infty - \infty - X \cos(x/(k+1)) - X 1 - 0 |f(x)| = -1$. $3 \le \sqrt{1.4.9}$. a) If $\sqrt{m} = k \ge 2$, then m = k by definition. Hence x = z/2 and y = z/2. Suppose xn > 2. It also diverges for $p \le 0$ by the Divergence Test. 10.1.5. a) Let a be the common limit point. c) Consider f(x) = 2x + 3x - 2. Since f(0) is continuous, it follows from the sign preserving lemma that there is an interval $I \subset (a, b)$ containing x0 such that $f(0) = -(\pi/(s^2 + \pi^2)) = 2s\pi/(s^2 + \pi^2) = 2s\pi/(s^$ set $\psi(u) = (f - 1(-u), -u)$ for $u \in [-f(a), -f(b)]$ and $\tau(u) = f - 1(-u)$. Thus $xn \in Br(a) \cap E$. But by (2), $\cos \theta = (a \times b) \cdot c/(ka \times bk \, kck)$. Hence (1, 1, 1) and (-1, -1, -1) are the only points where the tangent plane of H is parallel to x + y - z = 1. This function is continuous, and f(-1) = 1/e - 1 < 0 < 1 = f(0). Thus the point on the paraboloid where the tangent plane is parallel to x + y + z = 1 is (-1/2, -1/2, 1/2) and an equation of this tangent plane is 2x + 2y + 2z = -1. By elementary set algebra and Theorem 1.37, $(f - 1 (A) \cap B) \cup f - 1 (C) = f - 1 (A \cap B) \cup f - 1 (A \cap B) \cup$ $0 \pi \sin u \, du. c$) Since $\sqrt{\sqrt{p}} (k - k 2 + k) - k 1 \sqrt{\sqrt{k} - k2} + k = \rightarrow -2 k + k2 + k k + k2 + k \sqrt{as k} \rightarrow \infty$ and by l'H^opital's Rule, $k 1/k \rightarrow 0 = 1$ as $k \rightarrow \infty$, we see by Theorem 9.2 that (k - k 2 - k, k 1/k, -1/2, 1, 0) as $k \rightarrow \infty$, we see by Theorem 9.2 that (k - k 2 - k, k 1/k, -1/2, 1, 0) as $k \rightarrow \infty$, we see by Theorem 9.2 that (k - k 2 - k, k 1/k, -1/2, 1, 0) as $k \rightarrow \infty$, we see by Theorem 9.2 that (k - k 2 - k, k 1/k, -1/2, 1, 0) as $k \rightarrow \infty$. then, $2.9253226 = 2 \cdot (1.4626613) < -1$ ex dx < 2.9253626. Since H is compact, there exists a finite subset A0 of A such that $\{V\alpha\}_{\alpha \in A0}$ covers H. 11.3.10. Since it is nonnegative on ∂H , there is a point (x1, t1) < 0. By the Triangle Inequality, $n \ge N$ implies $|xn - a| + |yn - a| < \epsilon$. f (x) $\ge g(x)$., $x^3 = -.31766$. If $-\delta < x < 0$, then f (x) = -x/x = -1 so |f (x) - L| = $|-1 + 1| = \sqrt{0} < \varepsilon$. n $\rightarrow \infty$ n $\rightarrow \infty$ c) Let xn = (-1)n + 1. By Lebesgue's Theorem, f is almost everywhere continuous. If x = -1 then f (x, y) = -1 - 3y - y 3 which has no critical points. d) Let (x, y, z) = $\varphi(t)$. JN such that $\tau 0 > 0$ on each Jk0 and J = \cup N k=1 Jk. If P (x) = an xn + $\cdots + 1$ a0, then it follows from Theorem 4.10 that P 0 (x) = nan xn-1 + \cdots + a1 exists and is a polynomial. If a + b + c = 4 then b = (20 - 8a)/7, c = (8 + a)/7. 14.1.1. a) Since x 2 sin kx is odd, bk (x2) = 0 for k = 1, 2, . Hence by Exercise 3.18, f v g and f A g are absolutely integrable on (a, b). Since F (0, 0, 0) = 0 and $\partial F 3 = 2z + p 2 \partial z 2 sin(x + y 2) + 3z +$ 4 equals $3/4\ 6=0$ at (0, 0, 0), the expression has a differentiable solution near (0, 0, 0) by the Implicit Function Theorem. In particular, 1/xn < 2. Finally, since P0 is finer than P, U (f, P). By the argument in part b), (2, xn) are closed and bounded, $\sqrt{}$ but not compact. 8.3.10. The result is false if "open" is omitted. b) Let V be relatively open in f (E), i.e., $V = U \cap f$ (E) for some U open in Y. Thus the Taylor series contains only odd terms. 9.4.1. a) f (0, π) = (0, 1] is compact and connected as Theorems 9.29 and 9.30 say it should; f (-1, 1) = (- sin 1, sin 1) is open, big deal; f [-1, 1] = [- sin 1, sin 1] is compact and connected as Theorem 4.20 and 9.30 say it should; f (-1, 1) = (- sin 1, sin 1) is open, big deal; f [-1, 1] = [- sin 1, sin 1] is compact and connected as Theorem 4.20 and 9.30 say it should; f (-1, 1) = (- sin 1, sin 1) is open, big deal; f [-1, 1] = [- sin 1, sin 1] is compact and connected as Theorem 4.20 and 9.30 say it should; f (-1, 1) = (- sin 1, sin 1) is open, big deal; f [-1, 1] = [- sin 1, sin 1] is compact and connected as Theorem 4.20 and 9.30 say it should; f (-1, 1) = (- sin 1, sin 1) is open, big deal; f [-1, 1] = [- sin 1, sin 1] is compact and connected as Theorem 4.20 and 9.30 say it should; f (-1, 1) = (- sin 1, sin 1) is open, big deal; f [-1, 1] = [- sin 1, sin 1] is compact and connected as Theorem 4.20 and 9.30 say it should; f (-1, 1) = (- sin 1, sin 1) is open, big deal; f [-1, 1] = [- sin 1, sin 1] is compact and connected as Theorem 4.20 and 9.30 say it should; f (-1, 1) = (- sin 1, sin 1) is open, big deal; f [-1, 1] = [- sin 1, sin 1] is compact and connected as Theorem 4.20 and 9.30 say it should; f (-1, 1) = (- sin 1, sin 1) is open, big deal; f [-1, 1] = [- sin 1, sin 1] is compact and connected as Theorem 4.20 and 9.30 say it should; f (-1, 1) = (- sin 1, sin 1) is open, big deal; f [-1, 1] = [- sin 1, sin 1] is compact and connected as Theorem 4.20 and 9.30 say it should; f (-1, 1) = (- sin 1, sin 1) is open, big deal; f [-1, 1] = [- sin 1, sin 1] is compact and connected as Theorem 4.20 and 9.30 say it should; f (-1, 1) = (- sin 1, sin 1) is open, big deal; f [-1, 1] = [- sin 1, sin 1] is compact and connected as Theorem 4.20 and 9.30 say it should; f (-1, 1) = (- sin 1, sin 1) is compact and connected as Theorem 4.20 and 9.30 say it should (-1, sin 1) is compact and (-1, sin 1) is compact Theorems 9.29 and 9.30 say it should. Then $p \sqrt{\sqrt{F} \cdot \varphi_0} = (-1 + \cos 3t + 5, \sin t/2, 1) \cdot (0, -\sin t, \cos t/2) = (\cos t - \sin 2t)/2$. Thus $\lim x \to 0$ log x does not exist. b) By parts and part a), $Z \propto Z Z n \propto n-1$ -st $n! \propto -st n!$ the -st dt = t + 0 is $0 \le 0 \le 0$. Then $\hat{t} = t + 0$. Hence by the Comparison vertices the formula of the comparison of the c Theorem, f is not improperly integrable on [a, b). g) Clearly $xn \rightarrow \infty$ as $n \rightarrow \infty$. Since $\nabla fy(a, b) \cdot (b, th) - \nabla fy(a, b) + th) - fy(a, b) - \nabla fy(a, b) + th) - fy(a, b) - \nabla fy(a, b)$ sin x cos x $^{-\infty}$ dx = $-p^{-1} - pxp x Z 1 \infty cos x dx = cos(1) - pxp+1 Z 1 \infty cos x dx$. Since y = x2 + 2x - 5 is a quadratic in x, we $\sqrt{have x} = (-2 \pm 4 + 4(5 + \sqrt{y}))/2 = -1 \pm 6 + y$. If x0 < x < x0 + δ then P (x)/(x - x0) > m0 / $\delta \ge m0$ (M/m0) = M . 4.2.1. a) By the Product Rule, (f g)0 (2) = f 0 (2)g(2) + f (2)g 0 (2) = 3a + c. Hence f -1 (V) $\cap E$ is relatively open in E. By Exercise 12.2.3, we have E ZZ 1 $\Delta u(x0) = \lim 2 \Delta u \, dA = 0. d$) Applying L'H^oopital's Rule twice, we obtain $\sqrt[4]{\sqrt[4]{k-1}} = \lim \sqrt[4]{k-\infty} k \rightarrow \infty \log k \log k/k \sqrt[4]{k-1} = \lim \sqrt[4]{k-\infty} k \rightarrow \infty \log k \log k/k \sqrt[4]{k-1} = \lim \sqrt[4]{k-\infty} k \rightarrow \infty \log k \log k/k \sqrt[4]{k-1} = \lim \sqrt[4]{k-\infty} k \rightarrow \infty \log k \log k/k \sqrt[4]{k-1} = \lim \sqrt[4]{k-\infty} k \rightarrow \infty \log k \log k/k \sqrt[4]{k-1} = \lim \sqrt[4]{k-\infty} k \rightarrow \infty \log k \log k/k \sqrt[4]{k-1} = \lim \sqrt[4]{k-\infty} k \rightarrow \infty \log k \log k/k \sqrt[4]{k-1} = \lim \sqrt[4]{k-\infty} k \rightarrow \infty \log k \log k/k \sqrt[4]{k-1} = \lim \sqrt[4]{k-\infty} k \rightarrow \infty \log k \log k/k \sqrt[4]{k-1} = \lim \sqrt[4]{k-\infty} k \rightarrow \infty \log k \log k/k \sqrt[4]{k-1} = \lim \sqrt[4]{k-\infty} k \rightarrow \infty \log k \log k/k \sqrt[4]{k-1} = \lim \sqrt[4]{k-\infty} k \rightarrow \infty \log k \log k/k \sqrt[4]{k-1} = \lim \sqrt[4]{k-\infty} k \rightarrow \infty \log k \log k/k \sqrt[4]{k-1} = \lim \sqrt[4]{k-\infty} k \rightarrow \infty \log k \log k/k \sqrt[4]{k-1} = \lim \sqrt[4]{k-\infty} k \rightarrow \infty \log k \log k/k \sqrt[4]{k-1} = \lim \sqrt[4]{k-\infty} k \rightarrow \infty \log k \log k/k \sqrt[4]{k-1} = \lim \sqrt[4]{k-\infty} k \rightarrow \infty \log k \log k/k \sqrt[4]{k-1} = \lim \sqrt[4]{k-\infty} k \rightarrow \infty \log k \log k/k \sqrt[4]{k-1} = \lim \sqrt[4]{k-\infty} k \rightarrow \infty \log k \log k/k \sqrt[4]{k-1} = \lim \sqrt[4]{k-\infty} k \rightarrow \infty \log k \log k/k \sqrt[4]{k-1} = \lim \sqrt[4]{k-\infty} k \rightarrow \infty \log k \log k/k \sqrt[4]{k-1} = \lim \sqrt[4]{k-\infty} k \rightarrow \infty \log k \log k/k \sqrt[4]{k-1} = \lim \sqrt[4]{k-\infty} k \rightarrow \infty \log k \log k/k \sqrt[4]{k-1} = \lim \sqrt[4]{k-\infty} k \rightarrow \infty \log k \log k/k \sqrt[4]{k-1} = \lim \sqrt[4]{k-\infty} k \rightarrow \infty \log k \log k/k \sqrt[4]{k-1} = \lim \sqrt[4]{k-\infty} k \rightarrow \infty \log k \log k/k \sqrt[4]{k-1} = \lim \sqrt[4]{k-\infty} k \rightarrow \infty \log k \log k/k \sqrt[4]{k-1} = \lim \sqrt[4]{k-\infty} k \rightarrow \infty \log k \log k/k \sqrt[4]{k-1} = \lim \sqrt[4]{k-\infty} k \rightarrow \infty \log k \log k/k \sqrt[4]{k-1} = \lim \sqrt[4]{k-1} =$ 1)k = we have Z d Z A(S) = c b p 1 + (f 0 (u))2 , p 1 + (f 0 (u))2 du dv = (d - c)L(C) a by Definition 13.6. c) Parameterize the surface by $\varphi(u, v) = (u, f(u) \cos v, f(u) \sin v), E = [a, b] \times [0, 2\pi]$. In particular, $ex - 2x - 0.7 \ge f(1) = e - 2.7 > 0$. b) Parameterize the surface S using $\varphi(u, v) = (u, v, f(u)), E = [a, b] \times [c, d]$. 1.5.1. a) f is 1-1 since f 0 (x) = 3 > 0. Suppose to the contrary there is a subsequence {nk} of integers such that kfnk - f k $\infty \rightarrow 0$ as k $\rightarrow \infty$. 3(a + b) c) If $\varphi(t) = (2 \sin t, 4 \sin 2 t, 2 \cos t)$ and I = [0, 2 π], then k $\varphi(0)$ (t) = (2 sin t, 4 sin 2 t, 2 cos t) and I = [0, 2 π], then k $\varphi(0)$ (t) = (2 sin t, 4 sin 2 t, 2 cos t) and I = [0, 2 π], then k $\varphi(0)$ (t) = (2 sin t, 4 sin 2 t, 2 cos t) and I = [0, 2 π], then k $\varphi(0)$ (t) = (2 sin t, 4 sin 2 t, 2 cos t) and I = [0, 2 π], then k $\varphi(0)$ (t) = (2 sin t, 4 sin 2 t, 2 cos t) and I = [0, 2 π], then k $\varphi(0)$ (t) = (2 sin t, 4 sin 2 t, 2 cos t) and I = [0, 2 π], then k $\varphi(0)$ (t) = (2 sin t, 4 sin 2 t, 2 cos t) and I = [0, 2 π], then k $\varphi(0)$ (t) = (2 sin t, 4 sin 2 t, 2 cos t) and I = [0, 2 π], then k $\varphi(0)$ (t) = (2 sin t, 4 sin 2 t, 2 cos t) and I = [0, 2 π], then k $\varphi(0)$ (t) = (2 sin t, 4 sin 2 t, 2 cos t) and I = [0, 2 π], then k $\varphi(0)$ (t) = (2 sin t, 4 sin 2 t, 2 cos t) and I = [0, 2 π], then k $\varphi(0)$ (t) = (2 sin t, 4 sin 2 t, 2 cos t) and I = [0, 2 π], then k $\varphi(0)$ (t) = (2 sin t, 4 sin 2 t, 2 cos t) and I = [0, 2 π], then k $\varphi(0)$ (t) = (2 sin t, 4 sin 2 t, 2 cos t) and I = [0, 2\pi], then k $\varphi(0)$ (t) = (2 sin t, 4 sin 2 t, 2 cos t) and I = [0, 2\pi], then k $\varphi(0)$ (t) = (2 sin t, 4 sin 2 t, 2 cos t) and I = [0, 2\pi], then k $\varphi(0)$ (t) = (2 sin t, 4 sin 2 t, 2 cos t) and I = [0, 2\pi], then k $\varphi(0)$ (t) = (2 sin t, 4 sin 2 t, 2 cos t) and I = [0, 2\pi]. $\cos 2t/2$ and $\cos 2t = (1 + \cos 2t)/2$. Set S1 = $\varphi(E1)$ and S2 = $\varphi(E2 \setminus E1)$. Then ka + bk1 = n X |ak + bk| $\leq k=1$ n X |ak + bk| \leq E. Choose $r \in Q$ such $\sqrt{that a - 2} < r < b - 2$. + + - - + such that if b1 = a1, b2 = a2, But this is exactly what it means for x to be parallel to y. b) Let $\epsilon > 0$. $k=0 P \infty$ Taking the limit of this inequality as $^2 \rightarrow 0$, we conclude that k=0 ak $rk \rightarrow L$ as $r \rightarrow 1-$. Hence by Theorem 4.18, f has onesided limits at each point in [a, b]. On the other hand, if x < 0, then by what we just showed, $\sin 2 x = \sin 2(-x) \le 2(-x) = 2|x|$. Let $xn = n^2$ and yn = -n and note by Exercise 2.2.2a that $xn + yn \rightarrow \infty$ as $n \rightarrow \infty$. Then Qz = -p, Pz = q, and Qx - Py = r. Conversely, suppose f (x) does not converge to L as $x \rightarrow \infty$. Then Qz = -p, Pz = q, and Qx - Py = r. Conversely, suppose f (x) does not converge to L as $x \rightarrow \infty$. Then Qz = -p, Pz = q, and Qx - Py = r. convergent sequence (in the SUBSPACE sense) in the set stays in the set. We conclude that a < k < b. Taking the limit of this inequality as $n \rightarrow \infty$, we obtain lim inf xn + 1 in figure (x + yn). This contradiction proves that $f(x_0) = y_0$. Since $\delta < 1$, $|x - 1| < \delta$ implies $|x_0 + x_0| < 9$. 1)M for $0 \le k \le n$. Therefore, 2 < b = 1 + a - 1 < 1 + (a - 1) = a. Hence these surfaces intersect to form a circle in the plane z = z0 := $(-1 + 5)/2 \sqrt{\sqrt{v}}$ of radius z0. Since k=1 |fk (x) - PN0 f (x)| < 2/2 for all $n \ge N$ and $x \in E$. c) If f is 1-1 (respectively, onto), then it follows from part a) that $g \circ f$ is 1-1 (respectively, onto). Thus the pair U, V separates E, which contradicts the fact that E is connected in R2, but E o = B1 (0, 0) $\cup B1$ (3, 0) is not. d) If A = (0, 1) and B = [1, 2], then $\partial(A \cap B) = \emptyset 6 = \{1\} = \partial A \cap \partial B \subseteq (A \cap \partial B) \cup (\partial A \cap \partial B)$. Thus (P, Q, R) $\cdot \phi 0$ (t) = $(-2 \cos t, 0, 2 \sin t) \cdot (\cos t, 0, -\sin t) = -2 \cos 2 t - 2 \sin 2 t = -2$, and $(P, Q, R) \cdot \psi 0 (t) = (-8 \sin t, 0, 8 \cos t) \cdot (-2 \sin t, 0, 2 \cos t) = 16 \cos 2 t + 16 \sin 2 t = 16$. b) Since $\nabla f = (3x^2y - y^3, x^3 - 3xy^2) = (2, -2)$ at (1, 1), and the equation of the tangent plane is $z = f(1, 1) + \nabla(1, 1) \cdot (x - 1, y - 1)$, we have z = 2x - 2y. $\rho(xn-1, a)$. Thus $n \ge N$ implies $|3(1 + 1/n) - 3| \equiv |3/n| \leq 3/N < \epsilon$. 15 Copyright © 2010 Pearson Education, Inc. Since cn $\rightarrow \infty$ as n $\rightarrow \infty$, it follows from the Heine-Borel Theorem that E is compact. Thus f (x) = 0 for all x $\in [0, 1]$. b) Let V represent the volume of P and θ represent the angle between a × b and c. It follows from the Completeness Axiom that A(x) = sup Ex exists for every x \in R. 4 32 30 c) This region is the set of points "under" the paraboloid x = y 2 + z 2 which lies "over" the region is the set of points "under" the paraboloid x = y 2 + z 2 which lies "over" the region is the set of points "under" the paraboloid x = y 2 + z 2 which lies "over" the region is the set of points "under" the paraboloid x = y 2 + z 2 which lies "over" the region is the set of points "under" the paraboloid x = y 2 + z 2 which lies "over" the region is the set of points "under" the paraboloid x = y 2 + z 2 which lies "over" the region is the set of points "under" the paraboloid x = y 2 + z 2 which lies "over" the region is the set of points "under" the paraboloid x = y 2 + z 2 which lies "over" the region is the set of points "under" the paraboloid x = y 2 + z 2 which lies "over" the region is the set of points "under" the paraboloid x = y 2 + z 2 which lies "over" the region is the set of points "under" the paraboloid x = y 2 + z 2 which lies "over" the region is the set of points "under" the paraboloid x = y 2 + z 2 which lies "over" the region is the set of points "under" the paraboloid x = y 2 + z 2 which lies "over" the region is the set of points "under" the paraboloid x = y 2 + z 2 which lies "over" the paraboloid x = y 2 + z 2 which lies "over" the paraboloid x = y 2 + z 2 which lies "over" the paraboloid x = y 2 + z 2 which lies "over" the paraboloid x = y 2 + z 2 which lies "over" the paraboloid x = y 2 + z 2 which lies "over" the paraboloid x = y 2 + z 2 which lies "over" the paraboloid x = y 2 + z 2 which lies "over" the paraboloid x = y 2 + z 2 which lies "over" the paraboloid x = y 2 + z 2 which lies "over" the paraboloid x = y 2 + z 2 which lies "over" the paraboloid x = y 2 + z 2 which lies "over" the paraboloid x = y 2 + z 2 + z 2 which lies "over" the paraboloid x = y 2 + z 2 + z 2 + z 2 + z 2 + z 2 + z 2 + z 2 + z 2 + z 2 + z 2 + z 2 + z 2 + z 2 + z 2 + z 2 whose boundary is S., n} we have $|xk| \le kxk \le nkxk \infty$. $\sqrt{Thus f(k)} = f(a) + D(2) f(c; x - a)/2!$. This happens, by part b), if and only if Rn is not connected. 13.1.11. Suppose $c \in (a, b)$ is a point of discontinuity of f 0. 10.3.6 this series converges everywhere on R. Similarly, B has a supremum. If $x \in E$, then $x \in Irj(xj)$ for some j, so fxj (x) = 1. \sqrt{shows} that $xn+1 = 2 + xn - 2 > 2\sqrt{c}$) Take the limit of x = 2 + x as $n \rightarrow \infty$. Then by Theorem 8.37 Vol (E1 \cup E2 ; G) $- {}^2XX \ge |Rj| + Rj \cap E 1 6 = \emptyset X |Rj| - Rj \cap E 2 6 = \emptyset |Rj| - Rj \cap E 2 6 = \emptyset \ge Vol(E1) + Vol (E1 \cup E2)$ $(E2) - V(E1 \cap E2; G) - ^2 > Vol(E1) + Vol(E2) - 2^2$. Since 0 < a < 1, the Geometric series k=1 a converges. Hence by definition, P (x)/(x - x0) $\rightarrow \infty$ as x $\rightarrow x0 + .$ By Theorem 2.8, convergent sequences are bounded. $\pi - 2\pi - 2c$) Let T (1, 0, 0, 0) = (a, b). f (a) 53 Copyright © 2010 Pearson Education, Inc. If k > N, then k(k/(k + 1), sin(k 3)/k) - (1, 0, 0, 0) = (a, b). f (a) 53 Copyright © 2010 Pearson Education, Inc. If k > N, then k(k/(k + 1), sin(k 3)/k) - (1, 0, 0, 0) = (a, b). f (a) 53 Copyright © 2010 Pearson Education, Inc. If k > N, then k(k/(k + 1), sin(k 3)/k) - (1, 0, 0, 0) = (a, b). f (a) 53 Copyright © 2010 Pearson Education, Inc. If k > N, then k(k/(k + 1), sin(k 3)/k) - (1, 0, 0, 0) = (a, b). f (a) 53 Copyright © 2010 Pearson Education, Inc. If k > N, then k(k/(k + 1), sin(k 3)/k) - (1, 0, 0, 0) = (a, b). f (a) 53 Copyright © 2010 Pearson Education, Inc. If k > N, then k(k/(k + 1), sin(k 3)/k) - (1, 0, 0, 0) = (a, b). f (a) 53 Copyright © 2010 Pearson Education, Inc. If k > N, then k(k/(k + 1), sin(k 3)/k) - (1, 0, 0, 0) = (a, b). f (a) 53 Copyright © 2010 Pearson Education, Inc. If k > N (a) 53 Copyright © 2010 Pearson Education, Inc. If k > N (a) 53 Copyright © 2010 Pearson Education, Inc. If k > N (a) 53 Copyright © 2010 Pearson Education, Inc. If k > N (a) 53 Copyright © 2010 Pearson Education, Inc. If k > N (a) 53 Copyright © 2010 Pearson Education, Inc. If k > N (a) 53 Copyright © 2010 Pearson Education, Inc. If k > N (a) 53 Copyright © 2010 Pearson Education, Inc. If k > N (a) 53 Copyright © 2010 Pearson Education, Inc. If k > N (a) 53 Copyright © 2010 Pearson Education, Inc. If k > N (a) 53 Copyright © 2010 Pearson Education, Inc. If k > N (a) 53 Copyright © 2010 Pearson Education, Inc. If k > N (a) 53 Copyright © 2010 Pearson Education, Inc. If k > N (a) 53 Copyright © 2010 Pearson Education, Inc. If k > N (a) 53 Copyright © 2010 Pearson Education, Inc. If k > N (a) 53 Copyright © 2010 Pearson Education, Inc. If k > N (a) 53 Copyright © 2010 Pearson Education, Inc. If k > N (a) 5 $0/k^2 = 1/(k + 1)^2 + \sin^2(k^3)/k < 1/k^2 + 1/k < 2/k < \epsilon^2$. 2.5.2. By Theorem 1.20, lim inf (-xn) := lim (inf (-xh)) = - lim (sup xk) = - l continuous on E. 7.1.2. a) Since $(336 + 3)/N \rightarrow 0$ as $N \rightarrow \infty$, given $\varepsilon > 0$, we can choose $N \in N$ so that $0 < (336 + 3)/N < \varepsilon$. Then f is C p and $\varphi \circ g(V)$ coincides with the graph of z = f(x, y), $(x, y) \in V$. Taking the limit of xn+1 = xn - f(xn)/f 0 (a). Moreover, ak = (2/3)(4/5). Hence P by Dirichlet's Test, ∞ the series converges if and only if k=1 ak converges. On the other hand, if [a, b] \subset (0, ∞) then $Z N = -N y \leq e-N a \rightarrow 0$ $1 = -0 R \propto 0 e-xy dx R \propto$ independently of y. By Abel's Formula, n = n-1 $X = X = -N y \leq e-N a \rightarrow 0$ $1 = -0 R \propto 0 e-xy dx R \propto$ independently of y. By Abel's Formula, n = n-1 $X = X = -N y \leq e-N a \rightarrow 0$ $1 = -0 R \propto 0 e-xy dx R \propto independently of y. By Abel's Formula, <math>n = n-1$ $X = X = -N y \leq e-N a \rightarrow 0$ $1 = -0 R \propto 0 e-xy dx R \propto independently of y. By Abel's Formula, <math>n = n-1$ $X = -N y \leq e-N a \rightarrow 0$ $1 = -0 R \propto 0 e-xy dx R \propto independently of y. By Abel's Formula, <math>n = n-1$ $X = -N y \leq e-N a \rightarrow 0$ $1 = -0 R \propto 0 e-xy dx R \propto independently of y. By Abel's Formula, <math>n = n-1$ $X = -N y \leq e-N a \rightarrow 0$ $1 = -0 R \propto 0 e^{-xy} dx R \propto independently of y. By Abel's Formula, <math>n = n-1$ $X = -N y \leq e-N a \rightarrow 0$ $1 = -0 R \propto 0 e^{-xy} dx R \propto independently of y. By Abel's Formula, <math>n = n-1$ $X = -N y \leq e-N a \rightarrow 0$ $1 = -0 R \propto 0 e^{-xy} dx R \propto independently of y. By Abel's Formula, <math>n = n-1$ $X = -N y \leq e-N a \rightarrow 0$ $1 = -0 R \propto 0 e^{-xy} dx R \propto independently of y. By Abel's Formula, <math>n = -1$ $X = -N y \leq e-N a \rightarrow 0$ $1 = -0 R \propto 0 e^{-xy} dx R \propto independently of y. By Abel's Formula, <math>n = -1$ $X = -N y \leq e-N a \rightarrow 0$ $1 = -0 R \propto 0 e^{-xy} dx R \propto independently of y. By Abel's Formula, <math>n = -1$ $X = -N y \leq e-N a \rightarrow 0$ 1 = -1 2×-1 $2 \times$ 11.5.10. Since Theorem 4.32 implies that L is continuous on $(0, \infty)$, it follows from hypothesis that lim tL(1 + 1/t) = lim L((1 + 1/t) = lim Theorem 11.14. But by six applications of l'h^oopital's Rule, k 4 /ek k6 6! = lim k = lim k = 0. 1.6.7. a) Let q = k/j. b) By Taylor's Formula, there is a c between x and 1 such that $|\log x - Pn(x)| = |(-1)n(x-1)n+1|/(cn+1(n+1))$. By Theorem 4.24, ex = k=0 x2k /k! + ec xn /n! for some c between 0 and x. Thus set $\xi = r + 2$. $\kappa(x0) = \lim_{n \to \infty} \frac{1}{2} |x|^{n+1} |x|^{n+1}$. $s - s0 2 \sin(\theta s / 2) |s - s0| d)$ Since $\varphi(t) = \nu(\hat{t})$, we have by the Chain Rule that $\varphi(0(t) = \nu 0(\hat{t}) \cdot \hat{0}(t) = \nu 0(\hat{t})$ absolutely for all p > 0 by the Ratio Test, since $(k + 1)/e(k+1)p k+1 1 = \rightarrow p 0$. c) Since $|x - 2| \le |x| + 2$, $-3 \le x \le 2$ implies $|x2 + x - 6| = |x + 3| |x - 2| \le 6|x - 2|$. Apply the Mean Value Theorem and the Inverse Function Theorem to f -1. 0 Z Z (P, Q, R) \cdot T ds = C2 $2\pi \sqrt{(0, 8 \cos 2t, 0)} \cdot (8 \cos t, 0, -8 \sin t) dt = 0$. For n = 7, this last ratio is about 0.00069 still a little too big, but it's about 0.00023 < 0.0005 for n = 8. Let $E = \{(x, y, 0) : x2 \le y \le 1 \text{ and } -1 \le x \le 1\}$. $0 \ 0 \ b$) Let $p \le q$. Since f is C 3, its third partial derivatives are all bounded on Br (a, b). Conversely, if xn is Cauchy and xnk $\rightarrow a$, then given $\varepsilon > 0$ there is an N such that $n, k \ge N$ implies $\rho(xn, xnk) < \varepsilon/2$. Since D = 4ac - b2, f(0, 0)= 0 is a local minimum if a > 0 and b2 - 4ac < 0, a local maximum if a < 0 and b2 - 4ac < 0, a local maximum if a < 0 and b2 - 4ac < 0, a local maximum if a < 0 and b2 - 4ac < 0, a local maximum if a < 0 and b2 - 4ac < 0, and (0, 0) is a saddle point if b2 - 4ac < 0. $-\sin t$, $\cos t/3$ = 2 $\cos 2 t/3$. Let (x^2, y^2, z^2) be a point on Π different 78 Copyright © 2010 Pearson Education, Inc. Since $E \setminus U \subset E 0$ is compact and f is continuous on E 0, use uniform continuity to choose a $\delta > 0$ such that $kx - yk < \delta$ and $x, y \in E 0$ imply $|f(x) - f(y)| < \frac{2}{2}(2Vol(E))$. If this claim holds, then the estimates above yield $Z \setminus \mu \P \cap \frac{1}{2}$. it follows from the Intermediate Value Theorem that either f > 0 on (a, b) or f 0 < 0 on (a, b). b) Let $I = [a, \infty) \subset (0, \infty)$. c) Let $E = \{(x, y) : x^2 + y^2 = 1, x^2 =$ (-1, 0) = 0.5((3, 5) - (5, 3)) = (-1, 1) and T (0, 1, 0, 0) = 0.5(T (0, 1, 1, 0) + T (0, 1, -1, 0)) = 0.5((3, 5) + (5, 3)) = (4, 4). k→∞ log k 1/k lim By the Logarithmic Test, if $-\alpha > 1$, then this series converges absolutely. $\pi - \pi \pi - \pi ak$ (f + g) = 14.1.4. Integrating by parts, we have Z 1 $\pi 0$ f (t) cos kt dt $\pi - \pi \mu \P Z \pi^{-} \pi 1 = f(t) \cos kt^{-} - \pi + k f(t) \sin kt dt$ $\pi - \pi ak(f_0) = 0 + kbk(f)$ since f is periodic. 3.2.8. Let $\epsilon > 0.4.4.1.a$ If $f(x) = \cos x$, then $f(2n-1)(x) = (-1)n \cos x$ and $f(2n-1)(x) = (-1)n \cos x$. (U) f(x) dx. Since each fxj is nonnegative on Br(xj) (xj), it follows from the choice of the r(xj)'s that $f(y) \ge fxj(y) > 0$. If E is closed, then it is already bounded because it is a subset of K. By Theorem 1.37, $f(A \cap B) \subseteq f(A) \cap f(B)$. Since g(x) 6 = 0, it follows from the Intermediate Value Theorem that either g(x) > 0 or g(x) < 0 for all $x \in [a, b]$. By Remark 9.37, A and B are closed sets, so by Theorem 8.24, A \cap B is a closed subset of the compact set A. See Example 5.12. P \propto k=0 (-1/ $\pi \propto$ k+1 k-1 k c) Pk=2 4 /9 = 36 k=2 (4/9)P = 36(4/9)2 /(1 - 4/9) = 64/5. Apply S := Sx := (D\phi(x)) - 1 to the identity $\phi(x) - \phi(y) = D\phi(x)(x - y) + {}^{2}x - y + T(x, y)$ where T (x, y) := Sx := (D\phi(x)) - 1 to the identity $\phi(x) - \phi(y) = D\phi(x)(x - y) + {}^{2}x - y + T(x, y)$ where T (x, y) := Sx := (D\phi(x)) - 1 to the identity $\phi(x) - \phi(y) = D\phi(x)(x - y) + {}^{2}x - y + T(x, y)$ where T (x, y) := Sx := (D\phi(x)) - 1 to the identity $\phi(x) - \phi(y) = D\phi(x)(x - y) + {}^{2}x - y + T(x, y)$ where T (x, y) := Sx := (D\phi(x)) - 1 to the identity $\phi(x) - \phi(y) = D\phi(x)(x - y) + {}^{2}x - y + T(x, y)$ where T (x, y) := Sx := (D\phi(x)) - 1 to the identity $\phi(x) - \phi(y) = D\phi(x)(x - y) + {}^{2}x - y + T(x, y)$ where T (x, y) := Sx := (D\phi(x)) - 1 to the identity $\phi(x) - \phi(y) = D\phi(x)(x - y) + {}^{2}x - y + T(x, y)$ where T (x, y) := Sx := (D\phi(x)) - 1 $S(^2x-y(x))$. Therefore, Du f (a) = $k\nabla f$ (a)k cos θ . d) By part c), Vol (E1 \ E2) \cup E2) = Vol (E1 \ E2) (E1 \ E2) = Vol (E1 \ E2) (E1 \ E2) = Vol (E1 \ E2) (E1 $Z 2\pi (P, Q, R) \cdot T ds = C1 \text{ Similarly, } (0, 8 \text{ sin2 t, } \sqrt{\sqrt{8} \cos t}) \cdot (-8 \text{ sin t, } 0, 8 \cos t) dt = 8\pi. \text{ By Exercise 4.1.2, } y(x) \rightarrow nan-1 \text{ as } x \rightarrow a. \text{ Thus 2 ab } \leq a + b \text{ and } b) \leq A(a, b) \cdot \sqrt{48} \cos t + b \text{ and } b \leq A(a, b) \cdot \sqrt{48} \cos t + b \text{ and$ F (u, v, x, y) = (φ 1 (u, v) - x, φ 2 (u, v) - y). d) Since f is differentiable at a, f is continuous at a and I1 /khk \rightarrow 0/f 2 (a) = 0 as h \rightarrow 0. Then there is a sequence xk \in B such that f (xk) does NOT converge to f (a). b) Let $\epsilon > 0$ and choose (by Archimedes) an N \in N such that k > N implies 1/k $< \epsilon 2$ /2. $k \rightarrow \infty$ Moreover, E is convex, closed, and bounded, hence compact. , yn +zn) = x1 (y1 + z1) + · · · + xn (yn + zn) = x1 y1 + · · · + xn yn + x1 z1 + · · · + xn zn = x · y + x · z. b) Let E be sequentially compact. There are two cases. 7.4.9. Modifying the proof of Theorem 7.43, we see that f is analytic. If they're both even, then an-k bk is the product of two positive numbers, hence positive. 4.1.8. a) If f has a local maximum at x0 then $f(x0 + h) - f(x0) \le 0$ for h small. 80 Copyright © 2010 Pearson Education, Inc. Hence y = 5 + 2x + x2 is a semicircle mounted on a rectangle with base 2 and altitude 5. If $|x - y| < \delta$ then $2|P(x) - P(y)| \le |m||x - y| < |m| < 2$. Since D ϕ is continuous on the compact set R, there is an M > 0 (which depends only on ϕ and R) such that kSx k ≤ M for all x \in R. x 35 Copyright © 2010 Pearson Education, Inc. x1 139 Copyright © 2010 Pearson Edu (a, b) then given $x \in (a, b)$, f(x) = k=0 f(k) (x 0)(x - x 0) k/k! has a positive radius P ∞ of convergence. b) Given $x \in (0, 1)$, $nx \rightarrow \infty$, hence $1/(nx) \rightarrow 0$ as $n \rightarrow \infty$. It follows that $n \ge N$ implies |an + bn| > |M| > 0 > M. A similar argument establishes an analogous inequality for lower integrals. $n \rightarrow \infty$ $n \rightarrow \infty$. It follows that $n \ge N$ implies |an + bn| > |M| > 0 > M. A similar argument establishes an analogous inequality for lower integrals. $n \rightarrow \infty$ $n \rightarrow \infty$. It follows that $n \ge N$ implies |an + bn| > |M| > 0 > M. implies f(0) = 0 or 1. d) This is the set of points inside the two branches of the hyperbola $x^2 - y^2 = 1$ which lie above the line y = -1 and below the line y = -1 and below the line y = 1. Exponentiating this inequality, we have $1 \le 1 \le j \le k$. Since N $\varphi = (1, 0, 0)$ points toward the positive x axis, we have ZZ ZZ $\omega = -(Ry - Qz) dA T1 E 1 Z 1 - z Z = -Z Z 1 1 - y Ry dy dz + Z 0 Qz dz dy 0 0 0 Z 1 = -(Q(0, y, 1 - y) - Q(0, y, 0)) dy$. A similar argument establishes the opposite inequality for local maxima. CHAPTER 3 3.1 Two-Sided Limits. (k - 1)! (n + k - 1)! 2 Thus x2 Bn00 (x) + xBn0 (x) + (x2 + Z 0 Qz dz dy 0 0 0 Z 1 = -(Q(0, y, 1 - y) - Q(0, y, 0)) dy. A similar argument establishes the opposite inequality for local maxima. CHAPTER 3 3.1 Two-Sided Limits. (k - 1)! (n + k - 1)! 2 Thus x2 Bn00 (x) + xBn0 (x) + (x2 + Z 0 Qz dz dy 0 0 0 Z 1 = -(Q(0, y, 1 - y) - Q(0, y, 0)) dy. - n2)Bn (x) = 0 for all x \in R. Combining this with part a), we have M1 = kT k. The tetrahedron ∂E has four faces, S1 in z = 0, S2 in x = 0, S2 in x = 0, S2 in x = 0, S3 in y = 0, and the slanted face S4. $\sqrt{\sqrt{3} \times 2121/243}$ Copyright © 2010 Pearson Education, Inc. CHAPTER 6 6.1 Introduction. On the other hand, $\infty \cup \infty$ j=1 Bj \subseteq V since each Bj \subseteq V. α) Since f (x) = x is increasing, $M_j = x_j := j/n$ and $M_j = x_j - 1 := (j - 1)/n$. Since $Z \in dx = x \log p \times Z$ 1 ∞ du = ∞ up for $p \le 1$, this series diverges by the Integral Test. b) The tangent line at (a, b) is y = b + 6a(x - a). Now x1 - p has a finite limit as $x \to \infty$ if and only if $\overline{x} = 1 - p > 0$, i.e., p > 1. Multiplying the first inequality by c and the second by b, we have $0 \le ac \le bc$ and bc

Solution 10.13 b) Apply part a) with a = 0 and $\rho(a, b) = ka - bk$. Thus A := U = E \ V is clopen and V = $j\pi$, k, j \in Z. Set M = $-1 - 1/\epsilon$. c) If C = {(ϕj , Ij) : j = 1, . To parameterize C1, set $\phi(t) = (\cos t, \cos t, \sin t/3)$ and I = $[-\pi/2, \pi/2]$. Hence xn is bounded by Definition 10.13 b) Apply part a) with a = 0 and $\rho(a, b) = ka - bk$. Thus A := U = E \ V is clopen and $\emptyset \subset A \subset E$. In particular, ($\varphi 0$ (t0)× $\varphi 00$ (t0)/k $\varphi 0$ (t0)/k $\varphi 0$ (t0)/k $\varphi 0$ (t0)/k $\varphi 0$ (s0). By hypothesis, P $\propto 0 \leq ak \leq a$. Hence (arcsin x) = 1/1 - x. Notice, we really only need that one of the series is bounded and the other convergence. Hence $e^{-1/|x-y|} < 2$, i.e., $f(x, y) \rightarrow 0 = f(x0, y0)$ as $(x, y) \rightarrow (x0, y0)$. Similarly, mn-1 (pq-1) = mpn-1 q-1 = mp(nq)-1. Then one the components of N ϕ is nonzero, say $\partial(\phi 1, \phi 2)/\partial(u, v)$ $\delta = 0$. We conclude by Theorem 7.10 that Z lim $k \rightarrow \infty 1 \mu 3 x \cos 0 \log k + x k + x \P Z dx = 0 1 x 3 dx = 1$. Therefore, the original series diverges when $p = \log 2$ (e). It follows from the inductive hypothesis that $\psi \circ \varphi$ takes {1, 2, . Thus (1 - x/k)k is an increasing sequence of continuous functions. Hence f + g and f g are co and $a \in X$ such that $\rho(xn, a) > n$ for all $n \in N$. 13.6.7. a) Let θ be the angle between F and T. 10.1.2. a) Suppose xn is bounded. Since fx (x) > 0 and fx is continuous, choose by the Sign Preserving Property an r(x) > 0 and fx is continuous, choose by the Sign Preserving Property an r(x) > 0 and fx is continuous, choose by the Sign Preserving Property an r(x) > 0 and fx is continuous, choose by the Sign Preserving Property an r(x) > 0 and fx is continuous, choose by the Sign Preserving Property an r(x) > 0 and fx is continuous, choose by the Sign Preserving Property an r(x) > 0 and fx is continuous, choose by the Sign Preserving Property an r(x) > 0 and fx is continuous, choose by the Sign Preserving Property an r(x) > 0 and fx is continuous, choose by the Sign Preserving Property an r(x) > 0 and fx is continuous, choose by the Sign Preserving Property an r(x) > 0 and fx is continuous, choose by the Sign Preserving Property an r(x) > 0 and fx is continuous, choose by the Sign Preserving Property an r(x) > 0 and fx is continuous, choose by the Sign Preserving Property an r(x) > 0 and fx is continuous, choose by the Sign Preserving Property an r(x) > 0 and fx is continuous, choose by the Sign Preserving Property an r(x) > 0 and fx is continuous, choose by the Sign Preserving Property an r(x) > 0 and fx is continuous, choose by the Sign Preserving Property and r(x) > 0 and fx is continuous, choose by the Sign Preserving Property and r(x) > 0 and fx is continuous, choose by the Sign Preserving Property and r(x) > 0 and fx is continuous, choose by the Sign Preserving Property and r(x) > 0 and fx is continuous, choose by the Sign Preserving Property and r(x) > 0 and fx is continuous, choose by the Sign Preserving Property and r(x) > 0 and fx is continuous, choose by the Sign Preserving Property and r(x) > 0 and fx is continuous, choose by the Sign Preserving Property and r(x) > 0 and fx is continuous, choose by the Sign Preserving Property and r(x) > 0 and fx Q dz dx + R dx dy. 0 b) If Ry = 0 and Qz = -xy then R = q(x, z) and Q = -xyz + f(x, y). Since ak + b and a are both positive or all negative. Thus this system can be solved by Cramer's Rule as indicated. If $x, y \in E$ and $|x - y| < \delta$ then $|(fq)(x) - (fq)(y)| \le |q(y)| |f(x) - f(x)|$ $(y) + |f(x)| |g(x) - g(y)| < M^{2} + M = ^{2}$. a) If kxk = 1, Theorem 8.17 implies kT (x)k \leq kT k. s = 0. Therefore f -1 (x) = (x+7)/3. Hence, C = E \cap V c. But by definition, $\sigma(xn, a) < 1$ implies $\sigma(xn, a) < 1$ implies satisfies $|f 0(x)| \le M$ for all $x \in (a, b)$. Then by the Fundamental Theorem of Calculus, d dx 5.3.2. a) $1 \le x \le 4$ implies $1 \le Z 4 \ 1 \sqrt{Z} t f (t - x) dt = 0 \ d dx \ Z 0 f (u) du = f (-x)$. Therefore, $\mu 2 \P x - 1 \ x^2 y - 2xy + y - (x - 1)^2 \lim x = (0, 0)$. By Theorem 8.2, this can be rewritten as $((x, y, z) - a) \cdot ((b - a) \times (c - a)) = 0$ which is equivalent to $(x - a) dt = 0 \ d dx \ Z 0 f (u) du = f (-x)$. b1 - a1 c1 - a1 y - a2 b2 - a2 c2 - a2 z - a3 b3 - a3 z = 0 c3 - a3 by Theorem 8.9. 8.2.7. a) Let A represent the area of P and θ represent the area of P N, i.e., $\lim \sup n \to \infty$ (1/xn) = ∞ = 1/s. Let f (x) = 1/x - 1 for x 6 = 0 and f (0) = 0. If x 0 \in U then B² (x0) \subseteq U for some ² > 0, which contradicts the choice of t0. \sqrt{d} Substitute u = sin x, du = cos x dx, i.e., dx = du/1 - u2, to obtain Z I := Z 1 log(sin x) dx = 0 0 sin 1 log u \sqrt{d} du. Thus the ellipse is a piecewise smooth C ∞ surface. Hence both these sequences converge to the same value, i.e., the series $\infty k+1/k$ converges. Then $n \ge N$ and $x \in E$ imply $|(f + g)(x) - (fn + gn)(x)| \le |f(x) - fn(x)| + |g(x) - gn(x)| \le |\alpha| |f(x) - fn(x)| \le |\alpha| |f(x) - fn(x)| \le |\alpha| |x| + |x|$ Theorem, if x, $y \in (a, b)$ and $|x - y| < \delta$, then $|f(x) - f(y)| = |f 0(c)| |x - y| \le M |x - y| \le k$. Since $\frac{1}{2} |dj/dxj$ (cos kx)| = k j | sin kx| k j | cos kx| when j is odd when j is even, it follows from Theorem 14.23 that $|dj/dxj| \le 1/k 2$ for k large. 9.1.7. a) Let B be a closed ball of radius R and center a. Thus f is continuous on [0, 1]. Since $\frac{\partial E}{\partial E}$ and E0 are both of volume zero, choose cubes Q1, $\partial f1 / \partial x$ Hence by the Inverse Function Theorem, $\partial f1 - 1 / \partial x = \partial f2 / \partial y / \Delta f$, etc. It follows that Br (a) \cup Bs (b) \subseteq Bd (x). 4.3.3. If a and b are roots of f, then by the Mean Value Theorem, $\partial f1 - 1 / \partial x = \partial f2 / \partial y / \Delta f$, etc. It follows that Br (a) \cup Bs (b) \subseteq Bd (x). 4.3.3. If a and b are roots of f, then by the Mean Value Theorem, $\partial f1 - 1 / \partial x = \partial f2 / \partial y / \Delta f$, etc. It follows that Br (a) \cup Bs (b) \subseteq Bd (x). 4.3.3. If a and b are roots of f, then by the Mean Value Theorem, $\partial f1 - 1 / \partial x = \partial f2 / \partial y / \Delta f$, etc. It follows that Br (a) \cup Bs (b) \subseteq Bd (x). dv + gx(x, y) and 0 Z Py = z qy(x, y, v) dv + hy(x, y). Thus $f\alpha 0(0) = 0$ by the Squeeze Theorem. Thus f(x) = S(x) + f(a) - S(a) on V, so set c = f(a) - S(a) on V, so set c = f(a) - S(a). Since for each $x \in K$ there is a $C \infty$ function $fx \ge 0$ such that fx > 0 for $x \in Ix$ and fx = 0 for $x \in Ix$. It follows that each component of $S \circ \varphi$ satisfies $|(S \circ \varphi)|(x) - (S \circ \varphi)|(x)| \le |x_j - y||(1 + \sqrt{nM} \varepsilon)$. Letting $j \to \infty$ in the inequality kfk - f is continuous at a, then by Theorem 3.22, f q is too. Hence by the Squeeze ∞ Theorem 1 f (xn) dx $\rightarrow 0$ as $n \to \infty$. Suppose that $\delta = 1/(2N)$ and $|x_0 - x|| < \delta$. If they're both oduct of two negative numbers, hence positive., n} onto A. Hence f (x, y) = 1 - (x + 1) + (y - 1) + (x + 1)(y + 1)(y - 1) + (x + 1)(y - 1)(y - 1)(y Additive Property holds. $\partial(u, v, w) = 0$ and $g(t) = -\infty$ as $t \to \infty$, it follows that g(t), hence f 0 (t), is negative for large t. We obtain $\infty X \propto X$ at bk = k=1 at b + s. Suppose f is continuous on E and A is closed in Rm. Since E is closed, it follows from Theorem 10.16 that $x \in E$. Conversely, if E is compact then by part a), E is sequentially compact. Suppose distinct points $x_j \in E \cap Br$ (a) have been chosen for each $1 \le j < k$. By definition, $x_n \to a$ as $n \to \infty$. Since f is increasing, $M_j = f(x_j)$ and $m_j = f(x_j-1)$ for all j.
Let f(x) = x + 1 for $-1 \le x \le 1$. 0 and f(x) = 2x - 1 for $0 < x \le 1$. b) If g is not zero, then 1/g is continuous on E, hence bounded by the Extreme Value Theorem. It follows that $aN + j \le aN + j - 1$ r $0 \le aN + j - 2$ r $02 \le \cdots \le aN$ r0j, i.e., $ak \le aN + j - 2$ r $02 \le \cdots \le a$ directional derivative in all directions, then all first partials of f exist. Since f = g on $(-\pi, \pi)$ implies Sf = Sg, we conclude that Sf converges to f uniformly on $[a, b] \subset (-\pi, \pi)$ and pointwise on $(-\pi, \pi)$. Then $Z Z 2\pi P dx + Q dy = \partial E (-\sin 2 t - \cos 2 t) dt = -2\pi$. Hence $|xn + yn - (xm + ym)| \leq |xn - xm| + |yn - ym| < \varepsilon$ for $n, m \geq N$. We suppose for simplicity that x and y are both finite. Therefore, $-1 f(x) = 1/\log x$. By part d), $x\alpha + \beta = E((\alpha + \beta)L(x)) = E(\alpha - \alpha) = E(0) = 1$, i.e., $1/x\alpha = x - \alpha$., $x^2n \}$ is a partition of [0, 1]. n. c) The minimum of 1/k for $k \in \mathbb{N}$. On the other hand, since F = (x, y, z) on ∂E , we have $ZZ Z 2\pi Z \pi/2$ $F \cdot n d\sigma = \partial E 0$ (cos2 u cos3 v + sin2 u cos3 v + sin2 u cos v) dv du = 4\pi 6 = 0. Therefore, 3 < 2 + a - 2 = b < a. Then y = f(a) = f(b) for some $a \in A$ and $b \in B$. Consider the function $\varphi - 1 \circ f \circ \varphi$. 102 Copyright © 2010 Pearson Education, Inc. Since f is continuous, it follows that $f(xnk) \rightarrow f(a) = M$. It converges to the continuous function $x^2 f(0)$ as k $\rightarrow \infty$. By construction, xk+1 does not equal any xj for $1 \le j \le k$. b) f -1 (e) = 1. k=0 7.3.11. $\sqrt{(j)}$ (j) 10.1.3. a) By Remark 8.7, for all $j \in \{1, 2, ..., y \in U \ ...,$ Apply a) for n = 0 to choose x1 = w such that $x1/10 \le x < x1/10 + 1/10$. 12.2 Riemann Integration on Jordan Regions. The function g := -f has a proper local maximum at x0, hence by part a), such $x1 \neq 0 \le Q$ and observe by the definition T that S $\phi(x) = S \circ \phi(y_0) - y_0 + x + T(x, y_0) =: z + x + T(x, y_0)$ for $x \in Q$. 3.2.7. By Exercise 3.2.3b and symmetry, it suffices to prove P(x)/(x - x_0) $\rightarrow \infty$ as $x \to x_0 + k = 1$ (-1) Pn Pn PN Pn PN Pn PN 6.1.9. a) $|k=1 \ bk - b| = |k=1 \ (bk - b)| \le k = 1 \ |bk - b| + k = N + 1 \ |bk - b| \le k = 1 \ |bk - b| + M \ (n - N)$. Suppose it holds for some $n \ge 3$. 0 Therefore, ZZ Z $\omega = -1$ TZ1Z1R(0, 1 - z, z) dz + 00ZZ1 + 1Q(0, y, 1 - y) dy - Z1R(1 - z, 0, z) dz - 0 P(x, 0, 1 - x) dx 01Q(1 - y, y, 0) dy + 0 P(x, 1 - x, 0) dx. (k + 1)! a k=0 c) Clearly, E(0) = 1., cM } is a partition of [c, d], such that ZS(f; P) 0. By Theorem 9.16, $\mu \lim (x,y) \rightarrow (1,-1)x - 1$, x + 2y - 1 ¶ = (0, 3). It follows from the Squeeze Theorem that |f(x)| $\rightarrow |L|$ as x \rightarrow x0 through E. Checking the extreme points of H, f (1, 0) = 1, f (3, 0) = 9, and f (1, 2) = 17. For h so small that x ± h \in I, we have f (-x + h) = -f (x - h) and -f (-x) = f (x). }. Since the absolute value of the ratio of consecutive terms of the series defining Bn is $|x/2|/((k + 1)(n + k + 1)) \rightarrow 0$ as $k \rightarrow \infty$, Bn (x) converges by the Ratio Test. By definition, $xn - yn \rightarrow 0$ as $n \rightarrow \infty$. Taking the infimum of this inequality, we obtain Vol (E1) \leq Vol (E2). Then φ is the trivial parameterization of x = f - 1 (y), and $\varphi \circ \tau$ (u) = $\varphi(f - 1 (u)) = (f - 1 (u), f(f - 1 (u))) = (f 3 3 y \rightarrow 0 + 0 10 u 1 - x + y - 1 x 0 1 R \infty b$ Since $|e-xy \sin x/x| \le e-xy \le e-x/2$ for $y \in [1/2, 3/2]$, π (e-xy sin x/x) dx converges uniformly on [1/2, 3/2] by the Weierstrass-M Test. By c), $|x - y| \le (x1 - y1) \cdot 0 = 0$. ajnn as $x \rightarrow a$ for any nonnegative integers j1, . Thus ak+1 < ak. Hence by Theorems 4.32 and 4.33, E(x) := L-1 (x) is differentiable and strictly increasing on R with E 0 (x) = 1/L0 (y) for y = E(x). b) By definition, $x \in f - 1$ ($\cap \alpha \in A \ f = 1$ ($\cap \alpha \in A \ f = 1$ ($\cap \alpha \in A \ f = 1$) if and only if $x \in \cap \alpha \in A \ f = 1$ ($\cap \alpha \in A \ f = 1$). b) It diverges for all $\alpha \in A \ f = 1$ ($\cap \alpha \in A \ f = 1$) ($\cap \alpha \in A \ f = 1$ ($\cap \alpha \in A \ f = 1$) ($\cap \alpha \in A \ f = 1$). b) It diverges for all $\alpha \in A \ f = 1$ ($\cap \alpha \in A \ f = 1$) ($\cap \alpha \in A \ f = 1$). b) It diverges for all $\alpha \in A \ f = 1$ ($\cap \alpha \in A \ f = 1$). b) It diverges for all $\alpha \in A \ f = 1$ ($\cap \alpha \in A \ f = 1$). Comparison of the formula is expressed as the formula of the . 11.3.5. a) Let T = Df (a) and S = Dg(a). 5.1.1. a) L(f, P) = 0.5f (0) + 0.5f (0.5) + f (1) = 17/16. Thus k > N implies k(k/(k + 1), sin(k 3)/k) - (1, 0)k < \epsilon. By definition, $\rho(xn, yn) \rightarrow 0$ as $n \rightarrow \infty$. 8.1.10. Here, we must also consider the possibility that the approximating sequence $xn \rightarrow a$ approaches through ∂D . Thus P (x)/Q(x) converges to an /bm as $x \rightarrow \infty$. $\pm \infty$. 2.2.9. a) Let $E = \{k \in Z : k \ge 0 \text{ and } k \le 10n+1 y\}$. b) Suppose $x \in \partial(A \cap B)$; i.e., suppose Br (x) intersects $A \cap B$ and $(A \cap B)$ c for all r > 0. N +1 (SN f)(0) ≤ 159 Copyright \mathbb{C} 2010 Pearson Education, Inc. $\pi - \pi k = 0$ 14.1.3.
These formulas follow easily from the linear properties of integration. Suppose f 0 is not strictly monotone on [a, b], in fact, not partition of [1, 2] with norm 1/n, it is easy to see that log(an) is a Riemann sum of R2 converges to 1 log x dx. b) Let I = [a, b] and set τ (u) = (u - a)/(b - a). 11.7.7. a) The Lagrange equations are a = $-2Dx\lambda$, b = $-2Ey\lambda$, and c = λ . Similarly, by part b) and the Squeeze Theorem, I2 /khk \rightarrow 0/f 3 (a) = 0 as h \rightarrow 0. This happens, by Exercise 10.5.8, if and only if X is not connected. In fact, for any $x \in R$, $\|\infty \cap \infty \infty \mu X X \cap sin(x/(k+1)) \cap X \|x\| = \|x\| - \|x\| = \|x\|$. If k=1 bk converges, then it follows that $n^2 + 3n \in Q$. Then by Theorem 8.32, Br (x) $\subseteq E$ o so $x \in R$. E o, a contradiction. Then I and J are intervals by Theorem 10.56. The worst possible scenario is $dT/T = \pm 0.02$ and $dg/g = \mp 0.01$ so $dL/L = \pm 0.03$. k=1 k=1 j 0. Ponpital's Rule, $(1 - 1/k) P \rightarrow n c$) sn := $k=1 (k + 1)/k 2 \ge tn := k=1 1/k$. On the other hand, f(x) = 1/x is continuous on (0, 1) but not bounded there either. dxn $) = 2 x_{2j} dx_{1} . \infty c$) Suppose $k=1 |xj| < \infty$. d) True. Thus the claim holds for all $n \in N$. a) Use part b) with $A = \{1, 2\}$. Do this for all $a \in \partial E$ to define g on ∂E . g (1) f (0) $\pi 2 2 4.5.2$. a) By the Intermediate Value Theorem, f ((0, ∞)) = (0, ∞). 9.1.5. a) Repeat the proofs of Remark 2.4 and Theorem 2.6, replacing the absolute value by the norm sign. p If f (x) ≥ 1 , then f (x) $\leq f(x) < 1 + f(x) > 1$. (x). 14.3.3. By Theorem 14.23, k 2 ak (f) $\rightarrow 0$ as $k \rightarrow \infty$, in particular, $|ak(f)| \leq 1/k$ 2 for k large. 110 Copyright © 2010 Pearson Education, Inc. On the other hand, since $0 \leq (2k + 1)a2k+1 \neq 0$ as $k \rightarrow \infty$, it follows from the Squeeze Theorem that $(2k + 1)a2k+1 \neq 0$ as $k \rightarrow \infty$, it follows from the Squeeze Theorem that $(2k + 1)a2k+1 \neq 0$ as $k \rightarrow \infty$, it follows from the Squeeze Theorem that $(2k + 1)a2k+1 \neq 0$ as $k \rightarrow \infty$. a C p function if and only if $f \circ h-1$ is C p. c) Since F = (ey cos x, x2 z, x + y + z) implies div F = -ey sin x + 1, it follows from Gauss' Theorem that ZZ 3 Z 1 Z ω = S 0 0 $\pi/2$ Z (1 - ey sin x) dx dy dz = 3 0 1 (0 π - ey) dy = 3(1 - e) + $3\pi/2$. Set C := max{1, |M|, |m|}. To show the triangle inequality holds, let a = (a1, . r \rightarrow 0 A(Br (x0)) B(x) r 0 0) B(x) r 0 0 Since Qx - Py is continuous on E, it follows that Qx - Py = 0 everywhere on E.R y To find an f such that $\nabla f = F$, we must solve fx = P and fy = Q. 10.1.1. Since $\rho(a, b) = 0$. Then z = x and $y = 2 = x^3$. In particular, E contains more than two clopen sets. Thus, by definition, $xn \rightarrow a$ as n $\rightarrow \infty$. 11.4 The Chain Rule. Since $\phi_j = 1$ on V, it is also clear that j j c Pj $\phi_j \circ \phi - 1 = 1$ on $\phi(V)$. 11.7.1. a) 0 = fx = 2x - y and 0 = fy = -x + 3y 2 - 1 imply y = 2x and $12x^2 - x - 1 = 0$, i.e., x = 1/3, -1/4. $n \rightarrow \infty$ Similarly, (ax) y = axy and a - x = 1/ax. If $(x, y) \in G(f)$ then $x \in [x_j - 1, x_j]$ for some j and $|x - x_j| < \delta$. Therefore, 2N sin(u/2N) \uparrow u as N $\rightarrow \infty$ for each $u \in [0, \pi]$. Finally, by the Comparison Theorem, if $k \ge N$, then $Z \mid fk - f \mid dV \le E^2 Vol(E) < ^2$. 0 Similarly, ZZZ Z 0 1 Z 1 - z Qy dV = (Q(x, 1 - x - z, z) - Q(x, 0, z)) dx dz = (R(x, y, 1 - x - y) - R(x, y, 0)) dx dy. On the other hand, if P1 is a partition of [a, c] and P2 is a partition of [c, b] then $P = P1 \cup P2$ is a partition of [a, b] and Z b (U) f (x) dx \leq U (f, P) = U (f, P1) + U (f, P2). e) Case 1. Since E is closed, it is clear that B r \subseteq E. But xn+1 = 2 + xn \geq 0 by definition (all square roots are nonnegative), so the limit must be x = 2. Since every singleton is closed (see Remark 10.10), E = $\bigcup x \in E \{x\}$ is a decomposition of E into closed sets. In particular, both φ and $\varphi-1$ are continuous and must take open sets (respectively, compact sets) to open sets (respectively, compact sets). Choose ² so small that C² < 1 + $\eta/|\Delta\varphi|(x)|$. Now, repeating the proof of Theorem 8.32, we see that the largest relatively open set which is a subset of A is the union of all sets U \subset A such that U is relatively open in E, and the smallest relatively closed set which contains A is the intersection of all sets $B \supset A$ such that B is relatively closed in E. By Theorem 9.16 and L'H^opital's Rule, $\mu \parallel y \sin x x \lim$, $\tan x^2 + y^2 - xy = (1, 0, 1)$. $y/(y - x) 1/(x - y) x - y - y 1 \sqrt{\sqrt{c}}$ By part a), $x - y = s^2 - 4t \operatorname{so} x/(x - y) = s/s^2 - 4t + 1/2$ which is the partial of the first component of f -1 with d) E has no cluster points if E is finite. Since $\varphi 0$ (t) = (et (sin t + cos t), et (cos t - sin t), et), the arc length is given by Z $2\pi L(C) = k\varphi 0$ (t)k dt = $\sqrt{2} 3 0 2\pi et dt = \sqrt{3}(e^{2\pi} - 1)$. Since $3 - x \ge 3 - 2 = 1$, we conclude that $(x - 3)(3 - x - 2x^2) = (3 - x)/(2x^2 + x - 3) \ge M$. \sqrt{b} Since (k k/k) $\ge (1/k)$, this series diverges by the Comparison Test. Therefore, f(-x) = 1/f(x). Since Br is open, the first identity is trivial. Multiplying these inequalities, we have $xj yj \le (supk \ge n xk)(sup yk)$. 14.4.1. Define g(x) = f(x) for $x \in [0, 2\pi)$ and $g(2\pi) = f(0)$. b) By Taylor's Formula, there is a c between x and 0 such that $|\cos x - P2n(x)| = |(-1)n+1(\sin c)\cdot x^{2n+1}|/(2n+1)!$. Thus the integral diverges for p < 1 by part a). Thus vi) holds c) kx × yk = sin θ kxk kyk $\leq 1 \cdot$ kxk kyk. Since $x \in [\alpha n, \beta n)$, it is also clear that $\beta n - x x - \alpha n \leq 1$ and ≤ 1 . If x > 0 then by Example 2.2 and Theorem 2.6, $\lim x1/(2n-1) = \lim x1/m = 1$. We may suppose fxx (a, b) 6 = 0. 8.1.7. By symmetry, we may use any side of Q. b) By Exercise 5.3.7b, $E(x) \rightarrow \infty$ as $x \rightarrow \infty$, and $E(x) \rightarrow 0$ as $x \rightarrow -\infty$. If $yn-1 \ge n - 1$ and $xn-1 \ge 1$ then $yn = xn-1 + yn-1 \ge 1 + (n - 1) = n$ and $xn = xn-1 + 2yn-1 \ge 1$. 14.3.4. Fix $j \in N$. If $xj \in N$, $z = (2/(p\lambda))/(p-2)$. By looking at the graph, we see that f(E) = (-0.5, 0.5). By definition, then, B is finite. If we let $n \rightarrow \infty$, then $Z n \rightarrow \infty Z 1$ lim = 1, But since these sets are c c c nested, $H1c \subseteq \cdots \subseteq HN$, so the union of this covering is $HN \cdot 1 - 1A = \pi - 1$ b) (1, 0, 0) = a(1, 1, 0) + b(0, -1, 1) + c(1, 1, -1) implies b = c = 1 and a = 0.7.4.2. a) For |x| < 1 we have by the Geometric series and Theorem 7.33 that $\infty \propto k = 0$ k = 0 X X x = x (-x)5 = (-1)k x5k+1. P $\propto P \propto 6.1.2$. k=1 (1/k - 1/(k + 1)) = a(1, 1, 0) + b(0, -1, 1) + c(1, 1, -1) implies b = c = 1 and a = 0.7.4.2. a) For |x| < 1 we have by the Geometric series and Theorem 7.33 that $\infty \propto k = 0$ k = 0 X X x = x (-x)5 = (-1)k x5k+1. P $\propto P \propto 6.1.2$. k=1 (1/k - 1/(k + 1)) = a(1, 1, 0) + b(0, -1, 1) + c(1, 1, -1) implies b = c = 1 and a = 0.7.4.2. a) For |x| < 1 we have by the Geometric series and Theorem 7.33 that $\infty \propto k = 0$ k = 0 X X x = x (-x)5 = (-1)k x5k+1. $1 - \lim k \rightarrow \infty 1/k = 1$. ∞e) Since $|1 - \cos x|/x2 \le 2/x2$, the integral 1 (1 - $\cos x)/x2$ dx converges. 13.4.1. a) The boundary is $9 = x^2 + z^2$, y = 0, with counterclockwise orientation when viewed from far out the positive y axis. 1 5.3.4. a) Let $u = \log x$ and dv = f 0 (x) dx. Thus $x\alpha \sin(1/x)$ is uniformly continuous on (0, 1) for all $\alpha > 0$. Given $\varepsilon > 0$, choose $N \in N$ such that $n \ge N$ implies |xn - a| and |yn - a| are both $< \epsilon/2$. In particular, $xn \rightarrow a$ as $n \rightarrow \infty$. Conversely, if kfk $-fjk < \epsilon$ for k, $j \ge N$, then fk (x) is Cauchy in Y for each $x \in H$., n + 1 onto $\{1, 2, .., 11, 2.11, -j+k - j + k - k = 1, k =$ $(\psi(s, t) \cdot N\psi(s, t) d(s, t) = |\Delta \tau(s, t)|F(\phi(\tau(s, t))) \cdot N\phi(u, v) d(u, v)$. When x = 0, 6.4.9. By a sum angle formula and telescoping, we see that 2 sin $x \cos((2k + 2)x) = \cos(2x) - \cos((2n + 2)x)$. ∞) and $y \in \mathbb{R}$. 8.1.9. By the Cauchy-Schwarz Inequality, $xn := n X \tilde{A}$ [ak bk | = (|a1 |, . Since fk and f are bounded on E, choose PN1 N2 \in N such that N2 > N1 and k=0 [Sk (x) - f (x)] < 2N2 /2 for all $x \in \mathbb{E}$. 15.3.1. Since d(x3 dy dz dw + y 2 dx dz dw) = (3x2 - 2y) dx dy dz dw, we have by Stokes's Theorem and spherical coordinates that Z (x3 dy dz dw + y 2 dx dz dw) = (3x2 - 2y) dx dy
dz dw + y 2 dx dz dw) = (3x2 - 2y) dx dy dz dw + y 2 dx dz dw) = (3x2 - 2y) dx dz dw + y 2 dx dz dw) = (3x2 - 2y) dx dz dw + y 2 dx dz dw) = (3x2 - 2y) dx dz dw + y 2 dx dz dw) = (3x2 - 2y) dx dz dw + y 2 dx dz dw) = (3x2 - 2y) dx dz dw + y 2 dx dz dw) = (3x2 - 2y) dx dz dw + y 2 dx dz dw) = (3x2 - 2y) dx dy dz dw + y 2 dx dz dw) = (3x2 - 2y) dx dz dw + y 2 dx dz dw) = (3x2 - 2y) dx dy dz dw + y 2 dx dz dw) = (3x2 - 2y) dx dz dw + y 2 dx dz dw) = (3x2 - 2y) dx dz dw + y 2 dx dz dw) = (3x2 - 2y) dx dz dw $\phi \cos 2 \phi d\phi = .$ By assumption i), $\cos 2 x \le \cos x$. Similarly, $(\partial/\partial y) C(y) F \cdot T ds = Q(x, y)$ for any vertical line segment $C(y) \subset V$ terminating at (x, y). This means that τ is increasing on each Jk, so by the one-dimensional change of variables formula (Theorem 5.34), we have $Z F(\psi(u)) \cdot \psi 0(u) du = J = N Z X F \circ \phi \circ \tau (u) |\tau 0(u)| du k=1 Jk$ $k=1 \tau (Jk) NZXZF \circ \varphi(t) \cdot \varphi(t) dt ZF(\varphi(t)) \cdot \varphi(t) dt = -N \tau (Jk) k=1 F(\varphi(t)) \cdot \varphi(t) dt. 0 13.3.7.$ Suppose that (φ, E) is a C p parameterization of S which satisfies $(x0, y0, z0) = \varphi(u0, v0) = -0.$ Hence by Cantor's Theorem, the coefficients of S - T must be zero, i.e., S and T are the same series. 2.1.4. Suppose xn is bounded Since [a, b] \subset (0, 2 π), it follows 7.2.9. By Example 6.32, D k=1 e n (x)| \leq 1/| sin(c/2)| for c = max{|a|, |b|}. c) By b), xn + yn $\sqrt{xn + yn} xn - yn xn + 1 - yn + 1 = -xn yn < -yn = .$ $\partial E E b) By definition and Theorem 11.2, <math>\lceil$ i curl grad f = det $\lfloor \partial/\partial x fx j \partial/\partial y fy \rceil k \partial/\partial z \rfloor fz = (fzy - fyz, fxz - fzx, fyx - fxy) = (0, 0, 0)$. Thus yn+1 < xn+1 < · · · < x1. Hence integrating term by term, $\mu \P \P Z 1 Z 1 X \infty \infty \infty \mu^3 x'^2 X X 1 1 1^1 = f(x) dx = sin dx = (-k cos 1 - cos . 11.5.3. By the Mean Value Theorem and the assumption about Dg, f(g(x)) - f(g(a)) = Df(g(c))Dg(c)(x - a) = Df(g(c))Dg(c)$ hypothesis, DS(x) = B = Df(x) for all $x \in V$. 6.2.2. a) It diverges by the Limit Comparison Test since $(3k 3 + k - 4)/(5k 4 - k 2 + 1) 3 \rightarrow 6 = 0 1/k 5$ as $k \rightarrow \infty$. 2 2 b) If $f(x) \rightarrow L$ and $g(x) \rightarrow M$ as $x \rightarrow x0$ through E then $f(x) + g(x) \rightarrow L + M$ by Theorem 3.8, and $|f(x) + g(x)| \rightarrow |L + M|$ by Exercise 3.1.6. Thus by part a), $(f \lor g)(x) \rightarrow (L + M + |L - M|)/2 = L \lor M$ M as $x \to x0$ through E. It spirals around this cone from $\varphi(0) = (0, 1, 1)$ to $\varphi(2\pi) = (0, e^{2\pi}, e^{2\pi})$. If $x, y \in [0, N]$, then |f(x) - f(y)| < 2. SOLUTIONS TO EXERCISES Chapter 8 8.1 Algebraic Structure. By Theorem 14.29, S – Sg converges everywhere to zero. Compactness was used to prove f g is uniformly continuous since both functions need to be bounded. It is closed. b) We may suppose that E is nonempty. If k=1 bk diverges, then it follows from the comparison Test that Pb) ∞ k=1 ak diverges, then it follows from Theorem 11.58 that f (a, b) is either a local maximum or a local minimum, depending on the sign of fxx (a b). 8.3.7. b) Suppose E is not connected. Thus each point $x \in R$ is a cluster point of $R \setminus Q$. –x is an upper bound of –E and –x $\in -E$ so –x = sup(–E). 0 13.6.10., $|an| \cdot (|b1|, . Since 2|xy| \leq (x + y), |xy|/3 x^2 + y^2 \leq (x^2 + y^2)^{1/2-1/3} \rightarrow 0$ as (x, y) $\rightarrow (0, 0)$. k $\rightarrow \infty$ –1 k 2e2 –1 9.6.8. a) Since [0, 1] $\cap Q$ is countable, it can be covered by such a collection of intervals by Remark 9.42. Something went wrong. In particular, a is a cluster point of E. 4x2 - 1 > 0. Hence x1, In particular, in this case φ is both 1-1 and onto and there is nothing to prove. 8.4.4. First, we prove that relatively open sets are closed under arbitrary unions, and relatively closed sets are closed under arbitrary intersections. If c < 0 then $-c \in P$, so $ac - bc = (b - a)(-c) \in P$, i.e., ac > bc. Since f is continuous, f (xk) \rightarrow f (sup E). fm (x)]. c) Let $M \in R$ and choose by Archimedes an $N \in N$ such that N > M. CHAPTER 5 5.1 The Riemann Integral. b) Since y 4 /(x2 + y 4) ≤ 1 and $\alpha > 0$, $|f(x, y)| \leq |x\alpha| \rightarrow 0$ as (x, y) $\rightarrow (0, 0)$. h2 Combining this with part b), we conclude that fyx (a, b) = fxy (a, b). The root is less than or equal to 1/2 for $k \ge N = 2$. $\sqrt{5.1.9}$. Let $\varepsilon > 0$ and choose a partition $P = \{x0, ..., there is an a \in R such that given <math>r > 0$ there is an $N \in N$ such that $k \ge N$ implies $|xnk - a| < r. n \rightarrow \infty n \rightarrow \infty n \rightarrow \infty n \rightarrow \infty 3.3.10$. Suppose n = 1. If $E \cap U = \emptyset$ then since $A \cap U = \emptyset$, there exists a point $x \in U \cap (A \setminus E)$. c) Let $u = \log k$ and note that $u \to \infty$ as $k \to \infty$. $h \to 0+h \to 0+1+e1/h h f 0$ (0) = lim Since this limit is 1 as $h \to 0-$, f is not differentiable at x = 0. Then $xn-1-2 \le xn-1-2$, i.e., $xn = 2 + xn-1 - 2 \le xn-1 - 2 \le$ $(fx (\varphi(t))\psi 0 (t) + fy (\varphi(t))\sigma 0 (t)) dt = (f \circ \varphi(b) - f \circ \varphi(a). c)$ Suppose E is open and connected but not polygonally connected. Thus by Theorem 12.26 and the Intermediate Value Theorem 12.26 and valid on R. Conversely, if Br (x) \cap E c 6= \emptyset for all r > 0, then x \in / E o because E o is open. 0 Therefore, Z ZZ P dx + Q dy + R dz = ∂ T (Ry - Qz) dy dz + (Pz - Rx) dz dx + (Qx - Py) dx dy. Hence by the Mean Value Theorem there is an x1 \in (c, x0) such that 0 < f (x0) - f (c) = f 0 (x1). Hence, n = 1, 2. Conversely, if E is convex and (x1, y1), (x2, y2) dx dy dz + (Pz - Rx) dz dx + (Qx - Py) dx dy dz + (Pz - Rx) dz dx + (Qx - Py) dx dy dz $y_2 \in E$, then $L((x1, f(x1)); (x2, f(x2)) \subseteq E$. a) By Exercise 4.1.8, f(x) = 0. a) By definition, Dek $f(a) = \lim t \to 0$ $f(a + tek) - f(a) \partial f = (a)$. 3M 3M 3 3M But $|f(x)| \leq P \propto P \propto k = 1$ 7.2.8. a) Fix $n \geq 0$ and $x \in R$. $\sqrt{\sqrt{b}}$ Suppose 2 < a < 3. 13.5.8. The sum rules are obvious. By Exercise 10.1.10, E is closed and bounded. b) By the Archimedean Principle, given $\varepsilon > 0$ there is an N \in N such that N > $\pi 2/\varepsilon 2$. Let $\varepsilon > 0$ and set $\delta = 1$. Thus A \cup B is compact. Since f is continuous and periodic, choose $\delta \in (0, 2\pi)$ such that $t \in E\delta := [0, \delta] \cup [2\pi - \delta, 2\pi]$ implies |f(x - t) - f(x)| < 2 for all $x \in R$. Given 2 > 0 choose N so large that bk > 0 and |ck| < 2/2 for $k \ge N$. b) True. 9.6.7. a) Fix $x \in [0, \pi/2]$ and let f (t) p = 2t/(4t - 3x), t ≥ 2. xn = n does not converge, but xn /n = $1/n \rightarrow 0$ as $n \rightarrow \infty$. Hence by Dirichlet's Test, ak sin(2k+1)x converges for each x ∈ (0, π) \cup (π , 2π). In particular, there are uncountably many x0 ∈ (a, b) which satisfy DR f (x0) ≤ 0. 11.6.1. a) Since \cdot Df (u, v) = we have \cdot D -1 3 f (a, b) = 2 - 15, -1, 5 3 2, $-1 \cdot$, 5/17 1/17 =. Thus $\cap k \in N$ [-1/k, 1/k] = {0}. Then by (*) and (), $\alpha 2$ (2) $0 \le D(2)$ f (c)(h) = $\alpha 2 D(2)$ f (c)(h0) < D f (a)(h0) < 0, 2 a contradiction. Since f and g are absolutely integrable on (a, b), it follows from Theorem 5.42 and the Comparison Theorem 5.42 and the Comparison Theorem that any finite linear combination of f, g, and |f - g| is absolutely integrable on (a, b). $\beta n - \alpha n 75$ Copyright © 2010 Pearson Education, Inc. By the Distributive and Commutative Properties of real numbers, $x \cdot (y+z) = (x1, ... 15.3.3.$ Since d(n X dj . 5.1.8. Given $^2 > 0$, let P be any partition of [a, b] which satisfies kP k < $^2/((f (b) - f (a)))$. Thus n $\ge N$ implies $|(\pi xn - 2)/xn - (\pi - 2)| \equiv 2 |(xn - 1)/xn| < 4 |xn - 1| < \epsilon$. Therefore, f g and α f belong to $Cc \infty$ (Rn) when f and g do. Instructor's Solutions Manual An Introduction to Analysis Fourth Edition William R. Also notice that $(1/kq k)/|1/(ak + b)q k)| = |ak + b|/k \rightarrow |a| 6 = 0$. b) B = [1 - 1 1]. h $\rightarrow 0$ h 27 Copyright © 2010 Pearson Education, Inc. Hence by part a), $\varphi - 1 \circ f \circ \varphi$ is 1-1 if and only if it is onto. Conversely, if lim supn $\rightarrow \infty$ $|xn| \leq 0$, then $0 \leq \lim \inf |xn| \leq \lim \sup \varphi$ $|xn| \leq 0$, $n \rightarrow \infty$ implies that the limits supremum and infimum of |xn| are equal (to zero). Consider F (x, t) := u(t) - x. Then $|x - a| < \delta$ implies that the limit exists and equals 0. The function 1 + f is absolutely integrable on (a, b) by hypothesis and the fact that $b - a < \infty$. It is true for n = 0R by Weierstrass' Theorem. a) Fix a > 1 and for each $x \in R$ consider the set $Ex := \{aq : q \in Q \text{ and } q \le x\}$. Thus the absolute minimum of f on H is $f(-4/5, \pm 21/5) = -9/5$. Since f is C 2 on V and D(2) $f(c)(h) = D(2) f(a)(h) + \P 2 \mu X \partial 2f \partial 2f(c) - Q \partial 2f(a)(h) + \P 2 \mu X \partial 2f \partial 2f(c) - Q \partial 2f(a)(h) + \P 2 \mu X \partial 2f \partial 2f(c)(h) = 0$ (a) hj hk, $\partial xj \partial xk \partial xj \partial xk j, k=1$ choose $\delta > 0$ such that 1 1 D(2) f (c)(h0) < D(2) f (a)(h0) = D(2) f (a)(h0) < D(2) f (a)(h0) = D(2) f (a)(h0) < \odot 2010 Pearson Education, Inc. 9.4 Continuous Functions. Moreover, if Rj is not a rectangle that intersects ∂E or E0, then Mj $-mj < ^2$. Since $xn - xN \in Z$, it follows that xn = xN for all $n \ge N$. Since $xn - xN \in Z$, it follows that xn = xN for all $n \ge N$. function f which satisfies these three properties. Thus
n X | Pn k=1 sin(2k + 1)x = Pn k=1 (cos((2k)x) - sin(2k + 1)x] ≤ 2/| sin x| < ∞ k=1 P for each fixed x \in (0, π) \cup (π , 2π). P ∞ b) By Example 7.49, log2 x5 = 5 log x/ log 2 = 5 k=1 (-1)k+1 (x - 1)k/(k log 2) for 0 < x < 2. 23 Copyright © 2010 Pearson Education, Inc. 13.1.3. The trace of $\varphi(\theta) := (f(\theta) + 1)k + 1$ $\cos \theta$, f (θ) sin θ) on I := [0, 2 π] coincides with the graph r = f (θ). c) x 6 = 0 for x \in (0, 1], so f (x) := $e^{-1/x}$ is continuous on (0, 1] by Theorem 3.22. Since any M0 \in R can be written as α M for some M \in R, we see by definition that xn $\rightarrow -\infty$ as n $\rightarrow \infty$. Consequently, $\neg \neg$ sup Ek – inf Ek $\neg = |f(x*) - f(y*)| < 2$. On the other hand, if t \in J then t = f (x) for some $x \in [a, b]$ so by the choice of α and β , $\alpha \le t \le \beta$, i.e., $J \subseteq [\alpha, \beta]$. d) Since by L'H^opital's Rule $k(k/(k+1))k \rightarrow e-1$ as $k \rightarrow \infty$, this series converges by the Root Test. 1.6.2. By two applications of Theorem 1.42i, $Q \times Q$ is countable, hence $Q3 := (Q \times Q) \times Q$ is also countable. By Newton's method, x3 + 3x2 + 4xn - 1 + 12x3 + 3x2n - 1+ 1 xn = xn-1 - n-1 2 n-1 = n-1 . b) Since D(f \circ g)(a) = Df (g(a))Dg(a) and the determinant of a product is the product of the determinant, we have $\Delta f \circ g$ (a) = $\Delta f (g(a))\Delta g$ (a). Since L0 (y) = 1/y, it follows that E 0 (x) = E(x). 1-x 6.6 Additional tests. an = 1 is Cauchy and bn = (-1)n is bounded, but an bn = (-1)n does not converge, hence cannot be Cauchy by Theorem 2.29. b) Since sin $\pi k = 0$ for all $k \in N$ and $\cos(0) = 1$, (1, sin πk , $\cos(1/k)$) \rightarrow (1, 0, 1) as $k \rightarrow \infty$. By Corollary 10.59, f -1 (U) $\cap E$ is relatively open in E. Suppose 2 < xn < 3. Thus Z 0.3095 \approx 1 Z 1 (x2 - x6/3!) dx 0 then choose $^2 > 0$ so small that $M - ^2/2 > 0$. g(x) = |x| is not differentiable at x = 0, but g 2 (x) = x2 is., RN } such that X(Mj - mj)|Rj| 0. Hence, 8k 4 - 10k 2 + 1 > 0 for all $k \ge 2$. By L'H^oopital's Rule, $B\alpha \rightarrow 0$ as $\alpha \rightarrow 0+$ and $B\alpha \rightarrow 0$ as $\alpha \rightarrow \infty$. 11.6.7. By the Implicit Function Theorem, solutions gj(u(j)) exist for each j. If $x \in A$ and $y \in B$, then $x + y \le \sup A + \sup B$, so $sup(A + B) \le \sup A + \sup B$. This means that $S \circ \varphi(Q)$ is very nearly a translation, z + Q, of Q. Conversely, if L := 2n+1 X ak = k=2 as $n \rightarrow \infty$. Wade University of Tennessee, Knoxville The author and publisher of this book. 2.4.5. Let xn = Pn k k=1 (-1) / k S := for $n \in N$. It diverges at x = 1 (the harmonic series) and converges at x = -1 (an alternating series). If C(x, y) ends in a horizontal line segment, then by part a), fx = P . d) Since $\sqrt{1/k(k p - 1)kp} = -1 kp - 1 1/k p + 1/2 as k \rightarrow \infty$, it follows from the Limit Comparison Test and the p-Series Test that this series converges if and only if p + 1/2 > 1, i.e., p > 1/2. Since $ak \downarrow 0$ as $k \rightarrow \infty$, the radius of convergence of the power series $f(x) := \infty X(-1)k$ ak xk k=0 is greater than or equal P to 1 (see the proof of Exercise 7.3.5), i.e., f (x) converges for all $x \in [0, 1)$. Thus the original limit is e0 = 1.00147 Copyright © 2010 Pearson Education, Inc. Therefore, Z 1 Z Z 1 y 2 + z 2 Vol (E) = y2 - 1 Z 1 3 dx dz dy = 0 1 (1 + 3y 2 - 3y 4 - y 6) dy = -1 88. On the other hand, m0 \in E implies 2n b \leq m0, so a = b - (b - a) -a, i.e., n := -m \in E. b) Suppose that f is even. Finally, if $x \in V$, then $x \in J$ is interval. (x - n) $\rightarrow 0$ as $x \rightarrow 0$. Thus f (x) $\leq y * \leq y$, i.e., (x, y) $\in E$. Given r > 0. (by Theorem 3.8), it follows from the Squeeze Theorem that xn sin(x-n) $\rightarrow 0$ as $x \rightarrow 0$. Thus f (x) $\leq y * \leq y$, i.e., (x, y) $\in E$. Given r > 0. $0, \rho(xn, a)$ is eventually smaller than r, e.g., Br (a) \cap E contains xn1 for some n1. Therefore, b = 1 - 1 - a < 1 - (1 - a) = a. Using f (a) f (a + h) as a common denominator, we have 1 1 f (a) - f (a + h) - = . If A \cap B = \emptyset , then sup(A \cap B) := $-\infty$ and there is nothing to prove. e) Since $|yn| \le M/n \rightarrow 0$ as $n \rightarrow \infty$. For example, if a = 0, b = 1, then sup(A \cap B) is eventually smaller than r, e.g., Br (a) \cap E contains xn1 for some n1. Therefore, b = 1 - 1 - a < 1 - (1 - a) = a. Using f (a) f (a + h) as a common denominator, we have 1 1 f (a) - f (a + h) - = . If A \cap B = \emptyset , then sup(A \cap B) is eventually smaller than r, e.g., Br (a) \cap E contains xn1 for some n1. g(x) = x and f(x) = -x, then g(f(x)) does not exist, so the integral on the right side of part d) is not defined. Let a 6 = 0, $f(x) = g(x) = x^2$, and n = 2. $\geq C |Qj|$ Combining this estimate with the estimate in the previous paragraph, we conclude that $\neg \neg Vol(\varphi(Qj)) \neg \neg - |\Delta\varphi(x)| \neg \langle \eta \neg |Qj|$ for j large, i.e., $Vol(\varphi(Qj))/|Qj| \rightarrow |\Delta\varphi(x) - as j \rightarrow \infty$. 2.5.4. a) Since inf k > n xk + inf k > n yk is a lower bound of xj + yj for any j > n, we have inf k > n xk + inf k > n yk < inf j > n, we have inf k > n xk + inf k > n yk < inf j > n, we have inf k > n xk + inf k > n yk < inf j > n, we have inf k > n xk + inf k > n yk is a lower bound of xj + yj for any j > n, we have inf k > n xk + inf k > n yk < inf j > n, we have inf k > n xk + inf k > n yk < inf j > n, we have inf k > n xk + inf k > n yk < inf j > n, we have inf k > n xk + inf k > n yk < inf j > n, we have inf k > n xk + inf k > n yk < inf j > n, we have inf k > n xk + inf k > n yk < inf j > n, we have inf k > n xk + inf k > n yk < inf j > n, we have inf k > n xk + inf k > n yk < inf j > n, we have inf k > n xk + inf k > n yk < inf j > n, we have inf k > n xk + inf k > n yk < inf j > n, we have inf k > n xk + inf k > n yk < inf j > n, we have inf k > n xk + inf k > n yk < inf j > n, we have inf k > n xk + inf k closed in E. 15 S 0 0 b) By the calculation which follows Definition 13.28, Nq = (a2 cos u cos 2 v, a2 sin v cos v) points outward. If $\alpha > 1$ then $\overline{} = y$. 0 0 145 Copyright © 2010 Pearson Education, Inc. Thus ZZ Z Z $\alpha = S 0 = \pi 2$ ((v/2) cos u, (v/2) sin u, v 2) · ((v/2) cos u, (v/2) cos u) · ((v/2) cos u) $(v/2) \sin u, -v/4) dv du 0 Z 2 (v 2 - v 3) dv = -0 2\pi$. U (f, P) = 0.5f (0.5) + 0.5f (1) + f (2) = 137/16. By part a), f (x) = f (mx/m) = mf (x/m). Thus $\infty X (a_2k + a_2k+1) = \lim (a_2 + a_3) + \cdots + (a_2n + a_2n+1) = \lim (a_2 + a_3) + \cdots + (a_2n +$ But $a \in E$, so Bs (a) $\cap E = \{a\}$. Suppose n and m are even and m > n. By Theorems 10.56 and 10.62, f ([0, 1]) is connected. Hence, by the Extreme Value Theorem, there is an M > 0 such that $|f(x)| \leq M$ for all $x \in [a, b]$. By the Heine-Borel Theorem, there is an M > 0 such that $|f(x)| \leq M$ for all $x \in [a, b]$. By the Heine-Borel Theorem, there is an M > 0 such that $|f(x)| \leq M$ for all $x \in [a, b]$. By the Heine-Borel Theorem, there is an M > 0 such that $|f(x)| \leq M$ for all $x \in [a, b]$. By the Heine-Borel Theorem, there is an M > 0 such that $|f(x)| \leq M$ for all $x \in [a, b]$. $y_0 \mid \leq 2k(x, y) - (x_0, y_0)k < \delta$. Therefore, ZZ ZZ $\omega = (Px + Qy + Rz) dV$. f) Since n(1 + (-1)n) + n - 1((-1)n - 1) = 2n when n is odd, lim supn $\rightarrow \infty$ xn = ∞ and lim inf $n \rightarrow \infty$ xn = 0. Therefore, S is the Fourier series of f. Since $N \cup N k = 1 \tau (Jk) = \tau (Uk = 1 Jk) = \tau (J) = I$, we then ix x - x 0 + v 0 $\leq |x - x0| +$ conclude that Z Z g($\psi(u)$)k ψ 0 (u)k du = g($\varphi(t)$)k φ 0 (u)k dt. Let f (x) = 2 - x for x < 1, f (x) = 1 - x for x < 0 and g(x) = -x for x < 0 and integrable on [0, 1]. A similar argument handles the case L < 0. Thus $x^2k+1 \rightarrow 0$ as $k \rightarrow \infty$, but xn is NOT bounded. b) By definition, Z 1 Z 1 L(f, Pn) \leq (L) f (x) dx \leq U (f, Pn) \leq (L) f (x) dx \leq U (f, Pn) \leq (L) f (x) dx \leq U
(f, Pn) \leq (L) f (x) dx \leq U (f, Pn) \leq (L) f (x) dx \leq (L) f (x) dx = (L) f (x) dx \leq (L) f (x) dx = (L) f (x) dx \leq (L) f (x) dx \leq (L) f (x) dx \leq (L) f (x) dx = (L) f (x) dx \leq (L) f (x) dx = the limit of this last quotient is (by L'H^opital's Rule twice) lim $u \rightarrow \infty 2 \log u \cdot (1/u) 2 \log u = \lim = 0$. Choose $N \in N$ so large that N > (b - a)/N. Then $k \ge N$ implies $kyk - ak \le kxk - ak + kxk - ykk < \varepsilon$. P $\infty k k b$) True. Then Qx - Py = 2 and we have by Green's Theorem that $1 \ge ZZX x dy - y dx = \partial E dA = Area$ (E). Combining these statements, if $q \in Q$ then q = n/m so $f(qx) = f^3n \quad 3x \quad x = fn = (fn(x))1/m = fn/m(x) = fq(x) m m$ for $x \in R$. Conversely, if Bs (a) $\cap E = \{a\}$, then this set does not contain infinitely many points, so a is not a cluster point by definition. 2 13.3.3. Parameterize this ellipsoid using $\varphi(u, v) = (a \cos u \cos v, b \sin u \cos v, c \sin v)$ and $E = [0, a \cos u \cos v, b \sin u \cos v, c \sin v]$. $2\pi |x| - \pi/2$, $\pi/2 |$, $\sqrt{3}$ If y = x then 2x = 6x + x, $\sqrt{i.e.}$, $x \neq 0$ or $x = \pm 1/6$. 11.2.10. a Rb Rb Rb It follows that (U) a q(x) dx < a f (x) dx + 2.10.3.1. a) The closure is $E \cup \{0\}$, the interior is \emptyset , the boundary is $E \cup \{0\}$, the interior is \emptyset , the boundary is $E \cup \{0\}$, the interior is \emptyset , the boundary is $E \cup \{0\}$. Fix $j \in N$. Then $C \setminus B = \{0\}$ and f(C) = f(B) = [0, 1]. $C \cap R$ Suppose iii) holds and let $x_0 \in E$. By symmetry, we may suppose that $x = \infty$. Since $(2x - x)0 = 2x \log 2 - 1 > 1$ for all $x \ge 2$, i.e., 2x - x is increasing on $[2, \infty)$. Are there any others? 10.6.1. a) f $(0, \pi) = [0, 1]$ is compact and connected as Theorems 10.61 and 10.62 say it should; f $(-1, 1) = (-\sin 1, \sin 1)$ is open, big deal; f $[-1, 1] = [-\sin 1, \sin 1]$ is compact and connected as Theorems 10.61 and 10.62 say it should. 12.1.4. a) Since $\partial Br = Br \setminus Bro$, it suffices to show Bro (a) = E := {x : kx - ak \le r}. 10.4.8. a) Suppose not, i.e., $\cap Hk = \emptyset$. Let P be any partition which satisfies xj-1 = x0 - \delta and xj = x0 + \delta for some j. h $\rightarrow 0$ h $\rightarrow 000$ f[$(0,\infty)$ (0) = lim Thus f 000 (0) does not exist. a $\in H$ Since H is compact, there are points a1, a2, If y = -x then $-2x = -6x \sqrt{+x}$, i.e., $x = \sqrt{0}$ or $x = \pm 1/2$. Thus use Theorem 7.10. f (x) f (y) c2. It follows that Mj (f 1/m) - mj (f 1/m) $\leq (Mj (f) - mj (f))/c2$. Then $|f(xn) - L| \leq 2$ for all $n \geq N$, i.e., Then there is an $^{2}O > 0$ such that given n > 0 i.e., $|f(xn) - L| \geq 20$. Hence $(\varphi, [a, b])$ and $(\psi, [f(a), f(b)])$ are orientation equivalent. $s = -\infty$. P ∞ If an $/bn \rightarrow \infty$ then an \geq bn for n large. 6.4.3. a) Since $[(k + 1)3/(k + 2)!]/[k 3/(k + 2)!] \rightarrow 0$ as $k \rightarrow \infty$, this series converges absolutely by the Ratio Test. Thus, $x \in U \setminus U \circ \subseteq \partial U$. It is open. $\forall E = \{x \lor 1\} \cup \{(x, 1): -2 \leq x \leq 2\} \cup \{(x, -1): -2 \leq x \leq 2\}$. On the other hand, if p > 1, choose q > 0 such that p - 1 > q and a constant C such that $\log x \le xq/p$ for all $x \ge C$. $\partial E \ \partial E \ E \ E \ 13.6.6.$ Let $^2 > 0$. By part b), $n \ 2X - 1 \ n \ 2X - 1 \ (a2k \ (f) + b2k \ (f)) \le 2 \ k = 2n - 1 \ (a2k \ (f) + b2k \ (f)) \le 2 \ (a2k \ (f) + b2k \ (f)) = 2 \ (a2k \ (f) + b2k \ (f)) = 2 \ (a2k \ (f) + b2k \ (f)) = 2 \ (a2k \ (f) + b2k \ (f) + b2k \ (f) \ (a2k \ (f) + b2k \ (f)) = 2 \ (a2k \ (f) + b2k \ (f) + b2k \ (f) \ (a2k \ (f) + b2k \ (f) \ (a2k \ ($ 1 and 0 if j > 1. Thus [] 4 0]. e) Since 1/n + (-1)n = 1/n + 1 when n is even and 1/n - 1 when n is odd, inf E = -1 and sup E = 3/2. 11.5.4. Let B = [bij] be the n × n matrix that represents Df (a) and set S(x) = B(x). xv yv zv xv yv zv ($\varphi \circ \psi$) $0 \cdot (\varphi u \times \varphi v) = (xu ut + xv vt) 13.4$ Oriented Surfaces. 12.5.1. If (f g)(x) 6 = 0 and g(x) 6 = 0. If 1/b ≤ 0 then b = b2 (1/b) ≤ 0 a contradiction. If x, a $\in (0, 1)$ and $|x - a| < \delta$, then $|f(x) - f(a)| = |x - a| |x + a + 1| \leq 3|x - a| < 3 \epsilon = \epsilon$. Since $x \geq 0$, f 0 (x) = ex + cos $x \geq 1$ + cos $x \geq 0$, f 0 (x) = ex + cos $x \geq 0$, f 0 (x) = ex + cos $x \geq 1$ + cos $x \geq 0$, f 0 (x) = ex + cos $x \geq 0$, f 0 (x) = ex + cos $x \geq 0$, f 0 (x) = ex + cos $x \geq 0$, f 0 (x) = ex + cos $x \geq 0$, f 0 (x) = ex + cos $x \geq 0$, f 0 (x) = ex + cos $x \geq 0$, f 0 (x) = ex + cos $x \geq 0$, f 0 (x) = ex + cos $x \geq 0$, f 0 (x) = ex + cos $x \geq 0$, f 0 (x) = ex + cos $x \geq 0$. conclude that F and T are orthogonal. 6.2.8. Notice that ak + b 6= 0 for k \in N, since otherwise, b/a = $-k \in Z$. Define xj 's by 1 1 1 < x2 = $-\delta < x^2 = -\delta < x^2$ 6x + 3 = 0 and 12z 2 + 12z 3 = 0, i.e., x = 1 and z = 0, -1. Let r > 0 be so small that $rt1 < \frac{2}{2}$ and set w(x, t) = u(x, t) + r(t - t1). 26 Copyright © 2010 Pearson Education, f (2k) (0) = 0 for all $k \in N$. yn y yn yn $\frac{1}{2}xn x = 1$ and z = 0, -1. Let r > 0 be so small that $rt1 < \frac{2}{2}$ and set w(x, t) = u(x, t) + r(t - t1). 26 Copyright © 2010 Pearson Education, f (2k) (0) = 0 for all $k \in N$. yn y yn yn $\frac{1}{2}xn x = 1$ and z = 0, -1. Let r > 0 be so small that $rt1 < \frac{2}{2}$ and set w(x, t) = u(x, t) + r(t - t1). 26 Copyright © 2010 Pearson Education, f (2k) (0) = 0 for all $k \in N$. yn y yn yn $\frac{1}{2}xn x = 1$ and z = 0, -1. Let r > 0 be so small that $rt1 < \frac{2}{2}xn x = 1$ and z = 0. Corollary 5.23, so are f n for all $n \in N$., xk have been chosen so that $xk \in E \cap Br$ (a) and $s := \min\{\rho(x1, a), .$ It is neither open nor closed. 3-1-15.2.1.a |x + 1| = -x - 1 if $x \ge -1$ and |x + 1| = -x - 1 if $x \ge -1$ and |x + 1| = -x - 1 if $x \ge -1$ and |x + 1| = -x - 1 if $x \ge -1$. It follows from the inductive hypothesis that $\psi \circ \phi$ is 1-1 on $\{1, 2, . Since 0 \le ey/k \le eM/k, it follows that <math>|ey/k - ey/k \cos(x/k)| = |ey/k| |1$ $-\cos(x/k)| \le eM/k (1 - \cos(M/k)) \rightarrow 0$ uniformly on E as $k \rightarrow \infty$. 11.5.9. Let F (t) = f (a + tu) and observe by definition that F 0 (t) = lim h \rightarrow 0 F (t + h) - F (t) f (a + tu + hu) - f (a + tu) = lim = Du f (a + tu) + 0 $\rho(x, a) > r$ and $\{x \in X : \rho(x, a) < s\}$ are both open, hence $E := \{x \in X : s \le \rho(x, a) < r\} = \{x \in X : \rho(x, a) > r\}c \cap \{x \in X : \rho(x, a) < r\}$ are both open, hence $E := \{x \in X : s \le \rho(x, a) < s\}c$ is closed. $\sqrt{\sqrt{b}}$ b) Let $n \to \infty$ in the identity $y_n + 1 = x_n + 1$ yn. c) Repeat the proof of Theorem 3.8, replacing the absolute value by the norm sign. Solving these simultaneous equations, we have b = -1, c = 0, and a = d = -1, c = 0, and a = d = -1, c = 0, and a = d = -1, c = 0, and a = d = -1, c = 0, and a = d = -1, c = 0, and a = d = -1, c = 0, and a = d = -1, c = 0, and a = d = -1, c = 0, and a = d = -1, c = 0,
and a = d = -1, c = 0, and a = d = -1, c = 0, and a = d = -1, c = 0, and a = d = -1, c = 0, and a = d = -1, c = 0, and a = d = -1, c = 0, and a = d = -1, c = 0, and a = d = -1, c = 0, and a = d = -1, c = 0, a = -1, c = -1, 1, i.e., x - y + w = 1. Since $x^2 \cos kx$ is even, we can integrate by parts twice to verify $Z Z \pi 2 \pi 2 4 4(-1)k$ ak $(x^2) = x \cos kx \, dx = -x \sin kx \, dx = \pi 0 \, k\pi 0 \, k^2$ for k = 0. Thus $\cup k \in \mathbb{N}$ [-k, 1/k) = (- ∞ , 1). 10.5.1. a) Let $\mathbb{R} = [a, b] \times [c, d]$. e n (x) := Pn sin(kx) $\leq 1/|\sin(x/2)|$ for $x \in (0, 2\pi)$. 7.4.4. Since P (k) (x) = 0 for k > n and $x \in \mathbb{R}$, the Taylor series truncates. Thus f 0 (x) is analytic on (a, b). Clearly, $\partial g \partial f_1(x) = f_1(x_1) \cdots (x_1)$ for some open V, then C = E \cap V for some open V, then C = E \cap V for some open V, then C = E \cap V for some open V, then C = E \cap V for some open V, then C = E \cap V for some open V, then C = E \cap V for some open V, then C = E \cap V for some open V, then C = E \cap V for some open V, then C = E \cap V for some open V, then C = E \cap V for some open V, then C = E \cap V for some open V, then C = E \cap V for some open V for some op $-2 \cos v$, $-2 \sin v$) points inward, the wrong way. Since this set is closed, the limit, a low belongs to it. Thus the limit, a exists by xn+1 = 1 - 1 - xn, as $n \to \infty$, we have a = 1 - 1 - a, i.e., a = 0, 1. Therefore, $Z Z \pi/2 2 \pi F \cdot T ds = \sqrt{\cos 2 t} dt = \sqrt{1 - 3.4}$. If m is a lower bound of E then so is any m $e \le m$. Since $(n - 1)/(2n - 1) \rightarrow 1/2$ as $n \to \infty$, this factor is bounded. Thus k=0 P \propto k=0 (-1) k 2k x /(2k)! for x \in R. 2.1.6. If xn = a for all n, then |xn - a| = 0 is less than any positive ε for all n \in N. 8.3.1. a) This is the plane without the x-axis. 5.4.6. a) Choose a < b0 < b such that f (x)/g(x) < 2L + 1 for all b0 < x < b. Thus U = B1 (0, 0) \cap B $\sqrt{2}$ (2, 0). 3.3.3. By Exercise 3.1.6, |f| is continuous on [a, b]. a) Df (x, c) = 0 Therefore, $\lim \sup \to \infty$ I1 ≤ 2 . If ∇f (t0) 6= 0, then either u0 (t0) 6= 0 or v 0 (t0) 6= 0. Let a = -4, b = -1, and c = 2. Then by DeMorgan's Law, \cup Hkc = X, in particular, {Hkc} is an c open cover of H1 . 9.2.1. If E is compact, then by Heine-Borel, E is closed (and bounded). khk khk khk Thus by definition, T is differentiable at a and DT (a) = T . 1.5.2. a) f decreases and f (-1) = 5, f (2) = -4. Hence, w = st, i.e., E(x + y) = E(x)E(y). n 1 1 1 Taking the limit of this last inequality as b $\rightarrow \infty$, we see that f (xn) is absolutely integrable on $[1, \infty)$ for each n > 1. Thus the original series converges by the Comparison Test. 2|I| Now f is bounded on I by M := supx $\in I |g(x)| + |f(xk-1)| + |f(xk)|$. Let xn = (-1)n/n. Then A is finite, so we can choose a finite collection of rectangles PM {Rj : j = N + 1, . This function is continuous, and f (0) = -1 < 0 < 3 = f(1). Since Br (x) intersects A, it follows that $x \in \partial A$. Since f is continuous at any t 6 = 0, $\Omega f(t - h, t + h)$ gets smaller as $h \to 0$, so $\omega f(t) = 0$ for t 6 = 0. By Green's Theorem, Z ZZ Z F · T ds = $\pi/2$ Z 2 (v - 0) dA pieces is 4 + 1 = 5. Conversely, if $x \in f(f - 1(E))$, then x = f(a) for some $a \in f - 1(E)$. By Theorem 5.34, $f \circ \omega \cdot |\omega 0|$ is integrable on [a, b], xn = 1 converges and yn = (-1)n does not converge. 1.2.7. a) Since $|x + 2| \leq |x| + 2$, $|x| \leq 2$ implies $|x2 - 4| = |x + 2| |x - 2| \leq 4|x - 2|$. Pare $\infty \infty c$) Given $x \in E$, let x = k = 1 bk/3k. where bk 6= 1. Thus ap0 = A(p0). 5.6.8. a) Let $E := \{x \in [a, b] : f(x) > y0\}$., $xN \in K$ such that $K \subset j=1$ Ixj. It follows from the Squeeze Theorem that f is differentiable at $0 \in I$, and its derivative is zero. Hence k=1 ak converges absolutely by the Ratio Test, since $\mu \mu \P \P - 1$ ak +1 1 1 -1 ak +1 2 as $k \to \infty$ by L'H^opital's Rule. Therefore, the latter must be uncountable by the argument of Remark 1.43. Since supk > n × s + 2 for all n > N, i.e., lim supn $\rightarrow \infty$ xn = s. G3 149 Copyright © 2010 Pearson Education, Inc. Let $x \in \mathbb{R}$ and choose N such that $n \ge N$ implies $|yn| < \delta$. Checking the critical point f (0, 0) = 0, and extreme points of H, f (1, 1) = 3, f (1, -1) = -1, f (-1, 1) = -5, and f (-1, -1) = 3, we conclude that the absolute minimum of f on H is f (-1, 1) = -5. Conversely, let $a \in \mathbb{R}$, 2 > 0, and set I = (f(a) - 2, f(a) + 2). 10.2.3. If a is a cluster point for E, then let $xn \in (B1/n (a) \cap E) \setminus \{a\}$. Adding these inequalities, we obtain $f(x) + q(x) \le f(y) + q(y)$. 5.3.8. a) By Exercise 5.3.7, L is differentiable and strictly increasing, hence 1-1, on $(0, \infty)$, and takes $(0, \infty)$ onto R. Since q > 1, it follows from Raabe's Test that k=1 as converges absolutely., xk have been chosen so that $xk \in E \cap (a - r, a + r)$ and $s := \min\{|x1 - r| = 1 \}$. a|, 7.5.4. The line tangent to y = f(x) at (xn-1), f(xn-1) bas equation y = f(xn-1) + f(xn-1). Since f(a), $f(b) \in f(E)$, it follows that $[f(a), f(b)] \subset f(E)$. Fix $k \in \{1, 2, ..., 1, 2, ..., 1\}$ bas equation y = f(x) + f(x) + 1/2. Since f(a), $f(b) \in f(E)$, it follows that $[f(a), f(b)] \subset f(E)$. Fix $k \in \{1, 2, ..., 1\}$ bas equation y = f(x) + 1/2. Since f(a), $f(b) \in f(E)$, it follows that $[f(a), f(b)] \subset f(E)$. Fix $k \in \{1, 2, ..., 1\}$ bas equation y = f(x) + 1/2. Since f(a), $f(b) \in f(E)$, it follows that $[f(a), f(b)] \subset f(E)$. $-1 \text{ n} - 1 + pq - 1 \text{ nn} - 1 = (mq + pn)n - 1 q - 1 \cdot \cos x \cos 2 x 4.3$ The Mean Value Theorem. If x = 1 and z = -1, then 3 + y - 4 = 1 and -1 + 3 + w = 0, i.e., y = 2, w = -2. If x0 = 3 is the initial quess, then $|x0 - \pi| < 0.000136465$. Thus it converges if and only if $x \in 1$ and y = -2. If x = -2. If x[-1, 1). 0 Thus by hypothesis iii), Zr = Qx - Py = zrz(x, y, v) dv + gx - hy = r(x, y, z) - r(x, y, 0) + gx - hy. $g(0, \pi) = \{0, 1\}$ is connected as Theorem 10.62 says it should; $g[0, \pi] = \{0, 1\}$ is connected as Theorem 10.62 says it should; $g[0, \pi] = \{0, 1\}$ is connected as Theorem 10.62 says it should; $g[0, \pi] = \{0, 1\}$ is connected as Theorem 10.62 says it should; $g[0, \pi] = \{0, 1\}$ is connected as Theorem 10.62 says it should; $g[0, \pi] = \{0, 1\}$ is connected as Theorem 10.62 says it should; $g[0, \pi] = \{0, 1\}$ is connected as Theorem 10.62 says it should; $g[0, \pi] = \{0, 1\}$ is connected as Theorem 10.62 says it should; $g[0, \pi] = \{0, 1\}$ is connected as Theorem 10.62 says it should; $g[0, \pi] = \{0, 1\}$ is connected as Theorem 10.62 says it should; $g[0, \pi] = \{0, 1\}$ is connected as Theorem 10.62 says it should; $g[0, \pi] = \{0, 1\}$ is connected as Theorem 10.62 says it should; $g[0, \pi] = \{0, 1\}$ is connected as Theorem 10.62 says it should; $g[0, \pi] = \{0, 1\}$ is connected as Theorem 10.62 says it should; $g[0, \pi] = \{0, 1\}$ is connected as Theorem 10.62 says it should; $g[0, \pi] = \{0, 1\}$ is connected as Theorem 10.62 says it should; $g[0, \pi] = \{0, 1\}$ is connected as Theorem 10.62 says it should; $g[0, \pi] = \{0, 1\}$ is connected as Theorem 10.62 says it should; $g[0, \pi] = \{0, 1\}$ is connected as Theorem 10.62 says it should; $g[0, \pi] = \{0, 1\}$ is connected as Theorem 10.62 says it should; $g[0, \pi] = \{0, 1\}$ is connected as Theorem 10.62 says it should; $g[0, \pi] = \{0, 1\}$ is connected as Theorem 10.62 says it should; $g[0, \pi] = \{0, 1\}$ is connected as Theorem 10.62 says it should; $g[0, \pi] = \{0, 1\}$ is connected as Theorem 10.62 says it should; $g[0, \pi] = \{0, 1\}$ is connected as Theorem 10.62 says it should; $g[0, \pi] = \{0, 1\}$ is connected as Theorem 10.62 says it should; $g[0, \pi] = \{0, 1\}$ is connected as Theorem 10.62 says it should; $g[0, \pi] = \{0, 1\}$ is connected as Theorem 10.62 says it should; $g[0, \pi] = \{0, 1\}$ is connected as Theorem 10.62 says it should as Theorem 10.62 says it should as Theorem $1 = \{-1, 0, 1\}$ is compact but not connected-note that Theorem 10.61 does not apply since g is not continuous. Since f (100) = 0.00999, n = 100 terms will estimate the value to an accuracy of 10-2. g) It diverges by the Root Test, since has a limit supremum of $4/\pi$. Since xn, yn > 0, the limit must be 2. If $A \cap B = \emptyset$, then use the Monotone Property., xN such that N [K ⊂ Br(xj) (xj). Choose N ∈ N such that $n \ge N$ implies $xn \in (x - 2, x + 2)$. (1 + t3)2 (1 + t3) all $k \in N$. Since $\cos x 6 = 0$ for $x \in [0, 1]$, it follows from $\sqrt{x2}$ Theorem 3.22 that $e \sin x / \cos x$ is continuous on [0, 1]. Clearly, xn is Cauchy. Since this last inequality is satisfied only when xn = xN, it follows that xn = xN is continuous on [0, 1]. documentation contained in this book. x4 = -.3176721, b) It converges by the Comparison Test and the Geometric Series Test, since $0 \le (k-1)/(k2k) \le 1/2k$, P ∞ b) The partials sums of k=1 (-1)k assume only the values -1, 0, hence are bounded. Thus by Theorem 4.23, there is an $x0 \in (x1, x2)$ such that f 0 (x0) = f 0 (x3), c) $1/(n + 1) \le 0.0005$ implies $(n + 1) \ge 2000$, i.e., $n \ge 1999$. If b > 0, then for $\varepsilon = b$, $a < b - \varepsilon = 0$. Let $L = \sup\{f(x) : x \in (a, b)\}$. b) Set $F(x, y, z) = x^2 + y^2 + z^2 + \sin(x^2 + y^2) + 3z + 4 - 2$. b) Repeat the proof of Theorem 3.26. Therefore, f is continuous at a. The boundary is $y = \pm x$, z = 0, and $z = 1 - y^2$, $x = \pm 1$. 15.2.5. a) Suppose (U, q) and (V, h) are charts from different atlases of M. Since
$d - c \ge 2$, choose $x^2 \in [c, d]$ such that $x - x^2 = \pm 1$. If we let $s := \min\{r, \rho(x^1, a), \ldots$ Conversely, if $x \in V = U \cap Y$, then there is ball BX, open in X, such that $x \in BX \subset U$. Since F (u0, v0) = (0, 0) and $\partial(F1, F2) = \partial(\varphi 1, \varphi 2) = 6 = 0$, $\partial(u, v) = 0$, $\partial(u, v)$ containing (x0, y0) and a continuously differentiable function $q: V \rightarrow R2$ such that $\varphi 1$ (q(x, y)) = x and $\varphi 2$ (q($\lim x \to 0$ (3/x)/e1/x = $\lim x \to 0$ (3/x2)/(2e1/x /x3) = 0. By the Cauchy-Schwarz Inequality and hypothesis, Fk ($\varphi(t)$) · $\varphi(t)$ uniformly on I, as $k \to \infty$. 4.5.1. a) By the Inverse Function Theorem, (f -1)0 (2) = 1 1 = ., JN such that $\tau 0 6 = 0$ on each Jk0 and $J = \bigcup N k = 1$ Jk. c) By repeating the steps in b), we see that the original expression is rational if and only if n(n+7) = n2 + 7n = m2 for some $m \in N$. $\partial gm / \partial xn$ (a) Therefore, $\partial h / \partial xj$ (a) = $\nabla f(g(a)) \cdot \partial g / \partial xj$ (a). Then for x = 0 we have f(x)g(x) = 1/x which has no limit as $x \to 0$. By part a, then, $|f - 1(x)| \le 1/|f 0(c)| < \infty$ for all $x \in I$. Its speed is $k(-2a \sin 2t, 2a \cos 2t)k = 2a$. $P \approx 9.6.1$. Since $fk \ge 0$, the partial sums of k=1 fk are increasing on [a, b]. $\partial x_j \partial x_j$ Therefore, by Exercise 9.3.6 and hypothesis, g and gx are all continuous on V. b) The domain of f is all $(x, y) \in \mathbb{R}^2$ such that x = 0, y = 0, and x/y = 0, z(x) + 1/2 for $k \in \mathbb{Z}$ (for example, $tan(\pi/2)$ is undefined). By part a, $xq - aq xn - an = \cdot (xq(m-1) + \cdots + aq(m-1)) - 1 = :y(x) + z(x)$. For $s \ge b > a$, $|e - (s - a)t \phi(t)| \le M e^{-b}$, hence by ∞ the Weierstrass M-Test, $0 = -(s-a)t \varphi(t) dt$ converges uniformly on $[b, \infty)$. Hence by induction, xn > yn > 1 for all $n \in \mathbb{N}$. 2.4.6. By Exercise 1.4.4c, if $m \ge n$ then $|xm+1 - xn| = |m X (xk+1 - xk)| \le k = n \mu \|m X 1 1 1 1 = 1 - -(1 -)$. 2.5.9. If $xn \to 0$, then $|xm| \to 0$. Thus by induction this inequality holds for all $n \in \mathbb{N}$. 12.1.9. By Exercise 12.1.5, we may suppose E is closed. 12 2 1 3 + = . Hence, it follows from Exercise 1.4.4c and definition that n X 9 4 4 1 .4999 \cdots = + lim 10 n $\rightarrow \infty$ 10k 10 n \rightarrow $g(x) = x^2$. In R2, an `1 ball at the origin is $\{(x, y) : |x| + |y| < 1\}$. If c > 0 then $c \in P$ and it follows from ii) that $bc - ac = (b - a)c \in P$, i.e., bc > ac. Hence by the Comparison Theorem, f is improperly integrable on [a, b). If Ry = x and Qz = 2z, then R = xy + f(x, z) and $Q = z^2 + g(x, y)$. c) Since any number is either algebraic or transcendental, R is aL(x) = 1., x2n-2 } then Mk (f) = 1 for all odd k, M2 (f) = 1, and Mk (f) = 0 for all even k > 2. Conversely, suppose E \cap Br (a) | as nonempty for every r > 0. Similarly, $[i \nabla \cdot (F \times G) = \nabla \cdot det [F1 G1 j F2 G2] k F3] = (\nabla \times F) \cdot G - (\nabla \times G) \cdot F$. Thus limx $\rightarrow 0$ tan(1/x) does not exist. Since sup E \in / E, we also have x1 < sup E. $\partial g1 / \partial xn$ (a) | ... b) Fix $x \in R$. Hence Z 2 lim log(an) = n $\rightarrow \infty$ 2 log x dx = (x log x - x - = log 1 1 R2 1 log x dx. 11.6.9. Suppose Fz (a, b, c) 6= 0. 5.1.6. Let m be the number of points in E. Since the series is identically zero when x = 0, π , 2π , it converges everywhere on [0, 2π]. dxn j=1 we have by Stokes's Theorem that Z $\omega = 2 \partial Q$ n Z X j=1 (-1)j-1 xj dx1 . , xn } of [a, b]

such that U (f, P) - L(f, P) < 2 c ε . 14.2.6. Let Z $2\pi \Delta N$ (x) = f (x - t) φN (t) dt - f (x). Since $2k p \le k=1$ 1/ $k \sqrt{+1}$ converges if and $\sqrt{\sqrt{k} 2p + 1} + k p \le 2k (2p + 1) \le 1/(2k p + 1)$ Binomial Formula, the inductive hypothesis, and what we just proved, $a^{2n-1} + b^{2n-1} = (a^{n-1} + 2)^2 + (2a^{n-1} + 2)^2 = a^{2n-1} + (2a^{n-1} + 2)^$ c_2n . Thus $f(C \setminus B) = \{0\} = \{0\} = f(C) \setminus f(B)$. By using the change of variables u = x + h, du = dx, and a sum angle formula, we have $Z \mid \pi ak$ (f(x + h)) = $f(x + h) \cos kx \, dx \, \pi - \pi Z \pi 1 = f(u)(\cos ku \cos kh + \sin ku \sin kh) \, du \pi - \pi = ak$ (f(x + h)) = $f(x + h) \cos kx \, dx \pi - \pi Z \pi 1 = f(u)(\cos ku \cos kh + \sin ku \sin kh) \, du \pi - \pi = ak$ (f(x + h)) = $f(x + h) \cos kx \, dx \pi - \pi Z \pi 1 = f(u)(\cos ku \cos kh + \sin ku \sin kh) \, du \pi - \pi = ak$ (f(x + h)) = $f(x + h) \cos kx \, dx \pi - \pi Z \pi 1 = f(u)(\cos ku \cos kh + \sin ku \sin kh) \, du \pi - \pi = ak$ (f(x + h)) = $f(x + h) \cos kx \, dx \pi - \pi Z \pi 1 = f(u)(\cos ku \cos kh + \sin ku \sin kh) \, du \pi - \pi = ak$ (f(x + h)) = $f(x + h) \cos kx \, dx \pi - \pi Z \pi 1 = f(u)(\cos ku \cos kh + \sin ku \sin kh) \, du \pi - \pi = ak$ (f(x + h)) = $f(x + h) \cos kx \, dx \pi - \pi Z \pi 1 = f(u)(\cos ku \cos kh + \sin ku \sin kh) \, du \pi - \pi = ak$ (f(x + h)) = $f(x + h) \cos kx \, dx \pi - \pi Z \pi 1 = f(u)(\cos ku \cos kh + \sin ku \sin kh) \, du \pi - \pi = ak$ (f(x + h)) = $f(x + h) \cos kx \, dx \pi - \pi Z \pi 1 = f(u)(\cos ku \cos kh + \sin ku \sin kh) \, du \pi - \pi = ak$ (f(x + h)) = $f(x + h) \cos kx \, dx \pi - \pi Z \pi 1 = f(u)(\cos ku \cos kh + \sin ku \sin kh) \, du \pi - \pi = ak$ (f(x + h)) = $f(x + h) \cos kx \, dx \pi - \pi Z \pi 1 = f(u)(\cos ku \cos kh + \sin ku \sin kh) \, du \pi - \pi = ak$ (f(x + h)) = $f(x + h) \cos kx \, dx \pi - \pi Z \pi 1 = f(u)(\cos ku \cos kh + \sin ku \sin kh) \, du \pi - \pi = ak$ (f(x + h)) = $f(x + h) \cos kx \, dx \pi - \pi Z \pi 1 = f(u)(\cos ku \cos kh + \sin ku \sin kh) \, du \pi - \pi = ak$ (f(x + h)) = $f(x + h) \cos kx \, dx \pi - \pi Z \pi 1 = f(u)(\cos ku \cos kh + \sin ku \sin kh) \, du \pi - \pi = ak$ (f(x + h)) = $f(x + h) \cos kx \, dx \pi - \pi Z \pi 1 = f(u)(\cos ku \cos kh + \sin ku \sin kh) \, du \pi - \pi Z \pi 1 = f(u)(\cos ku \cos kh + \sin ku \sin kh) \, du \pi - \pi Z \pi 1 = f(u)(\cos ku \cos kh + \sin ku \sin kh) \, du \pi - \pi Z \pi 1 = f(u)(\cos ku \cos kh + \sin ku \sin kh) \, du \pi - \pi Z \pi 1 = f(u)(\cos ku \cos kh + \sin ku \sin kh) \, du \pi - \pi Z \pi 1 = f(u)(\cos ku \cos kh + \sin ku \sin kh) \, du \pi - \pi Z \pi 1 = f(u)(\cos ku \cos kh + \sin ku \sin kh) \, du \pi - \pi Z \pi 1 = f(u)(\cos ku \cos kh + \sin ku \sin kh) \, du \pi - \pi Z \pi 1 = f(u)(\cos ku \cos kh + \sin ku \sin kh) \, du \pi - \pi Z \pi 1 = f(u)(\cos ku \cos kh + \sin ku \sin kh) \, du \pi - \pi Z \pi 1 = f(u)(\cos ku \cos kh + \sin ku \sin kh) \, du \pi - \pi Z \pi 1 = f(u)(\cos ku$ $aq = aq \cdot (ar-q-1) < ax0 (a1/N-1) < \epsilon$. By hypothesis, g and f are differentiable on I \ {a}, the limit of g/f is of the form 0/0 or ∞/∞ , and the limit of g/f is of the form 0/0 or ∞/∞ , and the limit of g/f is of the form 0/0 or ∞/∞ , and the limit of g/f is of the form 0/0 or ∞/∞ , and the limit of g/f is of the form 0/0 or ∞/∞ , and the limit of g/f is of the form 0/0 or ∞/∞ , and the limit of g/f is of the form 0/0 or ∞/∞ , and the limit of g 0 /f 0 is zero. Since q < x and x < y imply q < y, it is clear by definition that $ax \le ay \cdot 4.4.3$. By Taylor's Formula, there is a c between x and 0 such that μ ¶ xn ec xn+1 ex - 1 + x + · · · + = . By part b), given any $x \in E$ the set Ux which can be polygonally connected to x through E is open. d) Since $\cdot 0 \mid Df(x, y, z) = 0 \mid xz \mid 2y \mid xy \mid -1$, and $D(f \cdot g)(x, y, z) = [y \mid 2(z + 1) \mid 2xyz + 2xy \mid -2yz \mid y \mid 2(x - 1)]$. Then f satisfies the hypotheses with $\alpha = 1$ and I = [-1, 1], but f is not differentiable at $0 \in [-1, 1]$. b) 0 = fx = 2x + 2y and 0 = fy = 2x + 6y imply x = y = 0. By Example 2.21, |I|1/n and $(b - a)1/n \rightarrow 1$ as $n \rightarrow \infty$. $\sqrt[4]{\sqrt[4]{12.6.a + b - 2}}$ ab = $(a - b)2 \ge 0$ for all $a, b \in [0, \infty)$. Choose N1 so large that $n \ge N1$ and $x \in E$ imply $|fn(x) - f(x)| < \frac{2}{3M}$. Choose so small that N sn < 2. If $n \ge N$ and $x \in E$ imply $|fn(x) - f(x)| < \frac{2}{3M}$. [a, b] then $\frac{1}{2}$ [fn (x)g(x) - f (x)g(x) = [fn (x) - f (x)] |g(x)| 0 choose N so large that $n \ge N$ and $x \in E$ imply $[fn (x) - f (x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{2, |\alpha| + 1\}}$ and $[gn (x) - g(x)] < \frac{2}{\max\{$ integral of φN is 1, notice that Z $2\pi \Delta N$ (x) = (f (x - t) - f (x)) φN (t) dt. Since fn \rightarrow f uniformly and n1/n \rightarrow 1, as n $\rightarrow \infty$, choose an N so large that n $\geq N$ implies |fn(x) - f(x)| 1/r for some r < R. If distinct points x1, . Thus the directional derivatives of f exist. Then g is periodic and of bounded variation on $[-\pi, \pi]$ and continuous on any interval [a, b] \subset $(-\pi, \pi)$. Therefore, $(f(h, k) - f(0, 0) - \nabla f(0, 0) \cdot (h, k))/k(h, k)k \rightarrow 0$ as $(h, k) \rightarrow (0, 0)$. If Qk is a cube of side s which contains xk then N X |Qk| = N sn <². d) Since $(x/3x)0 = (3x \cdot 1 - x \log 3)/3x < 0$ for $x \ge 1$, k/3k is decreasing. Then $n \ge N$ implies $|(f \cdot fn)(x) - f(2)(x)| = |f(x)| |fn(x) - f(x)| < \varepsilon$ for $x \in [a, b]$, i.e., $f \cdot fn \rightarrow f(2)$. uniformly on [a, b] as $n \rightarrow \infty$. By the Archimedean Principle, E is nonempty. 2 Therefore, $|xn+1 - xn| = |f(xn)/f(xn)| \le M |xn - xn - 1|P/20 < r02n+1 \le r0n+2$. Suppose $x_1 < x_2 < \cdots < x_n$ in E have been chosen so that sup $E - 1/n < x_n < \sup E$. Then $Ac := Y \setminus A$ is open in Y, so by Corollary 10.59, A0 := f - 1 (Ac) $\cap E$ is relatively open in E. Let $\varepsilon > 0$. 0. d) Let x = t2, y = t2, z = t, and let t run from 1 to 0. 4 d) Set $\delta = x/k$. Since c = 0 implies a = b = 0 and K has no tangent plane at the origin, there are no tangent plane at the origin, the o and suppose first that $x \ge 0$. Thus f/g is locally integrable and $|f|/|g| \le h/\epsilon$ holds everywhere on (a, b). If u satisfies the heat equation, then wxx - wt = -r < 0 on V. But by $\sqrt{(-1, 1)}$ with (arcsin x) = 1/0.2.2 trigonometry, cos y = $1 - x \cdot h \rightarrow 0 \rho h \rho$ h $\rightarrow 0 h fxj$ (ρx) = lim Thus by the Chain Rule, homogeneity, and the Power Rule, $\rho k - 1$ n X j=1 x j fxj (x) = n $X_{j=1} x_j fx_j (\rho x) = d d k (f(\rho x)) = (\rho f(x)) = k\rho k - 1 f(x)$. The sets $Am := \{ nk : k \in N \text{ and } - 2m \le k \le 2m \}$ are finite, hence at most countable. Fix $x \in [a, b]$. If E is sequentially compact, then by part b), E is closed and bounded. Let $f(x) = x\alpha / ex$. |t1 - t0| |t2 - t0| kbk2 Thus θ is 0 or π . Hence by Jensen's inequality, $\P \P 1/q \mu Z |f(x)| p dx \le 0.1 \P 1/q \rho (|f(x)| p dx = 0.1 \P 1/q \rho (|f(x)$ (x)|p dx 0 $\mu Z = 1$ $\P 1/q |f(x)|q dx$. On the
other hand, given $x \in E$, there is a sequence $xj \in Br$ such that $xj \to x$ as $j \to \infty$. 2 2 2 Thus $\{xn \}$ is increasing and bounded above, so $xn \to x$. 11.5.5. Let F be defined as in the proof of Theorem 11.35. Set f(y) = j=1 fxj (y). Let $xn \to \infty$ and set $yn = xn + \delta/2$. If 0 < x1 < x2 then Z x2 dt $x^2 - x^1 L(x^2) - L(x^1) = >$ > 0. Parameterize C1 by $\varphi 1$ (t) = (0, -t, 1 + 2t), I1 = [-1/2, 0]; C2 by $\varphi 2$ (t) = (t, 0, 1 - t), I2 = [0, 1]; and C3 by $\varphi 3$ (t) = (-t, (1 + t)/2, 0), I3 = [-1, 0]. Taking the limit of xn-1 + 1 - 1 = xn we see that a2 + a = 0, i.e., a = -1, 0. 12.2.2. If x \in [0, 1] \times \cdots \times [0, 1], then $0 \le x_{2j} \le 1$. d) DR g(x) = DR f(x) + 1/n > DR f(x) \ge 0 for all but countably many $x \in [0, 1] \times \cdots \times [0, 1]$, then $0 \le x_{2j} \le 1$. d) DR g(x) = DR f(x) + 1/n > DR f(x) \ge 0 for all but countably many $x \in [0, 1] \times \cdots \times [0, 1]$. (a, b). Thus $(x, y) \in R$ and it follows from Theorem 10.16 that R is closed. 2 Therefore, -2|x + 1| dx = 1/2(1) + 1/2(9) = 5. We check the boundary in four pieces. b) Suppose $\partial E = E$. 11.3.2. a) Since $\nabla f = (2x, 2y) = (2, -2)$ at (1, -1), and the equation of the tangent plane is $z = f(1, -1) + \nabla f(1, -1)$ $b_2 = 1.4.4$ Taylor's Theorem and l'H^o opital's Rule. Notice that $g(U \cap V) = f(U \cap$ = f (1). Differentiating term by term, we obtain E 0 (x) = solves the initial value problem y 0 - y = 0, y(0) = 1. Then U, V are nonempty open sets, U \cap V = \emptyset and U \cup V = E. parametrization of the surface φx (Ex). Thus E is sequentially compact by definition. 8.2.3. All we have to do is find two lines which lie in parallel planes. Then $3(n + 1)^2 + 3(n +$ 1) + 1 = $3n^2 + 3n + 1 + 6n + 6 \le 2 \cdot 3n + 6(n + 1)$. Taking the limit of this inequality as $^2 \rightarrow 0$, we obtain (U) a g(x) dx \le Rb Rb Rb f (x) dx. P ∞ Thus it follows that Sf is uniformly P ∞ Abel summable to f. 0 b) This boundary is a triangle (oriented in the clockwise direction when viewed from the origin) consisting of three line segments, C1 (which lies in the yz plane), C2 (which lies in the xz plane), C3 (which lies in the xz plane). c) By Fej er's Theorem, $\sigma N f \rightarrow f$ uniformly. Hence, 0 < 1 - x < 2 and $x^2 - 1 < 0$. $|x^1 - x^0| = -0 f(x^0) - 2074$ Copyright © 2010 Pearson Education, Inc. On the other hand, since f(a) is a local minimum we see by Taylor's Formula that $0 \le f(a + h) - f(a) = D(2) f(c)(h)$ () for $c \in L(a; a + h)$ and khk sufficiently small. Hence it follows from Dini's Theorem that $(1 - x/k)k \rightarrow e - x$ as $k \rightarrow \infty$ uniformly on any compact subset of R. 4.3.8. By the Mean Value Theorem, 0 < f(x1) - f(x2) = (x1 - x2)f 0(c1) for some x1 < c1 < x2 and 0 < f(x3) - f(x2) = (x3 - x2)f 0(c2) for some x2 < c2 < x3. $0 - \infty = \infty$ is not indeterminate. Hence by Theorem 2.29, xn is Cauchy. 90 Copyright © 2010 Pearson Education, Inc. Hence 1/M < 1/|g(x)| < 20 for all $x \in [a, b]$ and it follows that 1/g is bounded on [a, b] and it follows that 1/g is bounded on [a, b] and it follows that 1/g is bounded on [a, b] and it follows that 1/g is bounded on [a, b] and it follows that 1/g is bounded on [a, b]. Thus by the Monotone Convergence Theorem, $xn \to x$ for some $x \in \mathbb{R}$. b) If E = 1/20 for all $x \in [a, b]$, i.e., 1/gn is defined and bounded on [a, b]. $\alpha \in A$ Ea is empty or contains a single point, then E is connected by definition. Define f and g on [0, 1] by: f (x) = 1 when x \in Q and f (x) = 0 when x \in /Q, and g(x) = 1 + (x)-1. Also notice that n X |gk (x)| = k=1 Hence for each j, hj (x) = x22 + \cdots + x2n a^2 - x21 a - x1 = = . In particular, Z y Z x f (x, y) = Q(x, v) dv + P (u, 0) du. Then (y - ², y + ²) \cap $E \subset \{x1, . Thus (V, h) \text{ is a chart in } A \text{ which satisfies } h(V) = B1 (0) \text{ and } h(x) = 0. \text{ Hence } n - n - 1 - X - X - ak bk - \leq 2M |an| + 2M |ak+1 - ak|, \rho(xk, a)\}, \text{ then by hypothesis there is an } xk+1 \in E \cap Bs (a). 0 \text{ Similarly, } Z \cap Z \cap Z \cap Z \cap A + Q \cap A +$ x) dx + Q(1 - y, y, 0) dy + 0 1 P (x, 1 - x, 0) dx. If ak = 1/k 2 and bk = 1/k, then ak /bk \rightarrow 0 as k $\rightarrow \infty$ and k=1 ak converges, but k=1 bk does not. d) The closure is R, the interior is \emptyset , the boundary is R. Therefore, Sf = Sg, i.e., S is the Fourier series of f. π 4k 2 - 1 k=1 P2n -1 Hence it follows from parts a) and b) that k=1 $|\varphi(xk) - \varphi(xk-1)| > 2$ $\log(2n)/\pi$ for all $n \in N$. Since ta + x0 = ua + x0 implies t = u, C is simple. $\sqrt{7.3.9}$. The coefficients of this power series are given by ak = ((-1)k + 4)-k. ak = 1/k is strictly decreasing to 0 but k=1 1/k diverges. By the Product Rule and the Fundamental Theorem of Calculus, Z b Z b (f 0 (x)g(x) + f (x)g 0 (x)) dx = a (f (x)g(x))0, dx = f (b)g(b) - f (a)g(a)verified the first identity. c) Since p > 1, choose $\alpha > 0$ such that $p - \alpha > 1$. $n \rightarrow \infty$ $n \rightarrow \infty$ To obtain the reverse inequality, notice by the Approximation Property that for each $n \in N$ there is a jn > n such that $j - \alpha > 1$. $n \rightarrow \infty$ $n \rightarrow \infty$ To obtain the reverse inequality, notice by the Approximation Property that for each $n \in N$ there is a jn > n such that $j - \alpha > 1$. $n \rightarrow \infty$ $n \rightarrow \infty$ To obtain the reverse inequality, notice by the Approximation Property that for each $n \in N$ there is a jn > n such that $j - \alpha > 1$. $n \rightarrow \infty$ $n \rightarrow \infty$ To obtain the reverse inequality, notice by the Approximation Property that for each $n \in N$ there is a jn > n such that j = 0. connection with, or arising out of, the furnishing, performance, or use of these programs. Thus $\psi(u) \ge 0$ for all t > x. By the argument of the sequence xk chosen to approximate x. $|\partial g1 / \partial x1 (a)| \ge 0$ for all t > x. By the argument of Theorem 3.40, this definition is independent of the sequence xk chosen to approximate x. $|\partial g1 / \partial x1 (a)| \ge 0$ for all t > x. By the argument of Theorem 3.40, this definition is independent of the sequence xk chosen to approximate x. $|\partial g1 / \partial x1 (a)| \ge 0$ for all t > x. By the argument of Theorem 3.40, this definition is independent of the sequence xk chosen to approximate x. $|\partial g1 / \partial x1 (a)| \ge 0$ for all t > x. By the argument of Theorem 3.40, this definition is independent of the sequence xk chosen to approximate x. $|\partial g1 / \partial x1 (a)| \ge 0$ for all t > x. By the argument of Theorem 3.40, this definition is independent of the sequence xk chosen to approximate x. $|\partial g1 / \partial x1 (a)| \ge 0$ for all t > x. By the argument of Theorem 3.40, this definition is independent of the sequence xk chosen to approximate x. $|\partial g1 / \partial x1 (a)| \ge 0$ for all t > x. By the argument of Theorem 3.40, this definition is independent of the sequence xk chosen to approximate x. $|\partial g1 / \partial x1 (a)| \ge 0$ for all t > x. By the argument of Theorem 3.40, this definition is independent of the sequence xk chosen to approximate x. $|\partial g1 / \partial x1 (a)| \ge 0$ for all t > x. By the argument of Theorem 3.40, this definition is independent of the sequence xk chosen to approximate x. $|\partial g1 / \partial x1 (a)| \ge 0$ for all t > x. By the argument of Theorem 3.40, this definition is independent of the sequence xk chosen to approximate x. $|\partial g1 / \partial x1 (a)| \ge 0$ for all t > x. By the argument of Theorem 3.40, this definition is independent of the sequence xk chosen to approximate x. $|\partial g1 / \partial x1 (a)| \ge 0$ for all t > x. By the argument of Theorem 3.40, this definition to approximate xk chosen to appr contradicts the that H1 is nonempty. Thus by part a), a cannot be a cluster point of R. 5.4.7. a) Suppose L > 0.R If f (x) \rightarrow LRas x $\rightarrow \infty$, then choose N \in N such that f (x) > L/2 for x \geq N. b) Suppose that x < y. Hence ey/k cos(y/k) \rightarrow 1 uniformly on E as k $\rightarrow \infty$. Thus D2 F (x0) \geq 0. We claim that L = 0. Thus by Taylor's Formula, $\sqrt{x} + \sqrt{y} = 3 + x - 1 y - 4$ $(x - 1)2(y - 4)2(x - 1)3(y - 4)3 \sqrt{4} - - + + 2486416c516d5$ for some $(c, d) \in L((x, y); (1, 4))$. b) If E is a bounded infinite set, then it contains distinct points x1, x2, . Since f 0 (x) exists and is nonzero for all $x \in (0, \infty)$, it follows from Theorem 4.33 that f - 1 is differentiable on $(0, \infty)$ and (f - 1)0(x) = 1/f 0 (f - 1)(x). It is closed because its a bounded infinite set, then it contains distinct points x1, x2, . Since f 0 (x) exists and is nonzero for all $x \in (0, \infty)$, it follows from Theorem 4.33 that f - 1 is differentiable on $(0, \infty)$ and (f - 1)0(x) = 1/f 0 (f - 1)(x). complement {(x, y) : 2 x + 4y 2 > 1} is open. Iterating what we just proved, using the fact that the limit of the product is the product of the limit, we see that $g(x) \rightarrow f1$ (a1) · · · fn (an) as $x \rightarrow a$. a) Let $s = \hat{(t)}$. Then Z Z Z Z (x + y + z 3) ds = (x + y + z 3) ds + (x +t) dt + t dt = + +2 = . The converse follows similarly integrating term by term. c) Since (-1)n+1 + (-1)n/n = -1 + 1/n when n is odd, lim supn $\rightarrow \infty$ xn = 1 and lim inf $n\rightarrow\infty$ xn = -1. If x = 1 and z = 0, then 3 + y = 1 and z = 0, then 3 + y = 1 and z = 0, then 3 + y = 1 and z = 0, then 3 + y = 1 and z = 0, then 3 + y = 1 and z = 0, then 3 + y = 1 and z = 0, then 3 + y = 1 and z = 0, then 3 + y = 1 and z = 0, then 3 + y = 1 and z = 0, then 3 + y = 1 and z = 0, then 3 + y
= 1 and z = 0, then 3 + y = 1 and z 2010 Pearson Education, Inc. Thus Z 2π Z π /2 A(S) = 0 a2 | cos v| dv du = 4π a2. Now P (x) – P (y) = (x – y)Q(x, y) where Q(x, y) = an (xn-1 + ··· + y n-1) + ··· + a2 (x + y) + a1. When p = 1, the integral is log x $^{-0}$ which diverges. If c = max{x-², a} and d = min{x+², b} then c < d and (x-², x+²)∩[a, b) ⊇ (c, d). The inequality holds if 4k 2 > 4k 2 -1, i.e., if 1 > 0. c) Let $ak = (2 \cdot 4 \cdot 5 \cdot 2 \cdot 0 \cdot a)$ True. Thus M is not an upper bound for one of the sets A or B, a contradiction. ak a a a d) The formula holds for n = 1. If one of the pair x, y belongs to [0, N] and the other does not, for example, if $x \in [0, N]$ and $y \in /[0, N]$, then $|x - N| \le |x - y| < \delta$. C1 On C2, x = 2 hence dx = 0, and $0 \ge Z \ge 3 y dx + x dy$ = 2 dy = 4. Thus by the calculations in part b), choose n = 11., 8 but is a perfect square, namely 144 = 122, when n = 9. Then mj (|f|) $\geq nj$ (|f|, P) $\geq mj$ (|f|) $(x_j - x_j - 1) \geq ^2(2\delta) > 0$. And, if 1 , then the maximum is 1 and the minimum is <math>n(p-2)/p. Thus $\phi(t)$ is increasing for t > x, so $e\phi(k) = (1 - x/k)k$ e-x as $k \to \infty$ for all $x \in R$. j=1 Thus $f(H) \subset SN$ j=1 Vaj, i.e., f(H) is compact. By induction, there are infinitely many points in $E \cap Br(a)$. Thus $k \log k \ge k$ p for $k \ge 3$, and it follows from ∞ the Comparison Test that k=1 $k - \log k$ converges. ϵ) By definition, $|x \le 0| < 3x + 2 f(x) = x+2 0 2$. If k = 0 then nq = 1 is a root of the polynomial x - 1. N }, ψ and τ are C 1 and τ 0 > 0. other hand, if fxy (a, b) = 0 then either fxx (a, b) 6= 0 or fyy (a, b) 6= 0 or fyy (a, b) 6= 0 or fyy (a, b) 6= 0 or fy (a, means, electronic, mechanical, photocopying, recording, or otherwise, without the prior written permission of the publisher. If C(x, y) ends in a vertical line segment, then by part a), fy = Q. Thus xn + 1 = 2xn yn < xn. Since $|(1/\alpha)R_j| = (1/\alpha)n |R_j|$, it follows that X X Vol $(\alpha E) = \inf |\alpha R_j| = \alpha n$ Vol (E). Hence $x \in \partial E$. Hence $|k=1(1-\alpha)n|R_j|$, it follows that X X Vol $(\alpha E) = \inf |\alpha R_j| = \alpha n$ Vol (E). $\cos(1/k)| \le 0 |f(x)| dx \le 2$. The graph of f1 has no tangent at x = 0 because it oscillates between y = x and y = -x. In particular, $\{U\alpha\}_{\alpha \in A0}$ is a finite subcovering of E which covers H. Thus the original series diverges by the Divergence Test. is closed as Exercise 9.4.4 says it should; $f - 1(-1, 1) = R \setminus \{x : x = (2k + 1)\pi/2, k \in Z\}$ is open as Theorem 9.26 says it should; f - 1 [-1, 1] = R is closed as Exercise 9.4.4 says it should. 24m 24m Similarly, s(f; Gm) = (24m - 23m + 1 + 22m)/24m Consequently, s(f; Gm) = (24m - 23m + 1 + 22m)is a t \in E such that fx0 j (t) > 0 PN Thus f 0 (t) = k=1 fx0 k (t) \ge fx0 j (t) > 0. In the same way, we can prove that a0 (f (x + h) - f (x - h)) = -2ak (f) sin kh for $k \in N$. Now n2 + 3n is a perfect square when n = 1 but if n > 1 then (n + 1)2 = n2 + 2n + 1 < n2 + 2n + n = n22 + 3n = < n2 + 4n + 4 = (n + 2)2. When p = 1, the integral is log x 1 which diverges. Since k > sup E, k cannot belong to E, i.e., a < k. Since f (h, k) – f (0, 0) · (h, k) h4 + k4 = 2 ≤ 2(h2 + k2)3/2 - $\alpha \rightarrow 0$ k(h, k)k (h + k2) α + 1/2 as (h, k) → (0, 0), f is also differentiable at (0, 0). In particular, f (K) ⊂ {f (x1), . If x ∈ K, then x ∈ Ixj for some j, so f (x) ≥ 0 + · · · + fxj (x) + · · · + 0 > 0. Therefore $V = \bigcup x \in V B^2(x)$ as required. 4.1.4. Since $|f(x)| \le |x|\alpha$ for all $x \in I$, f(0) = 0. Thus this function has no limit as $(x, y) \to (0, 0)$. Since x = 1/2 by Theorem 2.36. Let $^2 > 0$ and choose $M \times n \to \infty$, choose $n \in X$ such that $n \ge N$ implies x = 1/2 by Theorem 2.36. Let $^2 > 0$ and choose $M \times n \to \infty$, choose $n \in X$ such that $n \ge N$ implies x = 1/2 by Theorem 2.36. Let $^2 > 0$ and choose $M \times n \to \infty$, choose $N \in N$ such that $n \ge N$ implies x = 1/2 by Theorem 2.36. Let $^2 > 0$ and choose $M \times n \to \infty$. $(xn) \rightarrow L$ as $n \rightarrow \infty$. 130 Copyright © 2010 Pearson Education, Inc. $\partial xn \partial x1 \partial xn - 1$ Fx1 Fx2 Fxn 11.6.8. By Theorem C.5, $\partial f1 / \partial x$ Df = $\partial f2 / \partial x \partial f1 / \partial y$ Af2 / $\partial y - \partial f2 / \partial y$, implies (Df) -1 = 1 $\Delta f \cdot \partial f2 / \partial y - \partial f2 / \partial x \partial f1 / \partial y$. Hence by Theorem 4.17i, f increases on [4, ∞). Since f is a uniform limit of continuous functions, f is continuous on [a, b]. Since A is nonempty, it follows from the Completeness Axiom that A has a supremum. Let $k \ge N$ and $x \in R$. Therefore, c f (x) $dx \ge {}^{2}0$ (d - c) > 0, a contradiction. 1.5.4. Suppose x belongs to the left side of (16), i.e., $x \in X$ and $x \in / \cap \alpha \in A$ Ea . $j \ge n k \ge n$ Taking the limit of this inequality as $n \to \infty$ establishes a). Hence DR f (x0) = limh $\rightarrow 0+$ (f (x0 + h) - f (x0))/h $\le 0.9.4.10$. $\mu \|k^-ak+1^-2p^- \equiv 2p \ k+1 \rightarrow ak^-k \ e \ P_{\infty} \ p \ as \ k \rightarrow \infty$, by L'H'opital's Rule, k=1 2kp k!/k k converges absolutely when $\sqrt{2} < e$, i.e., when p < log2 (e), and diverges when p > log2 (e). Finally, if x < y and 0 < b < 1, then -x > -y and by b), bx = (1/b) - x > (1/b) - y = by. 14.5.3. If SN \rightarrow f and TN \rightarrow f as N $\rightarrow \infty$, then S - T is a trigonometric series which converges to zero. All rights reserved. $h \rightarrow 0 h \rightarrow 0 h$ Thus f 0 (0) = 0 exists. Then f takes [-1, 1] onto [-1, 1] and f (0) = 1, but f -1 (f (0)) = f -1 (1) = {0, 1}. Therefore, this series converges if and only if p > 1. b) By the Squeeze Theorem, $x \cos(1/x^2) \rightarrow 0$ as $x \rightarrow 0$. 2x x 30 Copyright © 2010 Pearson Education, Inc. x+1 c) Let E = B2 (0, 0) B1 (0, 0). By assumption vi), $0 \le \cos x \le \sin x/x \le 1$. 46 Copyright © 2010 Pearson Education, Inc. Let $c := \min\{r0, s0\}$ and $d := \max\{2r, 2s\}$. If E contains two points, say a, b, then a, $b \in E\alpha$ for every $\alpha \in A$. 8.2.4. a) The columns of B are T (e1) = (0, 1, 1, 1), T (e2) = (0, 1, 0, 1), T (e3) = (0, 0, 0, 1). 1 5.3.3. a) If $x = \tan \theta$ then dx = sec2 θ d θ so Z 1 Z $\pi/4$ x3 f (x2 + 1) dx = 0 tan3 θ sec2 θ (sec2 θ) d
θ . c) Since cos2 x - sin2 x = cos(2x), it follows from Example 7.44 that cos2 x - sin2 x = ∞ X (-1)k (2x)2k k=0 (cos2 x - sin2 x = cos(2x), it follows from Example 7.44 that cos2 x - sin2 x = cos(2x), it follows from Example 7.44 that cos2 x - sin2 x = ∞ X (-1)k (2x)2k k=0 (cos2 x - sin2 x = cos(2x), it follows from Example 7.44 that cos2 x - sin2 x = ∞ X (-1)k (2x)2k k=0 (cos2 x - sin2 x = cos(2x), it follows from Example 7.44 that cos2 x - sin2 x = ∞ X (-1)k (2x)2k k=0 (cos2 x - sin2 x = cos(2x), it follows from Example 7.44 that cos 2 x - sin2 x = cos(2x), it follows from Example 7.44 that cos 2 x - sin2 x = cos(2x), it follows from Example 7.44 that co prove the left-most inequality, repeat the steps above, using part a) in place of Remark 6.22i, but with infimum in place of supremum and r1 < r in place of r0 > r, proves part c). Therefore, the solution is $\sqrt{(1/3, (3 - 1)/2)} \cup (1, \infty)$. If x, $a \in (0, 1)$ and $|x - a| < \delta$, then $|f(x) - f(a)| \le |x(\sin 2x - \sin 2a)| + |(x - a) \sin 2a| \le 2|\sin(x - a)| + |x - a| \le 3|x - (1/3, (3 - 1)/2) \cup (1, \infty)$. $a| < 3 \epsilon = \epsilon$. b) The ratio of consecutive terms of this series is (2k + 1)/(2k + 4) which converges to 1 as $k \to \infty$. A similar argument works for the case t < t0. Since both sets are nonempty and $\rho(x, y)$ is bounded below by 0, the dist (A, B) exists and is finite. j=1 Since each Braj (aj) contains xk for only finitely many k's and $x \in E$. 10.1.9. a) Repeat the argument of Remark 10.9 with $\varepsilon/2$ in place of ε . Since $F = \nabla f$, f must be C 2 on E, hence Qx = fyx = fxy = Py on E. Thus $1/yn \rightarrow 0$ as $n \rightarrow \infty$ and $xn \ge 1$ for all $n \in N$. Thus 5 < 2x + 3 < 7 and $0 < x - 1 < \delta$. Choose $N \in N$ such that $|ak| \le 1/2$ for $k \ge N$. If $\lambda \sqrt{=0}$ then x = 2y and the constraint implies $y = \pm 1/5$. Thus T(1, 0, 0) = T(0, -1, 1) + T(1, 1, -1) = (1, 0) + (1, 2) = (2, 2). dt = 3x2 f(x3) - 2xf(x2). PN0 7.1.10., BN, where $u(x, t) \ge -^2$ on $U := \bigcup N j=1$ Bj. Since $-1/x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $f = -1/x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $f = -1/x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $f = -1/x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $f = -1/x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $f = -1/x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $f = -1/x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $f = -1/x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $f = -1/x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $f = -1/x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $f = -1/x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $f = -1/x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $f = -1/x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $f = -1/x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $f = -1/x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $f = -1/x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $f = -1/x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $f = -1/x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $f = -1/x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $f = -1/x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $f = -1/x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $f = -1/x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $f = -1/x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $f = -1/x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $f = -1/x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $f = -1/x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $f = -1/x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $f = -1/x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $f = -1/x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $f = -1/x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $f = -1/x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $f = -1/x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $f = -1/x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $f = -1/x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $f = -1/x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $f = -1/x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $f = -1/x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $f = -1/x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $x \rightarrow -\infty$ as $x \rightarrow 0+$ implies that $x \rightarrow -\infty$ as $x \rightarrow -\infty$ a Inc. b) f - 1 (-1, 1) = [0, 1) is relatively open in $[0, \infty)$, the domain of f as Theorem 9.26 says it should; f - 1 [-1, 1] = [0, 1] is relatively closed in $[0, \infty)$ as Exercise 9.4.5a says it should. 11.3.3. By Theorem 11.22, the normal direction is given by (-2x, -2y, 1). We will choose two planes with normal (0, 0, 1), e.g., z = 0 and z = 1., fm (x) (-1, 1) = [0, 1] is relatively closed in $[0, \infty)$ as Exercise 9.4.5a says it should. 11.3.3. By Theorem 11.22, the normal direction is given by (-2x, -2y, 1). We will choose two planes with normal (0, 0, 1), e.g., z = 0 and z = 1., fm (x) (-2x, -2y, 1). the angle between this longest side and the "first" side of Q, y := (a - b, 0, . On the other hand, if $y \in /E$, then fx0 j (y) = 0 for all j. By Remark 1.41, the unit interval (0, 1) is uncountable, hence {x : x $\in (0, 1)$ } is an uncountable collection of pairwise disjoint nonempty sets which covers the unit interval (0, 1). , fxxxx = y 4 exy, fxxxy = (3y 2 + xy 3) exy $f(x) = (2+4xy + x^2y^2) exy$, fxyyy = $(3x^2 + x^3y) exy$, and fyyy = $x^4 exy$. If f (x0) 6 = 0 for some x0 \in [a, b], then by part a), a |f(x)| dx > 0, a contradiction. Thus |f(x) - f(y)| \leq |f(x) - f(y)| \leq |f(x) - f(y)| and |g(x)-g(y)| $<^2/2$. u d) Using the substitution u = t/x, we obtain Z x Z xy Z y dt dt du L(xy) = + = L(x) + = L(x φ_x (Exj). c) The graph of f is a parabola whose absolute maximum is 1 at x = 1. If we set H := {(0, y) : -1 \le y \le 1} then E \cup H is closed and bounded, hence compact. k=1 (kq) Pn 6.2.9. If sn := k=1 ak converges then so does s2n+1. p (2k)! (2k (2p)!/(2k)!. Conversely, if $E \cap Bs$ (a) \ {a} is always nonempty for all s > 0 and r > 0 is given, choose $x1 \in E \cap Br$ (a). Thus f is strictly increasing, hence 1-1, and $\left(\left| \begin{array}{c} x - 2 \right| & x < 2 \\ x <$ (2k)!. Conversely, if |xn| < C for all $n \in N$, then xn is bounded above by C and below by -C. d) Set F(x, y, z) = x + y + z + g(x, y). 0 13.3 Surfaces. We have $2 |f(x) - L| = |x3 \sin(ex)| < \delta 3 \cdot 1 = \varepsilon$ for every x which satisfies $0 < |x| < \delta$. 8.3.9. Suppose E is closed and $a \in A$. E, but inf $x \in E$ kx - ak = 0. Hence by Stokes's Theorem and Theorem 7.10, ZZ I lim curl Fk · n d\sigma = lim Fk · T ds k $\rightarrow \infty \lambda = 0$ K · may suppose and Theorem 7.10, ZZ I lim curl Fk · n d\sigma = lim Fk · T ds k $\rightarrow \infty \lambda = 0$ K · may suppose for the other hand k = 0 2k = 2n - 1 by induction. dx d) Let u = x - t so du = dx. Hence E = A $\cap Y$, where A = Y \ U is closed in Y. We may suppose that $B = +\infty$. But s < r, so Bs (a) \subset Br (a). xn = 1/2 - 1/n is strictly increasing and $|xn| \le 1/2 < 1 + 1/n$, but $xn \to 1/2$ as $n \to \infty$. $x \in Ac$. Thus by the Ratio Test, this series converges when 1 (i.e., when -3 < x < -1) and diverges when $P \propto |x + 2| < \sqrt{|x + 2|} > 1$. y does not depend on the sequence of Exercise 3.1.6). a) Since F (a, b) = n X ∞ X |xj| < ∞ . d) Let x, y \in R. 3 13.2.4. a) Since
τ 0 (u) = $\delta > 0$, (ψ , J) and (φ , I) are orientation equivalent by Definition 13.18. Hence x0 \in (c, d). Thus ZZ Z 1Z 1 14 $\omega = (x, y, x4 + y 2) \cdot (4x3, -2y, 1) dx dy = -$. Then $0 = f 0 (x) = x\alpha - 1 (\alpha - x)/ex$ implies $x = \alpha$. Moreover, since $B = \infty$, we know that f 0 (x) cannot be zero for large x. 6.5.1. a) Let $f(x) = \pi/2 - \arctan x$. c) Let $x-1 = (2k+1)\pi/2$ for $k \in N$ and notice that $\{xn, xn-1, . Since f is increasing, L - 2 < f(x0) \le L$ for all x0 < x < b. If x > 0 and y > 0, then by the Additive Property, x + y > 0 and by the First Multiplicative Property, x + y > 0. Since f is 1-1 from A onto B0, it follows from Theorem 1.30 that x = f(f - 1(x)) = f(f - 1(y)) = y. If n = 1 then f 0(x0) = 1 for all $x0 \in \mathbb{R}$. When x = -1 it diverges by the Limit Comparison Test (compare it with 1/k). Since g is differentiable at a, $I / khk \rightarrow 0$ as $h \rightarrow 0$. Since g = -1 it diverges by the Limit Comparison Test (compare it with 1/k). Copyright © 2010 Pearson Education, Inc. k=0 b) Let $^2 > 0$ and choose N \in N such that $k \ge N$ implies $|\sigma k - L| < ^2$. Moreover, if $g(t) = t/(t^2 + x)$, $t \ge 1$, then $g(t) = t/(t^2 + x)$, $t \ge 1$, then $g(t) = t/(t^2 + x)$, $t \ge 1$, then $g(t) = t/(t^2 + x)$. This formula holds for = 1. R1 c) No. If f(x) = x, then -1 f (x) dx = 0 but f (x) 6= 0 for all x 6= 0. By hypothesis, h(a) > 0 and h(b) < 0. Set $f(t) = (\log t)$. t + x/(t + x) and g(t) = 1 + x/t - log t - x, for t > 1. d) 0 = fx = 2ax + by and 0 = fy = bx + 2cy imply (b2 - 4ac)y = 0, i.e., x = y = 0. Let φ k be defined on Ik so its graph forms a triangle R1 with base Ik and height 2k+1. We conclude that f is differentiable on R2 . 2.3.0. a) False. 95 Copyright © 2010 Pearson Education, Inc. Since f is convex, f (x) \leq y * . 2 b) By Definition 1.1, if $a \geq 0$ then a + = (a + a)/2 = a and if a < 0 then a + = (a + a)/2 = a and if a < 0 then a + = (a + a)/2 = a and if a < 0 then a + = (a + a)/2 = a and if a < 0 then a + = (a + a)/2 = a and if a < 0 then a + = (a + a)/2 = a. Thus we need only check whether $f[0,\infty)(0) = 0.3.3.5$. Since M - f(x) > 0, it follows from the Sign Preserving Property that there is an interval I centered at x0 such that M - f(x) > 0, i.e., f(x) < M for all $x \in I$. Then (1, 0) is a boundary point of E which does not belong to E. $\partial gm / \partial x1$ (a) . 0 b) If (φ, E) is the parameterization given in Example 13.31, then k $\varphi u \times \varphi v = 1$. $k(a^2 \cos u \cos^2 v, a^2 \sin v \cos v)k = a^2 |\cos v|$. Since sin(1/tk) = (-1)k, it is clear that $k\phi(tk) - \phi(tk+1)k \ge 2$ for each $k \in \mathbb{N}$. Then $\sqrt{\sqrt{\sqrt{Y}} + \frac{1}{k}} = 1/k^2 + \frac{1}{k} + \frac{1}{k}$ $\leq |f(x) - fN(x)| + |fN(x) - fN(y)| + |fN(y) - f(y)| < ^2$. Thus the integral converges if and only if p > 1. Thus nq is algebraic. b) By part a) and the Dot Product Rule, $0 = (v \ 0 \ (s) \cdot v \ 0 \ (s)) 0 = 2v \ 0 \ (s) \cdot v \ 0 \ (s) 0 = 2v \ 0 \ (s) \ 0 \ (s) \ 0 = 2v \ 0 \ (s) \ 0 \ (s) \ 0 \ (s) \ 0 = 2v \ 0 \ (s) \ 0 \ (s) \ 0 \ (s) \ 0 \ (s) \ 0 \ 0 \ (s) \$ Integrals, there is a $c \in [a, b]$ such that $Z 0 = b Z b f(x)xn dx = f(c) a xn dx =: f(c) \cdot I.$ 14.1.2. By definition and a sum angle formula, 1 $2\pi (SN f)(x) = Z \pi f(t) dt - \pi Z N \mu X cos kx + \pi sin kx f(t) cos kt dt + \pi - \pi Z \pi \P f(t) a xn dx =: f(c) \cdot I.$ 14.1.2. By definition and a sum angle formula, 1 $2\pi (SN f)(x) = Z \pi f(t) dt - \pi Z N \mu X cos kx + \pi sin kx f(t) cos kt dt + \pi - \pi Z \pi \P f(t) a xn dx =: f(c) \cdot I.$ 14.1.2. By definition and a sum angle formula, 1 $2\pi (SN f)(x) = Z \pi f(t) dt - \pi Z N \mu X cos kx + \pi sin kx f(t) cos kt dt + \pi - \pi Z \pi \P f(t) a xn dx =: f(c) \cdot I.$ Theorem. The actual value is $\Delta w = f(1.01, 1.98, 1.03) - f(1, 2, 1) \approx 3.049798 - 3 = 0.049798$ some $\alpha \in A$, i.e., x belongs to the right side of (16). b) Using the substitution u = x3, dx = 3x2 dx, we have Z 0 2 x 3 x e $-\infty 1 dx/(1 + x) = 32 Z 0$ eu $du = -\infty 1 \cdot c$) Set F (x, y, z) = xyz(2 cos y - cos z) + z cos x - x cos y. 1.5.0. a) False. 1.6.4. By definition, there is an $n \in N$ and a 1-1 function φ which takes Z := {1, 2, . Hence by Dirichlet's Test, a (b - $\infty 1 dx/(1 + x) = 32 Z 0$ eu $du = -\infty 1 \cdot c$) Set F (x, y, z) = xyz(2 cos y - cos z) + z cos x - x cos y. 1.5.0. a) False. b) converges, say to s. 3.3 Continuity. Conversely, if f g is continuous at a and f is continuous at a hen g = f g/f is continuous at a by Theorem 3.22. But the angle between ν 0 and ν 00 is $\pi/2$ and $\sin(\pi/2) = 1$., xN. On the other hand, by homogeneity, $k(\alpha f)(a + h) - f(a) - T(h)k = |\alpha| khk khk Since this last$ term converge to $\alpha \cdot 0 = 0$ as khk $\rightarrow 0$, it follows that αf is differentiable, and $D(\alpha f)(a) = \alpha T$. If $y = e_1/x$ then log y = 1/x, i.e., $x = 1/\log y$. The estimates in part b) can be made uniform if σk converges uniformly. Consequently, f - 1 (f ({a})) \supseteq {a, b} \supset {a}, which contradicts c). If k = 1 | Ik | < 1 then some point of [0, 1] is PN uncovered. Therefore, f (E) = [1, 5]. Suppose f is integrable on [a, b]. 7.4 Analytic Functions. Now yn < xn implies 2yn < xn + yn., a) = $(b - a, . Hence the integral represents the area of that semicircle, i.e., Z p 1 a 2 - x 2 dx = <math>\pi a 2$. By the Approximation Property for Infima, choose xk \in A and yk \in B such that kxk - yk k 1/(4e). 113 Copyright © 2010 Pearson Education, Inc. Therefore, $Z^-1 2 n-1 X Z 1 x 2k^{-3} x e^{-1} \le 1.5.3$. a) The minimum of x-2 on [0, 1] is 2. We conclude that f (1, -2, 0, 1) = 2 is the minimum of x-2 on [0, 1] is 2. We conclude that f (1, -2, 0, 1) = 2 is the minimum of x-2 on [0, 1] is 2. We conclude that f (1, -2, 0, 1) = 2 is the minimum of x-2 on [0, 1] is 2. We conclude that f (1, -2, 0, 1) = 2 is the minimum of x-2 on [0, 1] is 2. We conclude that f (1, -2, 0, 1) = 2 is the minimum of x-2 on [0, 1] is 2. We conclude that f (1, -2, 0, 1) = 2 is the minimum of x-2 on [0, 1] is 2. We conclude that f (1, -2, 0, 1) = 2 is the minimum of x-2 on [0, 1] is 2. We conclude that f (1, -2, 0, 1) = 2 is the minimum of x-2 on [0, 1] is 2. We conclude that f (1, -2, 0, 1) = 2 is the minimum of x-2 on [0, 1] is 2. We conclude that f (1, -2, 0, 1) = 2 is the minimum of x-2 on [0, 1] is 2. We conclude that f (1, -2, 0, 1) = 2 is the minimum of x-2 on [0, 1] is 2. We conclude that f (1,
-2, 0, 1) = 2 is the minimum of x-2 on [0, 1] is 2. We conclude that f (1, -2, 0, 1) = 2 is the minimum of x-2 on [0, 1] is 2. We conclude that f (1, -2, 0, 1) = 2 is the minimum of x-2 on [0, 1] is 2. We conclude that f (1, -2, 0, 1) = 2 is the minimum of x-2 on [0, 1] is 2. We conclude that f (1, -2, 0, 1) = 2 is the minimum of x-2 on [0, 1] is 2. We conclude that f (1, -2, 0, 1) = 2 is the minimum of x-2 on [0, 1] is 2. We conclude that f (1, -2, 0, 1) = 2 is the minimum of x-2 on [0, 1] is 2. We conclude that f (1, -2, 0, 1) = 2 is the minimum of x-2 on [0, 1] is 2. We conclude that f (1, -2, 0, 1) = 2 is the minimum of x-2 on [0, 1] is 2. We conclude that f (1, -2, 0, 1) = 2 is the minimum of x-2 on [0, 1] is 2. We conclude that f (1, -2, 0) = 2 is the minimum of x-2 on [0, 1] is 2. We conclude that f (1, -2, 0) = 2 is the minimum of x-2 on [0, 1] is 2. We conclude that f (1, -2, 0) = 2 easy to see that if fn (x) = xn, then kfn k1 = $1/(n+1) \rightarrow 0$ as $n \rightarrow \infty$. Then $0 < k(x, y) - (a, b)k < \delta$ implies $|x - a| < \delta$, so $g(x, y) - f(a)| = |f(x) - L| < \epsilon$. Vol (Br (x0)) Br (x0) 12.2.4. a) Since U (f, G) - L(f, G) ≤ U (f - fN, G) + U (fN, G) - L(f - fN, G) + U (fN, G) = 11 + 12 and fN is integrable, we can show that f is integrable if we show I1 is small. Then both terms on the right side of part a) which end in $-\gamma$ converge to zero as $n \rightarrow \infty$. 2 x(x2 + z 2) dz dx = 0 1 3 Z 1 (x - x7) dx = 0 122 Copyright © 2010 Pearson Education, Inc. b) Let E = [0, 1]. n xn⁻ |xn| 2.1.3. a) If nk = 2k, then 3 - (-1)nk \equiv 4 converges to 4. Since a \in K, we conclude that f (x) = f(a) for all $x \in K$. By part a), f(a+) and f(b-) exist., 1). b) Define f on R by f(x) = x for x = 0 and f(0) = 1., $yn + xn = y \cdot x$. C2 E b) Since $Qx = (y - x^2)/(x^2 + y - 2)^2 = Py$, we can replace ∂E with any simple closed curve which surrounds (0, 0) and is disjoint from ∂E . 13.1.10. If it's true for some $n \ge 1$, then by the Inductive Hypothesis, definition, and the Sum Rule, (f + g)(n+1) = (f(n) + g(n))0 = f(n+1) + g(n+1) +or g(e) = 0. b) By the Quotient Rule, μ ¶0 ff 0 (3)g(3) - f (3)g 0 (3) 2b - d (3) = . Then by Exercise 1.2.5a, 2 < xn+1 < xn . If x < M, then x + 1 < -1/ ϵ < 0 so $|x + 1| = -(x + 1) > 1/\epsilon$. Choose a partition {x0, x1, . If p ≤ 0, this series diverges by the Divergence Test. Then xn, yn $\rightarrow \infty$ so choose N so large that Q(xN, yN) > 2/ δ . 3.2.2. a) x3 - x2 - 4 = $(x-2)(x^2+x+2)$ and $x^2-4 = (x+2)(x-2)$ so $(x^3-x^2-4)/(x^2-4) = (x^2+x+2)/(x+2) \rightarrow 8/4 = 2$ as $x \rightarrow 2-$. Ro ∞ c) 0 eat e-st dt = 0 e-t(s-a) dt = 1/(s - a) for s > a. In particular, $|x0 - t0| \le c - x0 < \delta$. Since [B $\alpha = E \cap \alpha \in A$ [V α , $\alpha \in A$ and the union of V α 's is open by Theorem 8.24, it is clear that the union of the B α 's is relatively open in E. But by Lindel of (or using rational centers and rational centers and rational radii as in the proof of the Borel Covering Lemma), we can find open balls $B_j := B^2_j$ (xj) $\infty \infty$ such that $V \subseteq \bigcup \infty j=1$ Bj. Hence it surely converges. Moreover, by L'H^opital's Rule, $\log(1/f(k)) f 0$ (k)/f (k) = $-\lim = -\alpha$. b) Since x2 - 2x + $\sqrt{3} > x2$ 2 implies x < 3/2, inf E = 0, sup E = 3/2. By Theorem 10.16, E must be closed. Either there is an $a \in H$ such that for each r > 0, Br (a) contains xk for infinitely many k's, or for each $a \in H$ there exists an ra > 0 such that Bra (a) contains xk for only finitely many k's. If f is continuous at x = 1 then $|f(x0 / yn)| \rightarrow |f(1)| = 0$ as $n \rightarrow \infty$, i.e., f is continuous at x 0. x0 < 1. $n \rightarrow \infty$ n $\rightarrow \infty$ b) It suffices the there exists an ra > 0 such that Bra (a) contains xk for only finitely many k's. to prove the first identity. $3\ 3\ 2\ 3\ 2\ 1\ +\ t\ (1\ +\ t\)\ 1\ +\ t\ (1\ +\ t\)\ 2\ 1\ 4\ 8\ Copyright\) = cD < 0$ and a minimum when cD > 0. It follows that G(f) is of volume zero. Let $\psi(x) := f(\varphi(x))$ for $x \in Z$. Similarly, $y \in Bs(b)$. d) Let s = E(x), t = E(y), and w = E(x + y). This last series converges since $\alpha > 1/2$. 2a \sqrt{We} claim that $e \le 2/(ae)$. a Thus 7.50 (with x0 = 0), Rnf, 0 (a) $\rightarrow 0$ as $n \rightarrow \infty$, for all $a \in \mathbb{R}$. 2 c) $-1/x^2 \rightarrow 0$ as $x \rightarrow -\infty$ so $e - 1/x \rightarrow e0 = 1$. It can be parameterized by $\varphi(t) = (\cos t/2, \cos t/2, \sin t)$, $t \in [0, \infty)$. 2π]. By part a), $V = ka \times bk \cdot h$, where $h = kck \cdot |\cos \theta|$. By the first derivative test, $\varphi(1) = 1/e$ is a local maximum. 9.5.4. Suppose A is uncountable. Since $\{xn\}$ is increasing, $n \ge N$ implies $xn \ge xN > M$. 2.5.1. a) Since 3 - (-1)n = 2 when n is even and 4 when n is odd, lim supn $\rightarrow \infty$ xn = 4 and lim inf $n \rightarrow \infty$ xn = 2. Since $n/\pi \rightarrow \infty$ as $n \rightarrow \infty$, we conclude that φ is not of bounded variation on [0, 1]. b) If $xn \rightarrow a$ in the discrete space, then for n large, $\sigma(xn, a) < 1$. 18 12.3.5. a) If f is continuous on R then f is integrable on [c, d] (also by Theorem 5.10). Taking the limit of (*), as $k_j \rightarrow \infty$, we see that $kb - ak \leq M$, i.e., $b \in B$. Since $\neg f(0 + h) - f(0) - \nabla f(0) + \neg f(0) + \neg f(0) - \nabla f(0) + \neg f(0$ Thus by the calculations p in part b), choose n = 14. 2.3.5. The result is obvious when x = 0. Hence integrating term by term, we obtain $\mu \parallel Z \pi/2 \times \pi/2 \times \infty \times X \perp 1 \ k\pi f(x) \ dx = cos(kx) \ dx = sin$. Since g is nonnegative, we have f(x) < (2L + 1)g(x) for x \in (b0, b). c) The columns of B are T (e1) = (1, -1), T (e2) = \cdots = T (en-1) = (0, 0), and T (en) = (1, -1), T (e2) = \cdots = T (en-1) = (0, 0), and T (en) = (1, -1), T (e2) = \cdots = T (en-1) = (1, -1), T (en-1) (-1, 1). 94 Copyright © 2010 Pearson Education, Inc. Copyright © 2010 Pearson Education, Inc. By definition, ZZ Z 1 Z 1 - y $\omega = -S2$ R(x, y, 0) dx dy, 0 Z and 0 1 Z $\omega = -S2$ R(x, y, 0) dx dy, 0 Z and 0 1 Z $\omega = -S3$ 1 - z Q(x, 0, z) dx dz. For example, Z 1 π (f (t) + g(t)) cos kt dt $\pi - \pi Z \pi Z 1 1 \pi = f$ (t) cos kt dt + g(t) cos kt dt = ak (f) + ak (g). 73 Copyright © 2010 Pearson Education, Inc. If $x \in [-M, M]$ then |fk(x)-f(x)| < 2. c) True. To show $E \subseteq f - 1$ (f (E)), let $x \in E$. Since f 0 = -g 0, it follows that f $0(x_1) < 0$ and f $0(x_2) > 0$. 8.2.1. a) By definition, a - b and a - c lie in the plane. 11.3.1. Since they are all C1 on their domains, they are all differentiable on their domains. By the choice of δ it follows that $|^2h(x)| \leq ^2khk$. Since the x axis lies to the left of the yz plane, we can parameterize this curve by $\varphi(t) = (3 \sin t, 0, 3 \cos t)$, $I = [0, 2\pi]$. 4.3.6. By the Mean Value Theorem, f(c) = f(c) - f(b) = (c - b)f 0 (x2) for some x1, $x \geq (a, b)$. 58 Copyright © 2010 Pearson Education, Inc. 3.4.6. a) Suppose I has endpoints a, b. 13.6.1. a) The trivial parameterization of z = -x, $x^2 + y^2 \le 1$, has normal (1, 0, 1), whose induced orientation on C is counterclockwise. Then $|xn - yn| = 1/n \rightarrow 0$ as $n \rightarrow \infty$, but neither xn nor yn converges. Since $m \le f(0) < f(x)$ for all $x \in /[-N, N]$, it follows that m is the absolute minimum of f on R. 11.4.1. By the Chain Rule, ∂w $\partial F \partial x \partial F \partial y \partial F \partial z = + +$, $\partial p \partial x \partial F \partial y \partial F \partial z = + +$, $\partial p \partial x \partial F \partial y \partial F \partial z = + +$. Since $xk \in E$ too, it follows from Lemma 1.40 that A is at most countable, a contradiction. b) By Gauss' Theorem and Exercise 13.5.8, ZZ ZZZ $u\nabla v \cdot n \, d\sigma = \nabla \cdot (u\nabla v) \, dV = (\nabla u \cdot \nabla v + u\Delta v) \, dV$. If f(x) < 1, then $f(x) < 1 \le 1 + f(x)$. Since in a bounded interval, E is bounded. c) If $\nabla f(a) = 0$ then Du f(a) = 0 and there is nothing to prove. On the other hand, since d/dt(te-(s-a)t) = e-(s-a)t(1-(s-a)t). -a)t), integration by parts yields $Z \propto L\{t(t)\}(s) = Z - t(s-a)$ te f (t) $= -1 \approx dt = -0 = -(s-a)t \phi(t)(1 - (s - a)t) dt$. Similarly, if $x \in Q$ and $r = -0 = -(s-a)t \phi(t)(1 - (s - a)t) dt$. Similarly, if $x \in Q$ and $r = -0 = -(s-a)t \phi(t)(1 - (s - a)t) dt$. $(t)^{2}$ (b) $(t)^{2}$ (b) $(t)^{2}$ (b) $(t)^{2}$ (b) $(t)^{2}$ (b) $(t)^{2}$ (b) $(t)^{2}$ (c) $(t)^{2}$ (c) M-Test, the j-th term by term derivative of Sf converges uniformly on R. Then div F = 0. Hence $---- \partial f = 0$, $2 - \cos(\pi/2 - x) = -\sin x$. 8.1.5. Let a, b, c denote the vertices of Δ , and C be the line segment between a and b.
Since f (15) = 0.0068, n = 15 terms will estimate the value to an accuracy of 10-2. 8.1.3. It is clear that equality holds if either x = 0 or y = 0. Since |x| is periodic and continuous on $[-\pi, \pi]$, it follows from Theorem 14.29 that this series converges to |x| uniformly on $[-\pi, \pi]$. 9.5.5. By Exercise 8.3.8, there exist $^2 := ^2x > 0$ such that $V = \bigcup x \in V B^2(x)$. 4 13.5.2. a) By Green's Theorem, Z Z d Z b $\omega = C(y-1) dx dy = (b-a)(c-d)(c+d-2)/2$. c) Using part b) and the product rule, $(f/g)0 = (f \cdot 1/g)0 = f 0 \cdot 1 g 0 g f 0 - f g 0 - f 2 = .11.5.11$. By definition, then, $xn \to a$ as $n \to \infty$. 1.2.10. (See the argument which appears in c) above.) 10.2.2. a) If a is not a cluster point, then some Br (a) contains only finitely many points of $E \setminus \{a\}$, say x1, . If $k := j - 1 \ge 0$ then $0 \le k < j$. Thus the original series diverges when q > 1. kuk2 In particular, the transitions are C p on $g(U \cap V)$. Since $\varphi 0$ (θ) = (f 0 (θ) cos $\theta - f(\theta)$ sin θ , f 0 (θ) sin $\theta + f(\theta)$ cos θ) we have $k\phi 0$ (θ) $k2 = |f 0 (<math>\theta$)| $2 + |f (\theta)|_2 6 = 0$. Since E and A are closed, it follows that C is closed. Moreover, it is obvious that they are nested. b) If f (b) < f (a) there is an x0 \in (a, b) such that y0 = f (x0) and DR f (x0) ≤ 0 . Then (x - ², x + ²) \cap E contains infinitely many points, so x is a cluster point of E. If (xk , yk) \in R converges to some (x, y) then a $\leq xk \leq b$ implies a $\leq x \leq b$ and similarly, c $\leq y \leq d$. By a similar argument, if k2 > r1 is least such that sk2 > y, then sr1 $< x s' \leq y + bk2$ for all r1 $\leq ` \leq k2$. 6.3.0. a) True. Also f + - f - = 2f/2 = f and f + f - = 2|f|/2 = |f|. Hence, if x, y \in R and kx - yk $< \delta$, then (*) kT (x, y)k $\leq M^2 \cdot kx - yk$. Since 2x3 - 3x + 1 = (x + 1) + (x + 1) -1)(2x + 2x - 1) implies that x = 1, $(-1 \pm 3)/2$, $\sqrt{\sqrt{the second inequality}}$ is equivalent to (-1 - 3)/2 < x < (-1 + 3)/2 or x > 1. a Rb Rc c Rb Thus (U) a f (x) dx + (U) c f (x) dx d) Integrating by parts twice, $Z \propto Z Z 1 b \propto -st -st \cos bt dt = -e \sin bt dt = -e \sin bt dt = -e \sin bt dt = -2 e \cos bt dt s s 0 s s 0 0 R \infty$ for s > 0. n a 7.1.9. a) By the Extreme Value Theorem, f is bounded on [a, b] and there are positive numbers $^{2}0$ and M such that $^{2}0 < |g(x)| < M$ for all $x \in [a, b]$. It follows that $\{xn \}$ is bounded. b) By Theorem 4.33, f -1 is differentiable on (c, d) and (f -1) (x) = 1/f (b) (x) = 1/f (c) (x) = 1/(f - 1 (x)). c) By Theorem 12.7 we must show that Vol $(E1 \cup E2) \ge Vol (E1) + Vol (E2)$. Since the ratio of successive coefficients is (2k - 1)(2k + 2) = 1 - (3/2)/(k + 1), it follows from Raabe's Test that the series converges absolutely at both endpoints. If k > 0 then nq is a root of the polynomial xj - nk. Then we can compute $\cos \theta$ two ways: $|w \cdot (a, b, b|) = 1 - (3/2)/(k + 1)$. c) $h = |\cos \theta| = .$ Thus by theorem 10.52, g is uniformly continuous on E. u2 2 c) Since div (x, y, z) = 3, we have by Gauss' Theorem that Z 1 Vol (E) = x dy dz + y dz dx + z dx dy. Define g(x) := limn \rightarrow \infty f (xn). Modify the proofs of Remark 2.4, Theorems 2.6, 2.8, and Remark 2.28 by replacing the absolute value signs by the metric ρ . By definition, there is a partition P² of [a, b] such that U (f, P²) - L(f, P²) < 2. b) The same estimates can be obtained. By Exercise 8 in 8.3, E \ C is relatively open in E, i.e., E \ C = E \cap V for some open V. Indeed, Z 1 Z 1 f (x, y) dx dy = 0, 0 but Z 1 Z \circ X 2-k+1 -k k=1 2 Z \circ X 2-k+1 k=1 2-k μ Z 1 ¶ f (x, y) dx dy 0 μ Z 1 ϕ k (y) ¶ (ϕ k (x) - ϕ k+1 (x)) dx dy = 0, 0 but Z 1 Z \circ 1 f (x, y) dy dx = 0 0 ∞ Z X k=1 Z 1 2-k+1 μ Z ¶ 1 f (x, y) dy 2-k Z 1 = φ 1 (x) φ 1 (y) dy dx + 1/2 dx 0 0 ∞ Z X k=2 μ Z 2-k+1 2-k φ k (x) ¶ 1 (φ k (y) - φ k+1 (y)) dy dx 0 = 1 + 0 = 1. It is relatively closed in B $\sqrt{2}$ (2, 0) because the limit of any convergent sequence (in the SUBSPACE sense) in the set stays in the set. Thus Rx = y + fx and we may set f = 0, 2 + 1 + 0 = 1. It is relatively closed in B $\sqrt{2}$ (2, 0) because the limit of any convergent sequence (in the SUBSPACE sense) in the set stays in the set. Thus Rx = y + fx and we may set f = 0, 2 + 1 + 0 = 1. It is relatively closed in B $\sqrt{2}$ (2, 0) because the limit of any convergent sequence (in the SUBSPACE sense) in the set stays in the set. Thus Rx = y + fx and we may set f = 0, 2 + 1 + 0 = 1. It is relatively closed in B $\sqrt{2}$ (2, 0) because the limit of any convergent sequence (in the SUBSPACE sense) in the set. Thus Rx = y + fx and we may set f = 0, 2 + 1 + 0 = 1. It is relatively closed in B $\sqrt{2}$ (2, 0) because the limit of any convergent sequence (in the SUBSPACE sense) in the set. Thus Rx = y + fx and we may set f = 0, 2 + 1 + 0 = 1. It is relatively closed in B $\sqrt{2}$ (2, 0) because the limit of any convergent sequence (in the SUBSPACE sense) in the set. Thus Rx = y + fx and we may set f = 0, 2 + 1 + 0 = 1. It is relatively closed in B $\sqrt{2}$ (2, 0) because the limit of any convergent sequence (in the SUBSPACE sense) in the set. Thus Rx = y + fx and we may set f = 0, 2 + 1 + 0 = 1. i.e., R = xy. u 2 (4 + 4x - 3x2) dx = 1. Thus the integral converges if and only if p < 1. Differentiating term by term, we obtain f 0 (x) = $\infty X k = 1 Rx \infty X k k - 1 k k k - 1 \leq =: g(x) ((-1)k + 4)k 3k k = 1 P \infty$ for $0 \leq x < 3$. 10.4.2. Let A, B be compact sets. 10.5.5. Suppose A is not connected. Therefore, $ur = gy \cos \theta - gx \sin \theta = v\theta / r$ and $vr = -fy \cos \theta + fx$. $\sin \theta = -u\theta /r$. 2 d) The boundary of S is given by $2x^2 + z^2 = 1$, y = x, oriented/in the counterclockwise direction. But Ea is an interval, hence (a, b) \subset Ea for all $a \in A$, i.e., $(a, b) \in E$. Hence by the Second Multiplicative Property and Theorem 1.20, inf E + 2 = -(sup(-E) - 2) > a > -sup(-E) = inf E. Since $f(t) \leq F(x0)$ for all $t \in [a, x0]$, it follows that $t0 \in (x0, c]$. From part a), we see that Pn this function has only one critical point: (a0, b0). By the Mean Value Theorem, f(n + 1) - f(n) = f 0 (cn) for some cn $\in (n, n + 1)$. 1), $n \in N$. We conclude that f(-2, 0) = -2 is the minimum, $f(1/2, \pm 15/2) = 17/4$ is the maximum (and f(2, 0) = 2 is a saddle point). b) Let $^2 > 0$ and $x \in [a, b]$. If a = b then a + c = b + c since + is a function. Therefore, it suffices to prove that given any continuous f on [a, b], there is a polynomial Q, with rational coefficients, such that $|f(x)-Q(x)| < \epsilon$ for all $x \in [a, b]$. $x \sqrt{\sqrt{This}}$ last quotient converges to 0 by Theorem 2.12. Thus V (E; Gm) = $2m \cdot 22m = 2 - m < \epsilon$. Since c is a point of discontinuity, it follows that f 0 (c-) < f 0 (c+). 10.4.6. Suppose that f is uniformly continuous on E., xN such that N [X = Bxj]. To prove this, combine the inequality in part b) above with the Comparison Theorem for improper integrals. It is clear by construction that f is 1-1. Since the geometric series k=0 (r0 /r) < ∞ , it follows from the Comparison Test that k=0 ak r0 converges, which contradicts the fact that r0 > R and R is the radius of convergence. If y = 0/then the constraint implies x = ±2. Thus the inequality holds for all k \in N. 9.3.1. a) The domain of f is all $(x, y) \in \mathbb{R}^2$ such that x = 1 and y = 1. d) The set is closed but not bounded (since $(n, 1/n) \in \mathbb{E}$ for all $n \in \mathbb{N}$). Hence by Theorem 10.56, $f(\mathbb{E})$ is an interval. 11.5.2. We must show $D(\hat{y} = 1, d)$ the set is closed but not bounded (since $(n, 1/n) \in \mathbb{E}$ for all $n \in \mathbb{N}$). Hence by Theorem 10.56, $f(\mathbb{E})$ is an interval. 11.5.2. We must show $D(\hat{y} = 1, d)$ the set is closed but not bounded (since $(n, 1/n) \in \mathbb{E}$ for all $n \in \mathbb{N}$). Hence by Theorem 10.56, $f(\mathbb{E})$ is an interval. 11.5.2. We must show $D(\hat{y} = 1, d)$ the set is closed but not bounded (since $(n, 1/n) \in \mathbb{E}$ for all $n \in \mathbb{N}$). Hence by Theorem 10.56, $f(\mathbb{E})$ is an interval. 11.5.2. We must show $D(\hat{y} = 1, d)$ the set is closed but not bounded (since $(n, 1/n) \in \mathbb{E}$ for all $n \in \mathbb{N}$). the graph of y = f(x), i.e., f is convex. b) Let xn = n and yn = n + 1/n. Rx 14.4.3. By the proof of Corollary 14.27, if a0 (f) = 0 then F(x) := 0 f(t) dt is continuous, periodic, of bounded variation, ak (F) = -ak (f)/k = 0 for $k \in N$. Thus $\cap x \in (0,1] [x - 1, x + 1] = (0, 1]$. 11.7.5. If fxy (a, b) 6 = 0 then fxx (a, b) = fyy (a, b) = 0 and it follows that D(2) f (a, b) = fxy (a, b)hk takes both positive and negative values as h, k range over R. $\sqrt{\sqrt{b}}$ Clearly, fx = 1/(4x3/2), fxy = 0, fyy = 1/(4x3/2), fxx = 3/(8x5/2), fyy = 3/(8y5/2), and all mixed third order partials are zero. When p < 0, the result is false, since ak = 1/k 1-p generates a convergent series by the p-Series Test (1 - p is GREATER than 1 in this case), but |ak|/k p = 1/k which generates the harmonic series, which diverges. In particular, it follows from assumption ii) that $(1 - \cos x)/x \rightarrow 0$ as $x \rightarrow 0$. 5.1.4. a) If f (x0) $\delta = 0$ then given 2 > 0 choose by the Sign Preserving Property a $\delta > 0$ such that $|f(x)| \geq 2$ for $|x - x0| \leq \delta$. Thus f is continuous on R2 $8.2.5. a) T(1, 0) = T(1, 1) - T(0, 1) = (-1, \pi, -1), and T(0, 1) = (4, 0, 1).$ Let $^2 > 0$ and choose N so large that $k=N+1 |ak|/k < ^2/2.$ If N > N2 then $|(S0(x) - f(x)) +
\cdots + (SN(x) - f(x)) + \cdots + (SN(x) - f(x)) + (-1, \pi, -1), and T(0, 1) = (4, 0, 1).$ Let $^2 > 0$ and choose N so large that $k=N+1 |ak|/k < ^2/2.$ If N > N2 then $|(S0(x) - f(x)) + \cdots + (SN(x) - f(x)) + \cdots + (SN(x)$ minimum), it follows that wxx (x2, t2) ≥ 0 . $\sqrt{\text{Note: If we replace } \phi \text{ by } g(x) = \sin x - 22x/\pi$, then the same argument shows $g(x) \geq 0$ for $x \in [0, \pi/4]$, and we obtain $Z^- \mu \P^- \pi/2^- \pi \sqrt{\pi \pi 1^-} - a \sin x e dx^- \leq e-a 2/2 + \sqrt{\leq \sqrt{+1}}$, $T^- 0^- 42a 22a 2e an improvement over the estimate we already obtained. If <math>n > 9$ then $(n + 3)^2 = n^2 + 6n + 9 < n^2 + 6n + 9 <$ k=1 Then 0 < x - sn < 1/10n, so by a) choose xn+1 such that $xn+1/10n+1 \le x - sn < xn+1/10n+1 + 1/10n+1 \le x - sn < xn+1/10n+1 \le x - sn < xn+1/10n+1$ combinations of 0's and 2's, hence the binary coefficients of f (x) exhibit all possible combinations of 0's and 1's. j=1 Let $x \in K$. If not, then some xn satisfies $|f(xnk)| > \epsilon 0$ for $k \in N$, so $|f(xnk)| > \epsilon 0$ for $k \in N$, so $|f(xnk)| > \epsilon 0$ for $k \in N$. If not, then some xn satisfies $|f(xnk)| > \epsilon 0$ for $k \in N$, so $|f(xnk)| > \epsilon 0$ for $k \in N$, so $|f(xnk)| > \epsilon 0$ for $k \in N$. $+ b2 + \cdots + br1 := sk1 - a - 1 - \cdots - ar1 - k1 < x$, and $sr1 \ge x + br1$. Printed in the United States of America. $x \in V$ 92 Copyright © 2010 Pearson Education, Inc. c) Let a = -1, b = 1, and f(x) = x3. Since $1/n \rightarrow 0$ as $n \rightarrow \infty$, it follows that xn satisfies the hypotheses of Exercise 2.4.4. Hence xn must converge to a finite real number. Since (|t|p) = 1, and f(x) = x3. p|t|p-1 exists for P all $t \in \mathbb{R}$ and n p > 1, it follows from Lagrange's Theorem that if f(x) is an extremum subject to the constraint k=1 |xk|p = 1 p-1 2 p then $2xj = p|xj|\lambda$. Thus E is sequentially compact. c) This set is bounded but not closed. 13.1.1. (ψ , I) runs clockwise. Then $f(x, y) = x^2 + 2x - y^2 = 5 \cos 2\theta + 4 \cos \theta - 1 =: h(\theta)$. k=n+2 (2ak /x A N X Moreover, by the claim and (*), |k - 1/x2| (ak /x)e -0 = 0. β) Since f (x) = x2 is increasing on $[0, \infty)$, Mj = x2j := (j/n)2 and mj = x2j - 1 := ((j - 1)/n)2 . 5.1.0. a) False. Let y \in Br (x) and let P be a polygonal path from x0 to x which lies in E. Since f (x) $\geq c > 0$ for all $x \in [a, b]$, it is easy to see that if x, $y \in [xj-1, xj]$, then f 1/m (x) $-f 1/m (y) \leq Mj (f) - 1/m (y) \leq Mj (f) = 1/m (y) \leq Mj (f) = 1/m (y) \leq Mj (f) = 1/m (y) = 1/m ($ mj (f) Mj (f) - mj (f) = Cm (m + 1)c(m-1)/m 45 Copyright © 2010 Pearson Education, Inc. It follows that 1/f is differentiable at a and its derivative is T. Choose $\delta > 0$ such that $|\varphi(t)| < 2$ for $0 \le t < \delta$. Hence, $\sqrt{\sqrt{P}}$ (P, Q, R) $\cdot \varphi 0$ (t) = (cos t sin 2 t/2, 0) \cdot (- sin t, cos t/2, cos t/2) $\sqrt{=}$ - sin 3 t cos t/4 - sin 2 t cos 2 t/(2 2). Then f 0 (x) = 3x2 +1 > 1 for all $x \in (0, \infty)$, but $x^2 / f(x) < 1/x \rightarrow 0$ as $x \rightarrow \infty$. Finally, $\sqrt{xn+1} - 1 - xn - 1 - (1 - xn) + 1 - 1 - xn - 1 - (1 - xn) + ($ Substituting 2x for x, we have $\cos(2x) = x^2 + \cos(2x) = 1 - x^2 + \infty X$ (-4)k x2k (2k)! k=2 for x \in R. By definition, h/(1 + e1/h) - 01 = lim = 0. V is open since it is a union of open intervals. Nevertheless, it is clear that $|x4 - \pi| < 0.0000000005$ which is much smaller than 0.000136465. b) dz = y cos(xy) dx + x cos(xy) dy. 13.5.3. a) By Gauss' Theorem since I is closed. Thus the formula is correct for this case. b) Repeat the proofs of Theorem 2.8 and Remark 2.28, replacing the absolute value by the norm sign. S E b) The boundary of S $\sqrt{}$ is given $\sqrt{}$ by x2 + y2 = 3, z = 0, oriented in the counterclockwise direction. Let $\varphi(t) = t \log(1 - x/t)$. 10.6 Continuous Functions. A similar argument shows that {x $\in \operatorname{Rn}: kx - ak > r$ and $\{x \in \operatorname{Rn}: kx - ak < s\}$ are both open, hence $E := \{x \in \operatorname{Rn}: kx - ak < s\}$ are both infima of E then $m \le m \in \mathbb{R}$ and $\{x \in \operatorname{Rn}: kx - ak < s\}$ are both open, hence $E := \{x \in \operatorname{Rn}: kx - ak < s\}$ are both infima of E then $m \le m \in \mathbb{R}$ and $\{x \in \operatorname{Rn}: kx - ak < s\}$ are both open, hence $E := \{x \in \operatorname{Rn}: kx - ak < s\}$ are both open. and $m \in m$, i.e., m = m. Thus f 0 (x) exists. $|t1| |t2| kak2 |t1| |t2| kak2 |t1| |t2| Hence \theta = 0$ or π . Let $^2 > 0$ and choose N so large that $n \ge N$ implies $|xn| < ^2 2 \cdot 14.4.5$. a) Fix $h \in R$ and $k \in N$. (It holds for n = 1, and if it holds for n = 1. $3n + 4 \cdot 3n = 2 \cdot 3n + 1$. On the other hand, the right side lies between 1/((2p + 1)(2p + 2)) = 0/(2p + 2) and 1/((2p + 1)(2p + 2)) = 1/((2p + 1)(2p + 2)) = 0/(2p + 2) = 0/(2p + 2)Theorem on Manifolds. 3 = 1.4.5. $0 \le an = .$ A similar argument works for f \land g. It follows from the definition of the operator norm that $|f(g(x)) - f(g(a))| \le kDf(g(c))k kx - ak$. $P \ge 6.2.6$. a) If an /bn $\rightarrow 0$ then an \le bn for n large. 3 Vol (E) = 13.5.6. a) Parameterize ∂E by $\varphi(t) = (\cos t, \sin t)$, $I = [0, 2\pi]$. k b) By L'H[^] e - 1 . b) f(x) = x is uniformly continuous on $[0, \infty)$ but not bounded there. CHAPTER 2 2.1 Limits of Sequences. By Exercise 1.2.9c, r + 2 is irrational. Then A - B = [-1, 1] so sup(A - B) = 1 6 = 0 = sup A - sup B. = °T ° kxk kxk ° c) Taking the supremum of this last inequality over all x = 0, we obtain $kT k \le M1 \cdot 2$ c) log(k(k + 2)/(k + 1)) - log(k/(k + 1)/(k + 2))5.3.10. CHAPTER 7 7.1 Uniform Convergence of Sequences. 10.1.8. a) If fn is Cauchy in C[a, b], then given $\varepsilon > 0$ there is an N such that m, $n \ge N$ implies $|fn(x) - fm(x)| \le kfn - fm k < \varepsilon$ for all $x \in [a, b]$. Also, by the homogeneous property of integration, $k\alpha f k 1 = |\alpha| kf k1$, so kf - gk1 is homogeneous. By parts iv) and v), $kx \times yk2 = (x \times y) \cdot (x \times y) = x \cdot (y \times y) \cdot (x \times y) \cdot (x \times y) = x \cdot (y \times y) \cdot (x \times y) \cdot (x \times y) = x \cdot (y \times y) \cdot (x \times y) \cdot (x \times y) = x \cdot (y \times y) \cdot (x \times y) \cdot (x \times y) \cdot (x \times y) = x \cdot (y \times y) \cdot (x \times y) \cdot (x \times y) = x \cdot (y \times y) \cdot (x \times y) \cdot (x \times y) = x \cdot (y \times y) \cdot (x \times y) \cdot (x \times y) = x \cdot (y \times y) \cdot (x \times y) \cdot (x \times y) = x \cdot (y \times y) \cdot (x \times y) \cdot (x \times y) \cdot (x \times y) = x \cdot (y \times y) \cdot (x \times y) \cdot (x \times y) \cdot (x \times y) = x \cdot (y \times y) \cdot (x \times y) \cdot (x \times y) \cdot (x \times y) \cdot (x \times y) = x \cdot (y \times y) \cdot (x \times$ $(x \times y) = x \cdot ((y \cdot y)x - (y \cdot x)y) = (y \cdot y)(x \cdot x) - (x \cdot y)2$. 8 Copyright © 2010 Pearson Education, Inc. b) Since $1 \pi Z \pi Z |f(x)|^2 dx = -\pi \pi x^2 dx$ -x 1 - x2) dx = 3 π . Therefore, $-C \le xn \le C$, i.e., |xn| < C for all $n \in N$. Hence, the
cited result follows immediately from the Second Mean Value Theorem for integrals. R ∞ R ∞ c) Using the substitution $u = \log x$, du = dx/x, we have $e dx/(x \log p x) = 1 du/up$. 6.1.4. Since ak+1 - 2ak + ak-1 = (ak+1 - ak) + (ak-1 - ak), this series is the sum of two telescopic series. Since f (a) and f (b) both belong to f (E), the interval (f (a), f (b)) is a subset of f (E). It follows (see Exercise 1.6.5a) that $g \circ f$ takes N onto B. Consequently, lim supn $\rightarrow \infty$ |Bn - b| $\leq ^2$., xn } such that U (f, P) - L(f, P) < min{Cm ², c-2 ²}, where Cm := (m + 1)c(m-1)/m . Since m0 := P (x0)/2 > 0 use 3.2.3b again to choose $\delta 0 > 0$ such that U (f, P) - L(f, P) < min{Cm ², c-2 ²}, where Cm := (m + 1)c(m-1)/m . Since m0 := P (x0)/2 > 0 use 3.2.3b again to choose $\delta 0 > 0$ such that U (f, P) - L(f, P) < min{Cm ², c-2 ²}, where Cm := (m + 1)c(m-1)/m . Since m0 := P (x0)/2 > 0 use 3.2.3b again to choose $\delta 0 > 0$ such that U (f, P) - L(f, P) < min{Cm ², c-2 ²}, where Cm := (m + 1)c(m-1)/m . Since m0 := P (x0)/2 > 0 use 3.2.3b again to choose $\delta 0 > 0$ such that U (f, P) - L(f, P) < min{Cm ², c-2 ²}, where Cm := (m + 1)c(m-1)/m . Since m0 := P (x0)/2 > 0 use 3.2.3b again to choose $\delta 0 > 0$ such that U (f, P) - L(f, P) < min{Cm ², c-2 ²}, where Cm := (m + 1)c(m-1)/m . Since m0 := P (x0)/2 > 0 that 0 < m0 < P(x) for $|x - x0| < \delta0$. Notice that $k 2 \le k 4$ for all $k \in N$. Therefore, $P \infty k=1$ (a2k n X n $\rightarrow \infty + a2k+1$) $\rightarrow L$ and k=1 P $\infty k=1$ 2n X k=2 ak = a1 + L converges. Since f(x0) is a proper local maximum, there is a $c \in (x0 - \delta, x0)$ such that f(c) < f(x0). 6.3 Absolute that f(c) < f(x0) = a^{2} + a^{2 Convergence. If they intersect, say $\varphi(t) = \psi(u)$, then t = 3u, t = 4u, and 0 = 1, a contradiction. Thus use Dini's Theorem. 13.3.4. a) Parameterize S by (φ , E), where $\varphi(u, v) = (u, v, 0)$. 4.1.0. a) False. Hence by Exercise 1.6.5a, g \circ f takes A onto N. 11.3 Derivatives, Differentials, and Tangent Planes. c) By the Chain Rule, (g \circ f)0 (3) = g 0 (f (3))f 0 (3) = bc. On the other hand, Vol (E 0) \leq Vol (E) by Exercise 12.1.6a. 2.4.7. a) Suppose a is a cluster point for some set E and let r > 0. f) Since 2 - (-1)n/n2 = 2 - 1/n2 when n is odd, inf E = 7/4 and sup E = 3. Rd Rd Rc 5.1.5. By hypothesis, c f (x) dx = a f (x) dx - a f (x) dx = 0 for all c, $d \in [a, b]$. Then by part c) and Exercise 12.4.9, $|\det(S)| \cdot \operatorname{Vol}(\varphi(Q)) = \operatorname{Vol}(S \circ \varphi(Qj)) \leq C^2 |Qj|$ for j large. By Theorem 8.32, E has no boundary if and only if $E = E \circ A$. By repeating the steps in Case 2, we conclude that xn decreases from $x_0 \geq 3$ to the limit 3. $-\infty < s < -\infty$. 10.5.2. a) It is relatively open in $\{(x, y) : y \geq 0\}$ because each of its points lies in a relative open ball which stays inside the set. b) Since $|x + 3| \le |x| + 3$, $|x| \le 1$ implies $|x^2 + 2x - 3| = |x + 3| |x - 1| \le 4|x - 1|$. 1. 2 p $\lambda j=1$ Pn Hence j=1 x2j=m(p-2)/p. 21 Copyright © 2010 Pearson Education, Inc. E Thus $\forall u = 0$, i.e., u is constant on E. Therefore, $k\nu \ 0$ (s0) $k = k\nu \ 00$ (s0) k = k $x E2 \times \cdots$ by taking each point (x1, x2, Notice that $n \ge 1$ implies $-3n \le -3$ so $1 - 3n \le -2$. In this case, Mj = mj = 0 when j < n/2, Mj = mj = 1 when j > n/2 + 1, and Mj (f) = 1 = mj (f) + 1 otherwise. 6.3.3. a) By the Integral Test (see Exercise 6.2.2d) it converges for all p > 1 and diverges for $0 . 4t 4t2 2t <math>\sqrt{2}$ b) If $x \ge a$ then $u(x, t) \le e-a$ $/4t / 4\pi t \rightarrow 0$ as $t \rightarrow 0+$ independently of x. Therefore, the series converges absolutely if and only if |p| > 1.2227.2 Uniform Convergence of Series. d) If $|bk+1|/|bk| = r = \lim \inf k \rightarrow \infty$, then by Remark 6.22iii and part b), $\lim \sup k \rightarrow \infty$, then by Remark 6.22iii and part b), $\lim \sup k \rightarrow \infty$ is evidently continuous for x = 0. Since $x \in [0, 2]$ implies ex /n \leq e4/n, it follows that |ex 2 /n for all x \in [0, 2] and n \geq N. Since nondegenerate intervals always contain infinitely many points, it follows that every point in [a, b] is a cluster point of [a, b). Therefore, k=0 ak rk diverges by the Divergence Test, which contradicts the fact that r < R and R is the radius of convergence. Since f (x) \rightarrow 0 as x \rightarrow ∞ , f (x) < 1 for large x. 16 Copyright \otimes 2010 Pearson Education, Inc. Thus {xn } is decreasing and bounded below by y1 and {yn } is increasing an relatively closed in f (E). Thus $x \in \partial E \cap U$. d) Clearly, 2 sin2 $x+2x-2x \cos 2 x = 2(x+1) \sin 2 x = (x+1)(1-\cos(2x))$. By Theorem 9.5, there is a subsequence xkj which converges, say to b. Thus the points are (a, b) = ((2k + 1)\pi/2) for $k \in Z$. b) Integrating term by term, we have Z b $E(x) dx = a \propto X xk+1^{-1}b = E(b) - E(a)$. 0 154 Copyright © 2010 Pearson Education, Inc. Since f(x) > g(x), we also have f(x) > M. Therefore, $\Delta N(x) \rightarrow 0$ uniformly in x, as $N \rightarrow \infty$. Hence f - 1(V) is relatively open in E. Thus the trace of $\varphi(t)$ lies in the second quadrant and is asymptotic to the line y = -x as $t \rightarrow -1+$. Thus log(an) $\mu \P 4$. xn = 1/4 + 1/(n + 4) is strictly decreasing and $|xn| \leq 1/4 + 1/(n + 4)$. 1/5 < 1/2, but xn $\rightarrow 1/4$ as n $\rightarrow \infty$. On C1, y = 1 hence dy = 0, and Z Z 1 y dx + x dy = 1 dx = 1. Hence given M > 0, choose N Pn-1 so large that sn $\geq M$ for n $\geq N \cdot 2 2$ b) Since μ Df (x, y) = x - y, the inverse exists for 0 < y < x and by the Inverse Function Theorem, $\mu \parallel \mu \parallel 1 x - 1 x/(x - y) 1/(y - x) D(f - 1)(f - 1)$ (x, y) = (Df(x, y)) - 1 = = . If $f(x) = x^2 + 1 = g(x)$, then $f(x) \rightarrow \infty$, g(x) > 0 for all x, but f(x)/g(x) = 1 does not converge to 0. c) By Remark 10.9, choose r0, s0 such that Br0 $(x) \subseteq Br(a)$ and Bs0 $(x) \subseteq$ proved in the text. If $x = y = \pm \infty$ or $-x = y = \infty$, there is nothing to prove. 4.2.4. By Exercise 4.1.2a, (xn)0 = nxn-1 for each $n \in N$. We can parameterize this intersection by $\varphi(t) = (z_0 \cos t, z_0 \sin t, z_0)$ and $I = [0, 2\pi]$. Since F is continuous on [c, d], choose $x_0 \in [c, d]$ such that F (x0) \geq F (u) for all $u \in [c, d]$. Therefore, the original integra converges. Then f (n) (x) = g (n-1) (x) is continuous but f (n+1) = g (n) exists nowhere on R. 5.1.7. a) Let P1 and P2 be partitions of [a, b], and let P = P1 \cup P2. Since $\beta k > 1$ and $xk > x\beta$, it follows from the Binomial Series expansion that $\mu \P \beta k (1 + x)\beta > 1 + x > 1 + x\beta$. 8.3.4. Since E1 is closed and E2 is open, and U = E1 \cap E2, it is clear by definition that U is relatively open in E1 and U is relatively closed in E2. To prove the right-most inequality, suppose that $r = \lim \sup k \rightarrow \infty ak+1 / ak$. Chapter 13 13.1 Curves. j! (p - 1)! 0 j=1 Since F (p) (t) = D(p) f (a + t(x - a); (x - a)), the result follows at once. Let m > N. Since $x4 + yp \le (x2p + y 2)x2 + (x2 + y 2)2 = (x2$ +y /(x +y) \leq x +y \rightarrow 0 as (x, y) \rightarrow (0, 0). Hence (φx , Ex) is a smooth 146 Copyright © 2010 Pearson Education, Inc. By hypothesis, k(1 - |ak+1 / ak |) > q for k large. b) Let (φ , I) be a piecewise smooth parameterization of ∂S . Summing over all j, we obtain n n X X 2 x2j = $p\lambda$ |xj |p = $p\lambda$, j=1 115 Copyright © 2010 Pearson Education, Inc. Thus an equation of the plane tangent to K perpendicular to x + z = 5 at a point (a, b, c) is $(1, 0, 1) \cdot (x - a, y, z - a) = 0$, i.e., x - z = 0., xN }, i.e., contains only finitely many points. Since $\{xn\} \subseteq E$, it is bounded. To decide which, look at the discriminant. The base of P is kbk and its altitude is kak sin θ . Since b > 0, $m0 \ge 1$. Thus by the Heine-Borel Theorem there exist SN x1... Given $^2 > 0$ choose N \in N
such that $n \ge N$ implies $xn \in (x - ^2, x + ^2)$, $R \propto R \propto 5.4.1$, a) 1 (1 + x)/x3 dx = 1 x - 3 dx + 1 x - 2 dx = 1/2 + 1 = 3/2, f (f (x0) Thus set x1 = f - 1 (x0), ka × bk kck 8.2.8. If (x0, v0, z0) lies on the plane II then the distance is zero, and by definition, ax0 + bv0 + cz0 - d = 0. Choose M > 0 so large that $|g1(x)| \leq M$ for $x \in E$. p = 11.4.8. Let $w = x^2 + y^2 + z^2$. 9.5.6. a) Suppose E is compact, and let $xk \in E$. 8.3 Topology of Rn . k(u, v)k Therefore, it follows from part a) that $\lim h \to 0$ $\Delta(h)$ fy $(a + h, b + th) - fy (a, b + th) = \lim fy (a, b)$. If $\{xn\}$ is bounded above, then there is an $x \in R$ such that $xn \to x$ (by the Monotone Convergence Theorem). k=2n-1Since f belongs to Lip α , it follows from part a) that n 2X -1 (a2k (f) + b2k (f)) n 2X -1 ≤ 2 k=2n-1 (a2k (f) + b2k (f)) sin 2 kh k=2n-1 α X 2 (a2k (f) + b2k (f)) sin 2 kh n=1 k=2n-1 α X 2 (a2k (f) + b2k (f)) sin 2 kh n=1 k=2n-1 α X 2 (a2k (f) + b2k (f)) sin 2 kh n=1 k=2n-1 α X 2 (a2k (f) + b2k (f)) sin 2 kh n=1 k=2n-1 α X 2 (a2k (f) + b2k (f)) sin 2 kh n=1 k=2n-1 α X 2 (a2k (f) + b2k (f)) sin 2 kh n=1 k=2n-1 α X 2 (a2k (f) + b2k (f)) sin 2 kh n=1 k=2n-1 α X 2 (a2k (f) + b2k (f)) sin 2 kh n=1 k=2n-1 α X 2 (a2k (f) + b2k (f)) sin 2 kh n=1 k=2n-1 α X 2 (a2k (f) + b2k (f)) sin 2 kh n=1 k=2n-1 α X 2 (a2k (f) + b2k (f)) sin 2 kh n=1 k=2n-1 α X 2 (a2k (f) + b2k (f)) sin 2 kh n=1 k=2n-1 α X 2 (a2k (f) + b2k (f)) sin 2 kh n=1 k=2n-1 α X 2 (a2k (f) + b2k (f)) sin 2 kh n=1 k=2n-1 α X 2 (a2k (f) + b2k (f)) sin 2 kh n=1 k=2n-1 α X 2 (a2k (f) + b2k (f)) sin 2 kh n=1 k=2n-1 α X 2 (a2k (f) + b2k (f)) sin 2 kh n=1 k=2n-1 α X 2 (a2k (f) + b2k (f)) sin 2 kh n=1 k=2n-1 α X 2 (a2k (f) + b2k (f)) sin 2 kh n=1 k=2n-1 \alpha consists of two pieces. 12.1.6. a) If $R_i \cap E_1 = \emptyset$ then $R_i \cap E_2 = \emptyset$. When x = 1 it converges by the Alternating Series Test. But (a/c, b/c, -1) < (1, 0, 1) = 0 implies that a = c. We conclude that $f(x) = 1/(x^2 - 1) < M$. Taking the limit of xn = (1 + xn - 1)/2 as $n \to \infty$, we see that x = (1 + x)/2, i.e., x = 1. We may suppose that r = 0. Hence by Cantor's and r = 0. Theorem, S = Sg. But f and g differ at at most finitely many points in $[-\pi, \pi]$. b) L(f, P) = 0.5f (0.5) + 0.5f (1) + f (2) = 11/8. In particular, f is increasing on 2 [0, a]. Hence Z 2 (|x + 1| + |x|) dx = 1 · (3 + 1)/2 + 1 + 2 · (5 + 1)/2 = 9.7.1.8. Choose N so large that [a, b] $\subset [-N, N]$. On the other hand, since ap0 \in Ep0, it is also the case that A(p0) \geq ap0 $s = \infty$. a) $\Omega f(t - h, t + h) = 1$ so $\omega f(t) = 1$ for all t. Also notice, since a point of E can belong to at most two intervals of the form [xj-1, xj], that the number of points in A is at most 2m. In particular, $\{V\alpha\}\alpha\in A0$ covers H and H is compact. We conclude by Stokes's Theorem that ZZ $2\pi F \cdot n d\sigma = -S \sqrt{\sqrt{(sin 3 t cos t/4 + sin 2 t cos 2 t/(2 2))}} dt = -\pi/(8 t cos 2 t cos 2 t/(2 2))$ 2). Since z = f(x, y) has a tangent plane at (x0, y0, z0) by Theorem 11.22, it follows that S has a tangent plane at (x0, y0, z0). Since a > b, it follows that Z Z $\pi \pi b(a + b \cos v)$ du dv = $4\pi 2 ab$. 4.2 Differentiability Theorems. x = 0. Since $1/inf k \ge n xk \ge 1/xj$ implies $1/inf k \ge n xk \ge 1/xj$. Let $\{x0, x1, y, x1, y, x1, y\}$. Then on $(0, 1) \setminus \{j/N : j = 1, .b\}$ Repeat the argument in part a), but this time, f(-x + h) = f(x - h) and f(-x) = f(x). Hence c is a root of f(0, 10.5.6). For each $x \in X$, f is constant on Bx. b) Let $A = \{(x, y) : y = 0\}$ and $B = \{(x, y) : y = 0\}$ and Thus the formula holds when x = 0. Suppose f - 1 (x) = f - 1 (y) for some x, $y \in B0$. Then U = A and $V = E \setminus A$ are nonempty relatively open subsets of E, $U \cap V = \emptyset$, and $E = U \cup V$. δ) Since f 0 (x) p = 2x + 2 < 0 for x < -6, f is 1-1 on $[-\infty, -6]$. If $xk \in f - 1$ (E) $\cap B$ and $xk \rightarrow a$, then $xk \in B$ and $f(xk) \in E$. 14.1.5. a) Since $f N(x) - f(x) \rightarrow 0$ uniformly on $[-\pi, \pi]$, it follows from Theorem 7.10 that $|ak(fN) - ak(f)| \le 1 \pi Z \pi |fN(t) - f(t)| |\cos kt| dt \le -\pi 1 \pi Z \pi |fN(t) - f(t)| dt -\pi converges to zero as N \rightarrow \infty$. 13.4.2. a) Use the trivial parameterization $\varphi(u, v) = (u, v, u^2 + v^2)$, E = B1(0, 0). 3.2.0. a) False. For example, f(x) = x for x < 0 and = 1 - x for x > 0 is 1-1 on $[-1, 0] \cup (0, 1]$ but increases on the left half interval and decreases on the right half interval. Since $1 - \cos(2x)$, it follows that $2 \sin 2x + 2x - 2x \cos 2x x + 1$ $1 = 3 = 1 - \cos(2x)$, it follows that $2 \sin 2x + 2x - 2x \cos 2x x + 1$ $1 = 3 = 1 - \cos(2x)$, it follows that $2 \sin 2x + 2x - 2x \cos 2x x + 1$ $1 = 3 = 1 - \cos(2x)$. $r \rightarrow 1 - \infty X$ ak $rk - L| \leq 2$. By part a), there is a $\delta > 0$ such that $|^2x - yk| < 2 \cdot kx - yk$ for all x, $y \in R$ which satisfy $kx - yk < \delta$. Since k ak = 1/3 if k is odd and 1/5 if k is even, the radius of convergence of this series is R = 3. 11.2.7. Clearly, f is continuous and has first-order partial derivatives at every point (x, y) 6 = (0, 0). Repeating this argument using lower sums and lower integrals, we obtain (L) a g(x) dx \geq a f (x) dx. Consider the quotient g/f. ak bk for $n \in N$. e) The maximum of 1/k for $k \in N$ is 1 and the minimum of -k for $k \in N$ is 1 and the minimum of 1/k for $k \in N$ is 2010 Pearson Education, Inc. When $p = \log 2$ (e), we compare the series with k., n + 1, so $\psi \circ \phi$ takes {1, 2, . In view of (2), this happens if and only if $a \cdot b = 0$. If {1/(nx)} were uniformly convergent, then there is an $N \in N$ such that $|1/(Nx)| \le 1$ for all $x \in (0, 1)$. By the Chain Rule, 0 = Fx(a, b) + Fy(a, b)(dy/dx), hence dy/dx = -Fx(a, b)/Fy(a, b). 12.3 Iterated Integrals. Then $w(x, t) \ge -^2 - rt1 = -^2/2$ for every $(x, t) \in H \setminus K$, i.e., is greater than the value of w at (x1, t1). Therefore, v = 0.65364. c) Since $(1 - \cos(1/x))0 =$ sin(1/x)/x2 < 0 for $x \ge 1, 1 - cos(1/k)$ is decreasing. By Theorem 9.8, then, f - 1 (E) \cap B is closed. Hence V (E1; G) $\le V$ (E2, G) for every grid G. P ∞ a) k=1 1/(k(k+2) - (2k+2)/(k+3) = -3 k=1 (2k/(k+2) - (2k+2)/(k+3) = -3 (2k/(k+3) = -3 ($|(f(x) - f(y))(f(x) + f(y))||f(x) - f(y)| \sqrt{|f(x) - f(y)|} \le = . P_{\infty \infty \infty} + 1 d) = 0$ (5 + (-3)k)/7k+2 = (5 k=0 (5/7)k + k=0 (-3/7)k)/72 = (5/2 + 1/10)/7 = 13/35. Therefore V = $\cup j=1$ Bj as required. 11.6.10. Similarly, (m/n)(n/m) = (mn)/(mn) = mn(mn)-1 = 1, so (m/n)-1 = n/m by the uniqueness of multiplicative inverses. Choose $n \in N$ so that 1 $bn0 \le 2/(2M)$ and N > n0 so large that |fn(x) - f(x)| < 2/2 for $n \ge N$ and $x \in [0, 1]$. $x \rightarrow 1 = 0$. Indeed, let $\varepsilon > 0$ and choose $\delta > 0$ such that $|x - a| < \delta$ implies $|f(x) - L| < \varepsilon$. If $\neg \infty R \infty p = 1$ then $0 dx/(1 + x) = log(1 + x) \neg 0$ which diverges. Thus the Taylor series contains only even terms. Thus g \circ f is almost everywhere continuous, i.e., g \circ f is integrable by Lebesgue's Theorem. Since F is C 1 and Ft (x0, t0) = u0 (t0) 6= 0, it follows from the Implicit Function Theorem that there is an open interval I0 containing x0 and a C 1 function g : I0 \rightarrow R such that g(x0) = t0 and 0 = F (x, g(x)) = u(g(x)) - x for all x \in I0. Suppose that xn-1 \geq 3 for some $\sqrt{n} \ge 1$. $\sqrt{\sqrt{b}}$ by the Power and Chain Rules, g 0 (x) = 2f (x) · f 0 (x)/(2x), so g 0
(4) = 2f (2) · f 0 (2)/(24) = π . 1.4.4. a) The formula holds for n = 1. Homogeneity and positive definiteness are obvious. Thus ak (f (x + h) - f (x - h)) = 2bk (f) sin kh for k \in \mathbb{N}. Suppose xn-1 < 1. 6.2.4. Since logp (k + 1) \ge logp k, we have $\infty X \propto X = 1 \le 1$. Therefore, the original limit is $e_1/6$. 3.1.1.a) Let $\varepsilon > 0$ and set $\delta := \min\{1, \varepsilon/7\}$., $xn \cdot (y1 + z1)$, z = 0 - 0 = 0. Let $t0 = \sup\{t \in (0, 1) : (1 - t)x + ty \in U\}$ and set $x0 = (1 - t0)x + ty \in U\}$ and set x = n(n + 1)/2. C a a Since C is closed, this last difference is zero. If f is also analytic, then by analytic continuation (Theorem 7.56), f (x) = 0 for all $x \in (-\infty, \infty)$. 5.5.1. a) Fix $k \in N$. Then, Z Z Z F · T ds = F · T ds + k=1 k p ak converges for some p > 1, then k p ak $\rightarrow 0$ as $k \rightarrow \infty$, P ∞ i.e., k |ak| < 1 for large k. Then f and g are uniformly continuous on R but (f g)(x) = x2 is not (see Example 3.36). 2 2, the limit is 3. Then by the Second Multiplicative Property, x > x - 1 so 0 > -1, i.e., every number from this case works. P ∞ P ∞ 6.2.0. a) False. 6.6.4. Since the range of f is positive, |f(k)| = f(k) for all $k \in N$. Using the parame $\sqrt{terization \varphi(t)} = (10 \sin t, 10 \cos t, 0)$, $t \in [0, 2\pi]$, we have by Stokes's Theorem that ZZ $Z \pi$ curl $F \cdot n d\sigma = S \sqrt{\sqrt{\sqrt{t}}} (10 \sin t, 10 \sin t,$) - L(f, Pn) = n X (xj - xj - 1)(xj - xj - 1) (xj - xj - 1) = j = 1 n 1 X 1 1 = $\rightarrow 0$ n2 j = 1 n as n $\rightarrow \infty$, so f is integrable by Definition 5.9. Since U (f, Pn) = n 1 X n(n + 1) 1 k = $\rightarrow n2$ 2n2 2 k = 1 R1 as n $\rightarrow \infty$, 0 x dx = 1/2. Taking the limit of this inequality as n $\rightarrow \infty$ and as $^2 \rightarrow 0$, we obtain lim sup(xn yn) \geq x lim sup yn. Since [0, 1] is uncountable, it follows from 1.6.0c that $E1 \times E2 \times \cdots$ is uncountable. $\partial h/\partial xn$ (a)] = $\nabla f(g(a)) \downarrow \downarrow$. In particular, xn = 2xn-1 yn-1 > yn-1. On the other hand, by part b), $I2 \leq M$ (s - a)/ $e\delta(s-a-1) \rightarrow R0$ as s $\rightarrow \infty$. 0 c) Let u = f(x) and dv = g 0 (x) dx. Consider the function f(x) = k=1 (bk/2)/2k. But x1 > 3 and induction + 3 - 2 = 3, so the limit must be x = 3. To find its x-intercept, set y = 0. Since $yn \rightarrow \infty$ implies yn > 0 for n large, we can apply Theorem 2.15 directly to obtain the conclusions when $\alpha > 0$. 6.3.1. a) Since $[1/(k + 1)!]/[1/k!] = 1/(k + 1) \rightarrow 0$ as $k \rightarrow \infty$, this series converges by the Ratio Test. The lower one is closer because $y = x^3$ is concave up on [0, 2], hence closer to the lower sum approximation than the upper sum approximation. b) By hypothesis, 2 < x1 < 3. It diverges for $p \le 0$ by the Divergence Test. c) Since $F = \nabla f$, we have $\nabla(f F) = \nabla f \cdot F + f \cdot \nabla F = F \cdot F + f \cdot \nabla \cdot \nabla f = fxx + fyy + fzz = 0$ by hypothesis. Since g is nonnegative, we have f(x) > M g(x) for $x \in (b0, b)$. By Exercise 13.5.8. But $\nabla \cdot \nabla f = fxx + fyy + fzz = 0$ by hypothesis. Since g is nonnegative, we have f(x) > M g(x) for $x \in (b0, b)$. By Exercise 13.5.8. But $\nabla \cdot \nabla f = fxx + fyy + fzz = 0$ by hypothesis. bounded in Rn . Each polynomial of degree n has at most n roots. Hence it follows from the Extreme Value Theorem that |f| is bounded on [a, b], i.e., supx \in [a,b] |f(x)| is finite. $\partial x \partial y$ Thus ZZ ZZ curl F \cdot n d $\sigma = 0.1.6.6.$ a) We prove this result by induction on n. Since the bn 's are increasing, bn ≤ 1 for all $n \in N$ and $n \geq n0$ imply that 1 – $bn \le 1 - bn0 \cdot 7.1.1.a$) Given $^2 > 0$ choose N so large that N > max{|a|, |b|}². Since g is increasing on [a, b], g 0 (x) ≥ 0 for all $x \in [a, b]$. 12 Copyright © 2010 Pearson Education, Inc. Moreover, $xn = 2 + xn - 1 - 2 \ge 2 + 1 = 3$. 11.6 The Inverse Function Theorem. R 2n P2n - 1 b) Clearly, 1 (1/t) dt > k = 1 1/(k + 1) > n/2 for $n \ge 1.0 k k k k k 0 0 k = 1$ $k=1 k=1 R1 1/k 2 \le 1 + k=2 1/(k(k-1)) = 2.00 On the other hand, using trivial parameterizations, we have ZZ 1 Z S4 Z 1 - u - v, v) dv du 0 Z 0 1 - v + R(u, v, 1 - u - v) du dv. 9.4.4.$ Suppose f is continuous on B, and that E is a closed subset of Rm. Hence, allow no more than an error of 3%. 10.3.7. a) If A = (0, 1) and B = [1, 2] then (A \cup B)o = (0, 2) but Ao \cup B o = (0, 2). c) 0 = fx = 3x2 + 3y and 0 = fy = 3x - 3y 2 imply y = 0 or y = -1, which correspond to the points (0, 0) and (1, -1). k=1 yk2 - 2a k=1 n X xk yk - 2b k=1 it is clear that Fa = -2 n X Fb = -2 yk + k=1 xk yk + 2a k=1 and n X n X yk + 2a xk2 + 2b k=1 $n \rightarrow \infty xn$. If $x \in E$ and $n \ge max\{N0, N\}$ then $\mu \| N0 N^{-1} X^{-1} X^{2} N0^{2} f(x) - f(x) + 1 - 4 = 2$. 10.4.1. a) Since $1/k \rightarrow 0$ as $k \rightarrow \infty$, this set is closed and bounded, hence compact. 1.6.3. Let g be a function that takes A onto B. Hence $x \in (A \cap \partial B) \cup (\partial A \cap \partial B)$. 11.6.3. Let F(x, y, u, v, w) = (u5 + xv 2 - y + w, v 5 + w). $y_{u2} - x + w, w4 + y_{5} - x4 - 1$ and observe that $F(1, 1, 1, 1, -1) = (0, 0, 0). k \pi | \cos ku | du \le \omega(f, -\pi \pi). n \to \infty 0$ Since these two limits are equal, it follows that $Z 1 Z (L) f(x) dx = \lim U (f, Pn) := I. f(x) dx =$ 2π b) By part a), $|ak(f)| \le \omega(f, \pi 1) k 2\pi Z \pi (f(u) - f(u + -\pi Z \pi)) \cos ku du. 0$ Similarly, $ZZZ 0Z 1 \omega = -0Z 1 (Q(1 - y, y, 0) - Q(0, y, 0)) dx + T2 and 1 (R(0, 1 - z, z) - R(0, 0, z)) dz + 0 1 (P(x, 1 - x, 0) - P(x, 0, 0)) dx$. 11.4.9. Let y = f(x) and take the derivative of F(x, f(x)) = 0 with respect to x. j=1 In particular, f is integrable on [a, b]. The upper one is closer because $y = 3-x^2$ is concave down on [0, 2], hence closer to the upper sum approximation. These lines are not parallel because their "direction vectors" (1, 1, 0) and (3, 4, 1) are not parallel. Since [a, b] $\subset (-1, 1)$ implies r < 1 and the ∞ k geometric series k=0 r converges, it follows from the Weierstrass M-Test that the original series converges uniformly on [a, b]. If $a \in U$, then by Theorem 9.7, $xk \in V$ for large k. Moreover, since $x \in [a, b]$ implies $|x|k/k| \le ck/k!$, where $c := max\{|a|, |b|\}$, it follows from the Weierstrass M-Test that the original series converges uniformly on [a, b]. Let curl (P, Q, R) = (x, -y, sin y). Hence Z 1 F · T ds = $\sqrt{2}$ C Z 2π 0 $\sqrt{-\pi}$ 2 (cos t - sin t) dt = . b) Let x, a \in (0, ∞) and q = n/m. On $\sqrt{G(a, \sqrt{the other hand}, since 0 \le a \le b \sqrt{we have A(a, b)} = (a + b)/2 \le 2b/2 = b$ and G(a, b) = ab $\ge a2 = a$. Since E \cap f -1 (A) = E \cap V c and V c is closed, it follows that f -1 (A) \cap E is relatively closed in E. 9.4.3. Recall that f - 1 (V) is relatively open in A if and only if f - 1 (E) = O \cap A for some open O in Rn. n m b) Fix x \in R. Hence by the Approximation Property for Suprema, choose xk \in E such that f (xk) \rightarrow M. Therefore, $\partial/\partial y(\propto \pi 100 \text{ Copyright } \mathbb{G} 2010 \text{ Pearson Education}$, Inc. c) If Vol (E) > 0 then Vol (E 0) > 0 by part b), hence E 0 cannot be empty. Thus the point (x, y, z) is a saddle point and ax + by + cz has no extrema subject to the constraint z = Dx2 + Ey2. $0 \le x < \infty$. A similar argument proves that if f - 1 (A) \cap E is relatively closed in E for all closed sets A in Y. then f is continuous on E. Then C is piecewise smooth and closed and it follows from hypothesis that $Z Z Z Z 0 = F \cdot T ds = F \cdot T ds = F \cdot T ds = F \cdot T ds$, i.e., $F
\cdot T ds = F \cdot T ds$. Given 2 > 0 choose by the Approximation Property an $x0 \in (a, b)$ such that L - 2 < f(x0). Hence by the Approximation Property an $x0 \in (a, b)$ such that L - 2 < f(x0). that $(\phi \circ \psi)0 = (xu ut + xv vt, yu ut + yv vt, zu ut + zv vt)$. If $C = \{\phi_j, [a_j, b_j]\}$ is piecewise smooth, then the integral over C breaks into a finite sum of smooth pieces., n}. b) Since $\log((\log k)p \log k)/\log k = p \log \log k \rightarrow \infty$ if p > 0, this series converges absolutely for all p > 0 by the Logarithmic Test. b) If DE < 0 then by part a), the discriminant is $\sin \theta \sin \phi + c \cos \phi$ $\sin \phi d\phi d\theta 0 Z 0 \pi/2^3 = 2abc' c\pi \cos \phi \sin \phi d\phi = \pi abc(a + b + c)/2$. Similarly, for each $j \ge n$, $\inf (xk + yk) \le xj + yj \le \sup xk + yj$. $x y (x,y) \rightarrow (0,1) 4 c$) The domain of f is all $(x, y) \in R2$ such that (x, y) = (0, 0). ak am an a - 1 k = n Thus $|xm+1 - xn| \le (1/an - 1/am)/(a - 1) \rightarrow 0$ as $n, m \rightarrow \infty$ since a > 1. $x^{2n+1} \}$ of [a, b] such that each $x \in E$ belongs to [x2k, x2k+1] for some $0 \le k \le n$, and $n X^2 | x2k - x2k+1 | = .$ By Definition 10.13, there is a $b \in X$ and an M > 0 such that $xn \in BM$ (b), i.e., $\rho(xn, b) < M$ for all $n \in N$. Thus ψ is 1-1 from Z onto B. e) Since sin(1/x2) is dominated by 1 and $tan x \to 0$ as $x \to 0$, it follows from Theorem 3.9 that this limit is zero. a) Notice x0 > y0 > 1.

Since Py = 0 implies P = h(x, z), we have Pz = hz and Rx = gx. 2 S B1 (0,0) 0 0 b) Let $\varphi(u, v) = (u, 2 \cos v, 2 \sin v)$, $E = [0, 1] \times [0, \pi]$. Thus $L(2n) \rightarrow \infty$ as $n \rightarrow \infty$. $n \rightarrow \infty$ $n \rightarrow \infty$ 1. Let $M \ge ak$ and note that $1/(k + 1)p \le 1/k p$ for all $k \in \mathbb{N}$. If (an + bn) - 1 converged to 0, then given any $M \in \mathbb{R}$, M = 0, there is a converged to 0, then given any $M \in \mathbb{R}$, M = 0, there is a converged to 0, then given any $M \in \mathbb{R}$, M = 0, there is a converged to 0, then given any $M \in \mathbb{R}$, M = 0, there is a converged to 0, then given any $M \in \mathbb{R}$, M = 0, there is a converged to 0, then given any $M \in \mathbb{R}$, M = 0, there is a converged to 0, then given any $M \in \mathbb{R}$ and note that $1/(k + 1)p \le 1/k p$ for all $k \in \mathbb{N}$. an N \in N such that n \ge N implies |an + bn |-1 < 1/|M|. P ∞ b) By Example 6.32, k=1 sin(kx) has bounded partial sums for all x \in R. Then ²0 /M = 0.8636912, r0 = .15 and r = 0.1736732. $\sqrt{\sqrt{c}}$ Let xn be irrational which satisfy xn \downarrow 2. P ∞ a) k=1 P ∞ 2 k b) P k=0 (-1)k-1/\pi 2k = -) = -1/(1 + 1/\pi 2) = -\pi 2/(\pi 2 + 1)., yn) = (x1 + y1, . Let E := {Ua $\alpha \in A$ be a relatively open covering of H. A similar argument shows f (x0) < 0 is also impossible. Then f (x) + g(x) = 1 and f (x)g(x) = 0 for all x \in R. dp dp 11.4.4. By the Chain Rule, ux = yf 0 (xy). c) Since |x| and |y| are $\leq x^2 + y^2$, |f (x, y)| $\leq 2(x^2 + y^2) 1/2 - \alpha \cdot 4.4.6$. a) Let f (x) = log x/x \alpha \cdot 1.4.0. a) False. Then f 0 (x) = cos x > 0 for all x = x + y^2, |f (x, y)| $\leq 2(x^2 + y^2) 1/2 - \alpha \cdot 4.4.6$. a) Let f (x) = log x/x \alpha \cdot 1.4.0. a) False. Then f 0 (x) = cos x > 0 for all x = x + y^2. $x \in (-\pi/2, \pi/2)$ and $f(-\pi/2, \pi/2) = (-1, 1)$. Thus by definition, ax is continuous on R when a > 1. Hence by Bernoulli's Inequality, $(1 + x/k)k/(k+1) \le 1 + x/(k+1)$ for all $x \ge -1$. 13.2.2. a) Let $\varphi(t) = (t, t^2)$ and I = [1, 3]. Since F (0, 0, 0) = 0 and $\partial F = xy + \cos(x + y + z)$ dz equals 1 = 0 at (0, 0, 0), the expression phase a differentiable solution near (0, 0, 0) = 0 and $\partial F = xy + \cos(x + y + z)$ dz equals 1 = 0 at (0, 0, 0), the expression phase a differentiable solution near (0, 0, 0) = 0 and $\partial F = xy + \cos(x + y + z)$ dz equals 1 = 0 at (0, 0, 0), the expression phase a differentiable solution near (0, 0, 0) = 0 and $\partial F = xy + \cos(x + y + z)$ dz equals 1 = 0 at (0, 0, 0), the expression phase a differentiable solution near (0, 0, 0) = 0 and $\partial F = xy + \cos(x + y + z)$ dz equals 1 = 0 at (0, 0, 0), the expression phase a differentiable solution near (0, 0, 0) = 0 and $\partial F = xy + \cos(x + y + z)$ dz equals 1 = 0 at (0, 0, 0), the expression phase a differentiable solution near (0, 0, 0) = 0 and $\theta = 0$ at (0, 0, 0) = 0 and $\theta = 0$ at (0, 0, 0), the expression phase a differentiable solution near (0, 0, 0) = 0 and $\theta = 0$ at (0, 0, 0) = 0 and $\theta = 0$ at (0, 0, 0) = 0 and (0, 0) = 0. 0) by the Implicit Function Theorem. Since g is continuous on [a, b], it follows from the Intermediate Value Theorem that there is a $c \in [a, b]$ such that f(c) = 0, i.e., such that f(c) = 0, i $\cos 2 u \cos 2 v$ is nonzero when v 6 = 0 and $v 6 = 2\pi$, this gives a smooth $C \propto parameterization of the ellipse except at the north and south poles, i.e., the points (0, 0, -c). If you want a more constructive proof, if <math>b \leq 0$ then $a < b - \varepsilon < 0 + 0 = 0$. 10.2.5. Modify the proofs of Remark 3.4, Theorems 3.6, 3.8, 3.9, and 3.10, replacing the absolute value signs with the metric ρ . Pn Pn-1 Pn 6.4.6. By Abel's Formula, k=m ak bk = Bn,m an - k=m Bk,m (ak+1 - ak) where Bn,m := k = m bk . 2 11.6.6. a) Notice $\sqrt{that s = x + y}$, t = xy, and $\sqrt{(x, y)} \in E$ imply s > 0, t > 0, x = s - y, and t = sy - y . x \rightarrow 1 x \rightarrow 1 x(x + 1) x(x + 1) x \rightarrow 1 x(x + 1) $\lim xn - 1 + \dots + x + 1 = 1 + \dots + 1 + 1 = n. \text{ Similarly, B is a closed set containing A, hence A \subseteq B. c) If A and B are as in part a), then <math>\partial(A \cup B) = \emptyset 6 = \{0, 1, 2\} = \partial A \cup \partial B.$ This proves i) $(\alpha x) \times y = (\alpha x 2 y_3 - \alpha x 3 y_2 - \alpha x 3 y_1 - \alpha x 1 y_3 - \alpha x 3 y_1 - \alpha x 3 y_1$ $(\alpha v3), x1(\alpha v2) - x2(\alpha v1) = x \times (\alpha v), so ii)$ holds. 7.1.7. Let $\varepsilon > 0$ and choose δ such that $|x - y| < \delta$ implies $|f(x) - f(y)| < \varepsilon$. 3 3.4.2. a) By L'H^oopital's Rule, sin $x/x \rightarrow 1$ as $x \rightarrow 0$. 9.1.6. a) Let $xk \in E$ converge to some point a. We conclude by part a) that $ah - 11 = \lim_{x \to 0} 11.3.6$. a) By modifying the proof of Lemma 3.28, we can prove that if f(a) 6=0, then |f(a + h)| > |f(a)|/2 > 0 for h small., h, Hence x = y. Hence by part a), $E(q) = E(1 \cdot q) = (E(1))q = eq$ for all $q \in Q$. Thus the function defined by g(x) = f(x), $x \in (a, b)$, g(a) = f(a+) and g(b) = f(b-) is continuous on [a, b]. In particular, $k \ge N$ implies $\infty |ak rk| \ge |ak-1 rk-1| \ge \cdots \ge |aN rN| > 0$. Then xn = 2 for all n, so the limit is 2. b) (0, a) = f(a+) and g(b) = f(b-) is continuous on [a, b]. In particular, $k \ge N$ implies $\infty |ak rk| \ge |ak-1 rk-1| \ge \cdots \ge |aN rN| > 0$. Then xn = 2 for all n, so the limit is 2. b) (0, a) = f(a+) and g(b) = f(b-) is continuous on [a, b]. 1) is bounded and $1/n \in (0, 1)$ has no convergent subsequence with limit in (0, 1). Thus the solution is $(-\infty, 1)$. Thus G has a tangent plane at (a, b, c) by Theorem 11.22. Thus $|an + bn| \rightarrow |x|$, NOT ∞ . b) There is an N \in N such that n, $m \ge N$ implies |xn - xm| and |yn - ym| are $< \varepsilon/2$. To examine the case when (x, y) = (0, 0), notice first that fx (0, 0) = (0, 0). $\lim h \to 0$ (f (h, 0) - f (0, 0))/h = $\lim h \to 0$ 0 = 0. x - $\alpha n \beta n - \alpha n b$) Let $\gamma = f 0$ (x). 2.4 Cauchy sequences. Moreover, by the Comparison Theorem, xn $\in E$ implies x0 $\in E$. Moreover, adding the two given identities, we have x(u2 + v 2) + y(u2 + v 2) = 9 + 7, i.e., (x + y)(u2 + v 2) = 16.0 11.6.2. a) Set F (x, y, z) = xyz + sin(x + y + z). c) For x $\geq R \infty 1$, $sin(1/x) = |sin(1/x)| R \propto \le 1/x$. h Similarly, fy (0, 0) = 0. xk \rightarrow sup E. thus by the Comparison Theorem, k=1 ak converges. 2.5.3. a) Since limn $\rightarrow \infty$ (supk $\ge n \times k < r$, i.e., xk < r for all $k \ge N$. 0 0 0 13.6.9. The proof that i) implies iii) is similar to the proof of Theorem 13.61. 9.3 Limits of Functions. If k=1 ak converges absolutely, then $|ak| \le 1$ for large k. If $-1 < x < -1 + \delta$, then -1 < x < 0 since $\delta \le 1$. By the argument above, $Q(x, y) \rightarrow \infty$ as x, $y \rightarrow \infty$. By the Approximation Property for Infima, choose $xk \in A$ and $yk \in B$ such that $\rho(xk, yk) \rightarrow dist (A, B)$. R k+1 d) False. 4.4.5. a) limx $\rightarrow 0$ since $\delta \le 1$. By the argument above, $Q(x, y) \rightarrow \infty$ as x, $y \rightarrow \infty$. = 25. c) Since |x| < 1 implies $t = |x2 - 1| = 1 - x2 \in (0, 1)$ and (-1)k+1(-1)k = -1, we have by Example 7.49 log $(|x2 - 1| - 1) = -\log(1 - x2) = -\infty X(-1)k+1(-x2)k = 1$ implies $n^2 (2 + \sin(n^3 + n + 1)) \ge n \sqrt{(-1)k+1}(-x^2)k = -1$. Then $2 + \sin \theta \ge 2 - 1 = 1$ implies $n^2 (2 + \sin(n^3 + n + 1)) \ge n \sqrt{(-1)k+1}(-x^2)k = -\infty X(-1)k+1$. -1 - 6 + x. $n \rightarrow \infty$ k $\geq n n \rightarrow \infty$ Case 2. Set $\epsilon n := \text{supk} \geq n | f(k + 1) - f(k) - L|$ and notice by hypothesis that $\epsilon n \rightarrow 0$ as $n \rightarrow \infty$. The boundary is
$x^2 + y^2 = 9$, z = 3/2. a) Let $x \in \partial E$. On the other hand, if m is not a perfect square, then by Remark 1.28, m is irrational. $g - 1 (0, \pi) = (0, \infty)$ is open, no big deal; $g - 1 [0, \pi] = [0, \infty)$ is closed-note that Exercise 10.6.3 does not apply since g is not continuous; $g - 1 (-1, 1) = \{0\}$ is not open and we don't expect it to be; g - 1 [-1, 1] = R is closed-note that Exercise 10.6.3 does not apply since g is not continuous. Then a < r + 2 < b. Thus either f is strictly increasing on (a, b) and takes (a, b) into (f (a+), f (b-)) or f is strictly decreasing. and takes (a, b) into (f (b-), f (a+)). Since all compact sets are closed, the limit of this subsequence must belong to E. Similar arguments prevail for all $k \in N$, so the series is dominated by k=1 [ak]. By the Generalized Mean Value Theorem, f (x) f (x) - f (x) + (a) f 0 (c) = = 0 g(x) g(x) - g(a) g (c) for some c between x and a. 0 RR On the other hand, $Qx = Py = (x^2 - y^2)/(x^2 + y^2)^2$, so E (Py - Qx) dA = 0. In particular, Py = fxy = fxy = fyx = Qx by Theorem 11.2. p $\sqrt{13.2.8}$. Clearly, a = 1, b = f(1) - f(0)/2. By Gauss' Theorem, ZZ ZZZ $\omega = S Z Z A - x^2 - z^2 3 dV = 3 (4x + 2z - x^2 - z^2)/(x^2 + y^2)/(x^2 + y^2)/2$, so E (Py - Qx) dA = 0. In particular, $Py = fxy = fxy = fxy = fxy = fxy = f(x - y^2)/(x^2 + y^$ 2 - 1 d(x, z). Suppose f is continuous on E and A is closed in Y. 2.5.8. It suffices to establish the first identity. The graph of f2 has a tangent at x = 0 because it is trapped between $y = x^2$ and $y = -x^2$, hence squeezed flat at x = 0. By Theorem 10.34, E has no boundary if and only if $E \setminus E \circ = \partial E = \emptyset$, i.e., if and only if $E = E \circ$. Thus set h(x) = c. e) Let $(x, y, z) = \varphi(t)$. Thus it is clear that $A(p0) \le ap0$. 3.4.3. If $\alpha > 0$ then $|x\alpha \sin(1/x)| \le x\alpha \rightarrow 0$ as $x \rightarrow 0+$. β f is 1-1 since f $0(x) = -e1/x/x^2 > 0$ for $x \in (0, \infty)$. To prove the reverse inequality, suppose f is 1-1 and $y \in f(A \setminus B)$. It remains to consider the case $x = \infty$ and $y = -\infty$. c) implies d). b) Multiplying top and bottom by $1/x^2$ we have $5x^2 + 3x - 25 + 3$ $3/x - 2/x = 3x^2 - 2x + 1 = 3x^2 - 2x^2 - 2x^$ $4k 2^{3}x'n+2k = k!(n+k)! n + 2k 2 k=0 = -x2 \infty X k=1^{3}x'n+2k-2 (-1)k-1 = -x2 Bn (x).$ 14.5.1. If F (x0) is a local minimum, then F (x0 + 2h) + F (x0 - 2h) - 2F (x0) ≥ 0 for all h $\in \mathbb{R}$. 3k k 3 k 3 b) Since $\sqrt{x} \sqrt{2} = (x/2)$, this series is geometric. Then a = x0 < x1 < \cdots < xN = b, and $|xk - xk-1| = (b - a)/N < \delta$ by the choice of N for all k. The upper one is closer because $y = \sin(x/5)$ is concave down on [0, 2], hence closer to the upper sum approximation. Since $V\alpha \cap V\beta = \emptyset$ for $\alpha \in A$ is uncountable. 3.2.5. Suppose f (x) $\rightarrow L$ as $x \rightarrow \infty$. 11.2.1. Let V denote the open cube (-1, 1) $\times \cdots \times (-1, 1)$. 1.3.6. a) Let $^2 > 0$ and $m = \inf E$. 2.2. $4 \cdots (2k) k=2$ The radius of convergence is R = 1, i.e., the endpoints of the interval of convergence are 0 and 2. Thus Q = x + h(y, z) and Qz = hz = 0, Ry = gy = 2y. b) Any continuous function f can be extended from a compact subset K of $(0, 2\pi)$ to be continuous and periodic on $[0, 2\pi]$. If ak = -1/k and bk = 1/k 2, then $ak \le bk$ for all $k \in N$ and k=1bk converges absolutely, but Pc) ∞ k=1 ak diverges. The inequality can be strict because if $\frac{1}{2}$ xn = 1 - yn = 0 1 n even n odd then lim supn $\rightarrow \infty$ (xn yn) = 0 < 1 = (lim supn $\rightarrow \infty$ yn)., yM }. 1 1 = e - 1 Z ∞ (1 + n + n(n - 1) + ··· + n!) + n!e-x dx 1 = e - 1 (1 + n + n(n - 1) + ··· + 2n!). c) By hypothesis, given $\varepsilon > 0$ there is an N \in N such that $n \ge N$ implies xn > 1/2 and $|xn - 1| < \varepsilon/(1 + 2e)$. 107 Copyright © 2010 Pearson Education, Inc. Since $c^2 > c^2$, it follows that f 00 (c) > 0. Thus by Theorem 2.36, lim supn $\rightarrow \infty$ (1/xn) = 0 = 1/s. However, $k \cdot (-1)k+1$ /k = (-1)k+1 does not converge to 0 as $k \rightarrow \infty$. 6.5 Estimation of series. Suppose it holds for some $n \ge 1$. By Theorem 6.35, Z Z $\infty - \infty f(x) dx \le sn - s \le f(n) - n f(x) dx$, n so Z $\infty |s - sn| \le f(n) + f(x) dx$. b) R is closed by Theorem 10.16. Since P (f) is a linear combination of f n's and constants, it too is integrable on [a, b] by Theorem 10.16. Since P (f) is a linear combination of f n's and constants, it too is integrable on [a, b] by Theorem 10.16. Since P (f) is a linear combination of f n's and constants, it too is integrable on [a, b] by Theorem 10.16. Since P (f) is a linear combination of f n's and constants, it too is integrable on [a, b] by Theorem 10.16. Since P (f) is a linear combination of f n's and constants, it too is integrable on [a, b] by Theorem 10.16. Since P (f) is a linear combination of f n's and constants, it too is integrable on [a, b] by Theorem 10.16. Since P (f) is a linear combination of f n's and constants, it too is integrable on [a, b] by Theorem 10.16. Since P (f) is a linear combination of f n's and constants, it too is integrable on [a, b] by Theorem 10.16. Since P (f) is a linear combination of f n's and constants, it too is integrable on [a, b] by Theorem 10.16. Since P (f) is a linear combination of f n's and constants, it too is integrable on [a, b] by Theorem 10.16. Since P (f) is a linear combination of f n's and constants, it too is integrable on [a, b] by Theorem 10.16. Since P (f) is a linear combination of f n's and constants, it too is integrable on [a, b] by Theorem 10.16. Since P (f) is a linear combination of f n's and constants, it too is integrable on [a, b] by Theorem 10.16. Since P (f) is a linear combination of f n's and constants, it too is integrable on [a, b] by Theorem 10.16. Since P (f) is a linear combination of f n's and constants, it too is integrable on [a, b] by Theorem 10.16. Since P (f) is a linear combination of f n's and constants, it too is integrable on [a, b] by Theorem 10.16. Since P (f) is a linear combination of f n's and constants, it too is integrable on [a, b] by Theorem 10.16. Since P (f) is a linear combination of f n's and constants, it too is integr must be at most countable by Theorem 1.42ii. d) The Lagrange equations are $3 = 6\lambda x - 3\mu x^2$, $1 = \mu$. Then nk < $\rho(xnk, b) + \rho(a, b) < 1 +$ converges, so f (1) converges. 129 Copyright © 2010 Pearson Education, Inc. 2 2 2 0 0 0 13.1.7. a) If $gk \rightarrow g$ uniformly on $\varphi(I)$, then $gk(\varphi(t))k\varphi(0)(t)k \rightarrow g(\varphi(t))k\varphi(0)(t)k \rightarrow g(\varphi(t))k\varphi(0)(t)k\varphi(0)(t)k \rightarrow g(\varphi(t))k\varphi(0)(t)k\varphi($ dx + (x + y) dy = C2 Z (b + y) dy = b(d - c) + c Z a xy dx + (x + y) dy = C3 c(b2 - a2), 2 d·x dx = b d2 - c2, 2 -d(b2 - a2), 2 138 Copyright © 2010 Pearson Education, Inc. 8.4.8. a) By Remark 8.23, Ø and Rn are clopen. Since $k(x, y)k \le 1 + x^2 + y^2 = kN\varphi k$, it follows that $ZZZZ | f(x, y) - f(0, 0) | d\sigma \le S 2\pi 2 Z \sqrt{8} (1 + x + y) d(x, y) = E 0 (1 + r^2) r$ dr $d\theta = 40\pi$. A similar argument proves that if f - 1 (A) $\cap E$ is relatively closed in E for all closed sets. It follows that $ZZZZ | f(x, y) - f(0, 0) | d\sigma \le S 2\pi 2 Z \sqrt{8} (1 + x + y) d(x, y) = E 0 (1 + r^2) r$ dr $d\theta = 40\pi$. A similar argument proves that if f - 1 (A) $\cap E$ is relatively closed in E for all closed sets. It follows that $ZZZZ | f(x, y) = E 0 (1 + r^2) r$ dr $d\theta = 40\pi$. $1/n \mu$ b $3^{2'} 3^{2'} 1/n M + n M - 2 = \xi M - \langle M - |I| \leq |f(x)| dx 0$ there is a $\delta > 0$ such that $2(*) |x - y| < \delta$ and $x, y \in I$ imply |g(x) - g(y)| < .6.6.1. a) The ratio of successive terms of this series is 2k + 3 > 1. But by hypothesis, $Z b f(x) \cdot f(x) dx = a$ for all $n \in N$. d) Since f(1) = 1, f(0) = 1/2, and f(n) (1) = -1/22 and f(n) (1)3)/2n for $n \ge 2$, we have $\sqrt{\infty} x = 1 + x - 1 X (-1)k - 1 1 \cdot 3 \cdots (2k - 3) + (x - 1)k \cdot 4.5.7$. By 4.5.6a, f - 1 is
differentiable on [c, d]. f) $\lim_{x \to 0^+} (1/(x \log x)/(-1/x^2) = \lim_{x \to 0^+} (1/(x \log x)/(-1/x^2)) = \lim_{$ then $\tilde{A} ! 2/(p-2) \mu \P 2/(p-2) n X 2 1 2 x j = m = m Pn$. Then A and B are connected in R2 but $A \cap B = \{(-1, 1), (1, 1)\}$ T is not connected. b) limx $\rightarrow 0 + (-\sin x - ex)/(\log(1 + x2)) = -\infty$. 5.1.3. a) Let $^2 > 0$ and suppose that f is bounded on [a, b], say by M > 0, and continuous on [a, b] except at a finite set E. 8 13.4.3. a) Using the trivial parameterization z = x4 + y2, we see that N $\varphi = (4x3, -2y, 1)$ points upward. c) By Gauss' Theorem, ZZ Z Z 1 - x2 Z $z F \cdot n d\sigma = 3 dx dy dz = S x 2 - 1032 Z 1 ((2 - x2) 2 - (x2) 2) dx = 8.13.3.1. a)$ If (φ , E) is the parameterization given in Example 13.33, then $\sqrt{k\varphi u \times \varphi v} k = k(v \cos u, v \sin u, -v)k = 2v$. Hence (P, Q, R) = (0, z 2, xy). Finally, let $G = \{R1, .$ Then |ka2k| < 2 for $k \ge N$, i.e., $2k a2k \rightarrow 0$ as $k \rightarrow \infty$. Choose $\alpha 0 \in A$ such that $V \cap E\alpha 0$ $6 = \emptyset$. bk bk bn bk+1 bk $=m k=m Now 1/bn \rightarrow 0$ as $n \rightarrow \infty$ so ∞X ak $= \lim n \rightarrow \infty k=m n-1 X \mu$ (ck - ck+1) k=m 1 bk +1 - 1 bk \P and this limit must exist. Hence 0 by Theorem 4.33, $f -1\sqrt{x}$ ($x := \arcsin x$ is differentiable or $N \rightarrow \infty$). cos y for x = sin y. $\sqrt{The function f might take [a, b]}$ to something outside the domain of g. But X is separable, so it follows from Lindel" of s Theorem that there exist open balls Bj := B² (xj) such that $V \subseteq \bigcup \infty$ j=1 Bj. k=1 6.1.5. By telescoping, $\left\{ \left| \left| \left| \left| -1 (x^2k - x^2(k-1)) \right| = (-1 + \lim x^2k) \right| = 0 \right| k \rightarrow \infty \right| k=1 diverges \infty X |x| < 1 |x| = 1 |x| > 1$. Thus G \circ f is C p on M by definition. c) Suppose F is conservative, i.e., F = (fx, fy) for some C 1 function f on V. Therefore, $fk \rightarrow f$ uniformly on R. Since $(f(x) - f(x0))/(x - x0) = (xn - xn0)/(x - x0) = xn - 1 + \cdots + xn - 1$, it is clear that f 0 (x0) = nxn - 1 . c) Let xn = n and yn = n + 1/n. Pn 7.2.7. Let Fn,m := k = m fk. Thus $n \ge N$ implies $|(2 - 1/n) - 2| \equiv |1/n| \le 1/N < \epsilon$. But if an and bn are Cauchy, then by Theorem 2.29, an +bn $\rightarrow x$ where $x \in \mathbb{R}$. Let f and g be as in the solution to 3.3.0d. Let A = B = [0, 1]. If $(x, y) \rightarrow (1, b)$ for some b > 0, we can assume that $0 < y0 \le y \le y1$ for some y0 < b < y1, so $|(y + 1/n)| \le 1/N < \epsilon$. But if an and bn are Cauchy, then by Theorem 2.29, an +bn $\rightarrow x$ where $x \in \mathbb{R}$. Let f and g be as in the solution to 3.3.0d. Let A = B = [0, 1]. If $(x, y) \rightarrow (1, b)$ for some b > 0, we can assume that $0 < y0 \le y \le y1$ for some y0 < b < y1, so $|(y + 1/n)| \le 1/N < \epsilon$. $1/y| \le (1 + y1)/y0 =: M$. Then $ZZ 1 P dx + Q dy + R dz = (P, Q, R) \cdot (-1, 0, 1) dt C1 0 ZZ 1 = -1 P (1 - t, 0, t) dt + 0 Z R(1 - t, 0, t) dt 0 Z 1 = -R(1 - z, 0, z) dz$. Suppose that E is a bounded, nonempty subset of Z. 2.1.5. If C = 0, there is nothing to prove. The trace spirals around the elliptical cylinder y 2 + 9z 2 = 9. I 13.2.6. Since f is continuously differentiable and nonzero on [a, b], we have by the Intermediate Value Theorem that either f = 0 on [a, b] or f = 0. Therefore, lim supn $\rightarrow \infty$ (xn yn) = ∞ . 1 – u2 Since sin 1 < 1, this last integral is improper only at u = 0. h h b) If f is differentiable at x0 then taking the limit of both inequalities in part a), we obtain estimates of the derivative of f at x0. Thus $n \ge N$ implies $|(2n2 + 1)/(3n2) - 2/3| = |1/(3n2)| \le 1/(3n2) - 2/3| = |1/(3n2)| = 1/(3n2) - 2/3| = 1/(3n2)$ expansion is valid on (0, 2]. If f is not 1-1 then there exist a, $b \in X$ such that a 6= b and y := f (a) = f (b). 86 Copyright © 2010 Pearson Education, Inc. But $n-1 \neq -1$ and uniqueness of multiplicative inverses implies (nq)-1 = $n-1 \neq -1$ and uniqueness of multiplicative inverses implies (nq)-1 = $n-1 \neq -1$ and uniqueness of multiplicative inverses implies (nq)-1 = $n-1 \neq -1$ and uniqueness of multiplicative inverses implies (nq)-1 = $n-1 \neq -1$ and uniqueness of multiplicative inverses implies (nq)-1 = $n-1 \neq -1$ and uniqueness of multiplicative inverses implies (nq)-1 = $n-1 \neq -1$ and uniqueness of multiplicative inverses implies (nq)-1 = $n-1 \neq -1$ and uniqueness of multiplicative inverses implies (nq)-1 = $n-1 \neq -1$ and uniqueness of multiplicative inverses implies (nq)-1 = $n-1 \neq -1$ and uniqueness of multiplicative inverses implies (nq)-1 = $n-1 \neq -1$ and uniqueness of multiplicative inverses implies (nq)-1 = $n-1 \neq -1$ and uniqueness of multiplicative inverses implies (nq)-1 = $n-1 \neq -1$ and uniqueness of multiplicative inverses implies (nq)-1 = $n-1 \neq -1$ and uniqueness of multiplicative inverses implies (nq)-1 = $n-1 \neq -1$ and uniqueness of multiplicative inverses implies (nq)-1 = $n-1 \neq -1$ and uniqueness of multiplicative inverses implies (nq)-1 = $n-1 \neq -1$ and uniqueness of multiplicative inverses implies (nq)-1 = $n-1 \neq -1$ and uniqueness of multiplicative inverses implies (nq)-1 = $n-1 \neq -1$ and uniqueness of multiplicative inverses implies (nq)-1 = $n-1 \neq -1$ and uniqueness of multiplicative inverses implies (nq)-1 = $n-1 \neq -1$ and uniqueness of multiplicative inverses implies (nq)-1 = $n-1 \neq -1$ and uniqueness of multiplicative inverses implies (nq)-1 = $n-1 \neq -1$ and uniqueness of multiplicative inverses implies (nq)-1 = $n-1 \neq -1$ and uniqueness of multiplicative inverses implies (nq)-1 = $n-1 \neq -1$ and uniqueness of multiplicative inverses implies (nq)-1 = $n-1 \neq -1$ and uniqueness of multiplicative inverses implies (nq)-1 = $n-1 \neq -1$ and uniqueness of multiplicative inverses implies x and lim supn $\rightarrow \infty$ sn = y. However, it is connected. $\partial x1 \partial xj$ i=1 j=1 Since H is compact and all these partial derivatives are continuous on H, it follows that there is a C > 0 such that n X n X |D(2) f (c; x - a)| $\leq C |xi - ai| |xj - aj| \leq n2 Ckx - ak2$. k=1 P ∞ P ∞ 6.1.1. (-1)k+1 /ek-1 = k=0 (-1/e)k = 1/(1 + 1/e) = e/(1 + e)., N } PN which covers E such that j=1 |Rj| < 2. 8.1.1 a) $kx - yk \le kx - zk + kz - yk < 2 + 3 = 5$. Thus by part a), Area (E) = 1 2 Z ∞ 0 9t2 dt 3 = (1 + t3) 2 2 Z ∞ 1 du 3 = . Since x = y 3/2 implies x0 = 3 y/2, we have by the explicit form (see the formula which follows (3)) that Z L(C) = 2 1 Z p 1 + 9y/4 dy = 0 1 $\sqrt{2}(133 - 1) 4 + 9y dy =$. By Theorems 8.30 and 9.30, f ([0, 1]) is connected. Therefore, f is continuous at each point $x \in [0, 1]$. $0 < s < \infty$. 9.4.7. a) Since f is continuous, so is kf k. $\pi - \pi$ And, by a sum angle formula, $\sqrt{2} \sqrt{\pi 2} \pi \cos kx \, dx \pi - \pi \sqrt{2} = (\cos(2 + k)x + \cos(2 - k)x) \, dx^2 - \pi \tilde{A}! \sqrt{\sqrt{2}} \sin (2 + k)\pi \sin (2 - k)\pi \sqrt{\sqrt{2}} = (2 + k)\pi \sqrt{\sqrt{2}} \sin (2 - k)\pi \sqrt{2} \sin (2 - k)\pi \sqrt{2} = (2 + k)\pi \sqrt{2} \sin (2 - k)\pi \sqrt{2} \sin (2 - k)\pi \sqrt{2} = (2 + k)\pi \sqrt{2} \sin (2 - k)\pi \sqrt{2} \sin (2 - k)\pi \sqrt{2} = (2 + k)\pi \sqrt{2} \sin (2 - k)\pi \sqrt{2} \sin (2$ $k \sqrt{2} (-1) \sin 2\pi 4(-1)k \sin 2\pi = 2 = .(2n + 1)! (2n + 2)!$ Since $x \in (0, \pi)$ implies xn+1 > 0 and sin c > 0, it follows that R > 0 when n + 1 is odd, i.e., when n = 2m. Hence m S(f; Gm) = m 2 2 1 XX 24m jk = j = 1 k = 1 22m (2m + 1) 24m + 22m = . Since $|cos(\theta)|| \le 1$ for any θ , it follows that R > 0 when n + 1 is odd, i.e., when n = 2m. Hence m S(f; Gm) = m 2 2 1 XX 24m jk = j = 1 k = 1 22m (2m + 1) 24m + 22m = .that $|f(x) - L| = -|\cos(\tan x)|/(x + 1) \le -1/(x + 1) \le \rho(x + 1) \le \rho($ 153 Copyright © 2010 Pearson Education, Inc. If P were uniformly continuous on R, then given 0 < 2 < 1 there is a $\delta > 0$ such that $|x - y| < \delta$ implies |P(x) - P(y)| <
2. If a = 0 then the inequalities are trivial. Since $\nu(s) = \phi(1 - 1 (s))$, it follows from the Chain Rule that $\nu 0(s) := d\nu d\phi dt 1 \phi 0(t) (s) = (t) = 0 \cdot \phi 0(t) = 0$. Hence by Theorem 10.39, $\partial E = 0$ $E \setminus E 0 = \partial(E 0) = \partial(E)$. for each fixed $x \in (0, 2\pi)$. γ f is 1-1 on ($\pi/2, 3\pi/2$) because f 0 (x) = sec2 x > 0 there. 6.3.2. a) The Ratio Test gives 1, but the series converges by the Comparison Test since k > e5 implies log k > 5 so k3 k3 1 < < 2. Thus E is nonempty. Since f is continuous and f (xn) > y0 for all $n \in N$, we have f (x0) $\geq y0$. Thus the integral converges for all p > 1 by part a). Then $|f| \le g$ and g is absolutely integrable on [0, 1] (the integral has value 3), but f is NOT even locally integrable on (0, 1) much less improperly integrable on [0, 1] (the integral has value 3), but f is NOT even locally integrable on [0, 1] (the integral has value 3), but f is NOT even locally integrable on (0, 1) much less improperly integrable on [0, 1] (the integral has value 3), but f is NOT even locally integrable on [0, 1] (the integral has value 3), but f is NOT even locally integrable on [0, 1] (the integral has value 3), but f is NOT even locally integrable on [0, 1] (the integral has value 3), but f is NOT even locally integrable on [0, 1] (the integral has value 3), but f is NOT even locally integrable on [0, 1] (the integral has value 3), but f is NOT even locally integrable on [0, 1] (the integral has value 3), but f is NOT even locally integrable on [0, 1] (the integral has value 3), but f is NOT even locally integrable on [0, 1] (the integral has value 3), but f is NOT even locally integrable on [0, 1] (the integral has value 3), but f is NOT even locally integrable on [0, 1] (the integral has value 3), but f is NOT even locally integrable on [0, 1] (the integral has value 3), but f is NOT even locally integrable on [0, 1] (the integral has value 3), but f is NOT even locally integrable on [0, 1] (the integral has value 3), but f is NOT even locally integrable on [0, 1] (the integral has value 3), but f is NOT even locally integrable on [0, 1] (the integral has value 3), but f is NOT even locally integrable on [0, 1] (the integral has value 3), but f is NOT even locally integrable on [0, 1] (the integral has value 3), but f is NOT even locally integral has value 3), but f is NOT even locally integral has value 3). We have f is NOT even locally integral has value 3), but f is NOT even locally integral has value 3), but f is NOT even locally integral has value 3). We have f is NOT even locally integral has v $(t1 - t0)(t2 - t0)kbk2 = \pm 1.602\pi$ Hence it suffices to prove that $R/r2 \rightarrow 0$ as $r \rightarrow 0$. If a = b then ac = bc since \cdot is a function. 71 Copyright © 2010 Pearson Education, Inc. b) By (2) in 5.1 and part a), $cos \theta = \nabla f(a) \cdot u/(k\nabla f(a)k, kuk) = Du f(a)/k\nabla f(a)k$. 2 Hence $\cdot = 0.0^{-2} \sin(\theta s/2) = -0.0^{-2} \sin(\theta s/2$ f (E) is connected in R, which by Theorem 8.30 means f (E) is an interval. a) $\sigma n = k=0$ (1 - k/n)ak = k=0 (n - k)ak /n = (na0 + (n - 1)a1 + \cdots + an-1)/n = (s1 + \cdots + sn)/n. Then by part b) (with p = 1) and the Comparison Theorem, $\tilde{A} \propto X \cdot 1/2 x_2 j \le k=1 \cdot 11 \cdot 7 \cdot 10$. abj . b) Since E is bounded, choose M so large that |x| < M and |y| < M for all $(x, y) \in E$. f) The maximum of (k-1)/k and the minimum of (k+1)/k for $k \in N$ is 1. But $x \in E$ implies $x \in A$ or $x \in B$. 4M $k=0 \cup nk=1$ [x2k-1, x2k] Let E0 := $\delta > 0$ such that and observe that f is continuous (hence, uniformly continuous) on E0. By Remark 8.14, $S \in L(Rn ; Rn)$. By the Mean Value Theorem, F(x) - F(y) = F 0 (c)(x - y) = $\mu \P f 0$ (c) 1 - 0 (x - y). Therefore, Z 1 Z Z 2x2 Vol (E) = 2 Z 0 x 2 / 4 Z 3 - x (x + y) dy dx = (x 2 / 4 I 0 7x3 63x4 91 +) dx = . Define ψ (respectively τ (t) = ψ) (t) (respectively τ (t) = ψ) on [0, 1] by ψ (t) = ψ (t) (respectively τ (t) = 0 otherwise. Let $xk \in E$ such that ψ (t) = ψ (t) (respectively τ (t) = ψ) (t) (respectively τ (t) = 0 otherwise. Let $xk \in E$ such that ψ (t) = ψ (t) (respectively τ (t) = ψ) (t) (respectively τ (t) (r $xk \rightarrow sup \ E \ as \ k \rightarrow \infty$. b) Let $M \in \mathbb{R}$ and choose by Archimedes an $N \in \mathbb{N}$ such that j=N+1 $|\mathbb{R}j| < ^2/2$. $n \rightarrow \infty \ 10k \ 9n \ .999 \cdots = \lim k=1 \ 2.3$ The Bolzano-Weierstrass Theorem. Since $\psi \ 0$ (u) = u/(1 - u)2, ψ has an absolute minimum of 0 at 0. d) Since the range of tan x on $(-\pi/2, \pi/2)$ is $(-\infty, \infty)$, $c = -\infty$ and d $= \infty$., VN to cover A and VN +1, . b) Since $|\sinh(x/k 2)| \le |x|/k 2 \le \max\{|a|, |b|\}/k 2$ for any $x \in [a, b]$, this series converges uniformly on [a, b] by the Weierstrass M-Test. By definition, $a \in f - 1$ (E) so $x \equiv f(a) \in f(f - 1(E))$. b) By factoring, we see that the inequality is equivalent to 1/(2n + 1) < 1/40, i.e., 2n + 1 > 40. Finally, since x0 = sup E we have $f(x0 + h) \le y0 = f(x0)$ for any h > 0. This verifies the first identity. We obtain $kT(x)k \le M2$ kxk for all $x \in Rn$. c) Repeat the proof of Theorem 2.12, replacing the absolute value by the norm sign. Hence by Dini's Theorem and Theorem 7.10, Z 1³ Z 1 x 'k x e4 - 1 lim 1+ e dx = e2x dx = . - - ≤ k k(k + 1) k k+1 k=1 67 Copyright © 2010 Pearson Education, Inc. k(h, k)k h2 + k 2 $\sqrt{\text{Along the path } \text{H}} = 0$, this expression is 0 and along the path H = 0, this expression is 0 and A, then it follows from b) that $f(A) = f(A \setminus \{x\}) = f(A) \setminus f(\{x\})$, i.e., $f(x) \in /f(A)$, a contradiction. c) If nk = 2k, then $(nk - (-1)nk nk - 1)/nk \equiv -1/(2k)$ converges to 0; if nk = 2k+1, then $(nk - (-1)nk nk - 1)/nk \equiv (2nk - 1)/nk = (4k + 1)/(2k + 1)$ converges to 2. Consequently, (-1)nk nk - 1/(2k) = 1/(2k) + 1/(2k + 1) $f(h, k) - f(0, 0) - \nabla f(0, 0) \cdot (h, k) = (hk)\alpha \log(h2 + k2)$ $(h^2 + k^2) 1/2 = k(h, k)k \le 1 2$ $(h + k^2) \alpha - 1/2 \log 2\alpha \mu 1 h^2 + k^2 \P$. Then $\cos x/(mx + b) \rightarrow \cos 1/0 - = -\infty$ as $x \rightarrow 1 -$, so by Theorem 3.40, this function cannot possibly be uniformly continuous on (0, 1). Repeating the argument above, we find a normal of the form (1, Fy/Fx, Fz/Fx) which again is parallel to n = (Fx (a, b, c), Fy (a, b, c), Fz (a, b, c), Fz (a, b, c))c)). dx j=1 n X $\frac{1}{2}(-1)j-1$ dx1. Thus by Theorem 8.9vii, d is a normal to the plane. $n \rightarrow \infty$ $n \rightarrow \infty$ $1 \rightarrow \infty$ $n \rightarrow \infty$ $1 \rightarrow \infty$ $n \rightarrow \infty$ nand observe that H covers $R \cap Q = \{b\} \times [a2, b2] \times \cdots \times [an, bn]$. Then σ takes B δ (g(x)) onto B1 (0), and by part a), (V, h) belongs to A. Thus the graph of y = |x + 1| for $x \in [-2, 2]$ consists of Rtwo triangles, the left one with base 1 and altitude 1, and the right one with base 3 and altitude 3. g(0, $\pi) = \{1\}$ is connected as Theorem 9.30 says it should; $g[0, \pi] = \{0, 1\}$ is compact but not connected-note that Theorem 9.29 does not apply since g is not continuous; $g(-1, 1) = \{-1, 0, 1\}$ is compact but not connected-note that Theorem 9.29 does not apply since g is not continuous; $g(-1, 1) = \{-1, 0, 1\}$ is compact but not connected-note that Theorem 9.29 does not apply since g is not continuous; $g(-1, 1) = \{-1, 0, 1\}$ is compact but not connected-note that Theorem 9.29 does not apply since g is not continuous. We check the boundary in three pieces. Thus $\{\phi_i\}$ $\varphi(W)$ is a partition of unity on $\varphi(V)$ subordinate to the covering $\{\varphi(W)\}$. The series converges at x = 2 by the alternating series test but diverges at x = 0. (see Exercise 4.2.7) and that 1 1 Mj (f) - mj (f) - \leq . By the Inverse Function Theorem, $\varphi(V)$ is C 1
on $\varphi(V)$. $O Z \pi Z y + 1 dy = 0.28$. If X = [0, 2], A = [0, 1] and $B = \{1\}$, then B \ A = \emptyset but (A \ B)c = [0, 1)c = [1, 2]. b) The Lagrange equations are $2x \sqrt{-4y} = 2x\lambda$ and $-4x + 8y = 2y\lambda$, i.e., $(2x + y)\lambda = 0$., n + 1. But by the Extreme Value Theorem, there exist x*, $y* \in [xk-1, xk]$ such that f (x *) = sup Ek and f (y *) = inf Ek. Since curl F = (zey, 1, y), it follows from Stokes's Theorem that Z ZZ (y 3 ey, 1, y) $\cdot (0, -3y)$ 2, 1) dA F · T ds = $-B\sqrt{3}(0,0)\sqrt{Z} 2\pi 3 CZ$ (3r3 sin $\theta - r2 sin \theta$) dr d $\theta = 0Z = 3\pi 0\sqrt{3} r3 dr + 0 = 27\pi/4$. Thus kf -gk1 is positive definite. a) By Example 7.44, cos x = P7.4.1. ∞ k 2k (-4) x /(2k)! for x \in R., n}, hence $\psi \circ \varphi$ is 1-1 from {1, 2, . Since f (x) = -1 implies x = 1 and f (x) = 2 implies x = 0, we also have f -1 (E) = (0, 1). k=1 This yields two equations in the two unknowns a, b: (n X x2k)a + (k=1 n X xk)b = k=1 (n X xk)a + nb = k=1 (n X xk)a + nb = k=1 n X xk yk yk k=1 so the matrix of coefficients has determinant d0. However, by Exercise 7.1.1b, fn (x)/gn (x) = 1/(nx) does not converges uniformly on (0, 1). 0 c) By the Chain and Product Rules F 0 (x) = x cos xf (x cos x) d (x cos x) = x cos xf (x cos x) d (x cos x) = x cos xf (x cos x) d (x cos x) = x cos xf (x cos x) d (x cos x) = x cos xf (x cos x) d (x cos x) = x cos xf (x cos x) d (x cos x) = x cos xf (x cos x) d (x cos x) = x cos xf (x cos x) d (x cos x) = x cos xf (x cos x) d (x cos x) = x cos xf (x cos x) d (x cos x) = x cos xf (x cos x) d (x cos x) d (x cos x) d (x cos x) = x cos xf (x cos x) d (x cos x) $(x \cos x)(\cos x - x \sin x)$. Now $n3 \le 3n$ holds by inspection for $n = 1, 2, 3, k \ 10\ 10k\ 10\ k = 1\ c)$ Combine b) with the Squeeze Theorem. By assumption iv) and part a), sin $x = \sin((x-x0) + x0) = \sin(x-x0)$ is $x \to x0$. 6.2.1. a) It converges by the Limit Comparison Test, since $(2k + 5)/(3k\ 3 + 2k - 1)\ 2 \to 6 = \sin x0$ as $x \to x0$. $0 \frac{1}{k} 2 3 \text{ as } k \rightarrow \infty$. $2\alpha n = 1 162 \text{ Copyright } \mathbb{C} 2010 \text{ Pearson Education, Inc. } 12.1.7. a)$ Fix $x \in \text{Rn} \cdot xn - x = (xn - x) \sqrt{=\sqrt{xn + x xn + x \sqrt{Since xn}}} + \frac{1}{2} \sqrt{(xn - x)} \sqrt{(xn - x)} + \frac{1}{2} \sqrt{(xn - x)} \sqrt{(xn - x)$ this surely holds for k = 0. Hence $Z Z \pi/2 Z \pi/2 \pi/4 \pi -a\sqrt{2/2} - -a \sin x - a \sin x - a$ Squeeze Theorem, $|f(x)| \le kxk \rightarrow 0$ as $x \rightarrow 0$ so f(0) = 0. Hence $f(x0 + h) - f(x0) \le 0$ when h > 0 and ≥ 0 ||k|. c) Let y = E(xq) and t = E(x). 96 Copyright © 2010 Pearson Education, Inc. Finally, by the Comparison Theorem for Integrals, $|f - g| \le |f - h| + |h - g|$ implies that $kf - gk1 \le kf - hk1 + kh - gk1$, so $kf - gk1 \le kf - hk1 + kh - gk1 \le kf - hk1 + kh - gk1$, so $kf - gk1 \le kf - hk1 + kh - gk1 \le kf - hk1 + kh - gk1 \le kf - hk1 + kh - gk1$. additive inverses, -(m/n) = (-m)/n. In particular, it cannot be rational. d) Let $\varepsilon > 0$ and set $\delta := 3 \varepsilon$. It follows that U is relatively open in B1 (0, 0) and relatively closed in B $\sqrt{2}$ (2, 0). It converges at x = -1/e by the Alternating Series Test but evidently does not converge absolutely. Then |ax0 - ax| = ax - ax0 < aq - ar = ar (aq - r - 1) < ax0 (a1/N - ax) = -1/e1) $\leq \epsilon$. Then $0 = \varphi 0$ (x) = x(-e-x) + e-x implies x = 1. k=0 (Either apply Abel's Transformation to the Geometric Series, or use the techniques introduced in Section 7.3.) Thus by part a), $|\infty X \propto X ak rk - L| = (1 - r)2 |k=0| (k + 1)(\sigma k - L)rk |k=0| (k + 1)(\sigma k - L$ L|rk + 2. b) x - 1 = 0 for $x \in [0, 1)$, so $f(x) := (x^2 + x - 2)/(x - 1)$ is continuous on [0, 1) by Theorem 3.22. We examine the trace of $\varphi(t) = 1$. By repeating the proof of Lemma 3.38 we can show that $f(x_1)$ is Cauchy. k k = 0 k = 2 Since C is a sum of positive numbers, the promised inequality follows at once. 2.2.0. a) False. b) By the Comparison Theorem, Z 2n 1 2X -1 Z k+1 2X -1 Z 1 1 1 X1 dx = dx < < x x k k n n k=1 n k=1 for all $n \in N$. If $1 < x < 1 + \delta$, then $1 < x \le 2$ since $\delta \le 1$. c) The set A given in Example 12.2 is countable, hence of Lebesgue measure zero, but not a Jordan region. A similar argument works for the infimum as well. b) Let $(x, y, z) = \varphi(t)$. Also, $(\lambda x = \sqrt{q}) = 1 + 1 = 1$. (3k - 2)(3k + 1) = -1 - 2k + 2 = - - - - 3k + 1 - 3 < 1, (-1)(-3). and bounded above by 3. b) Let L = 0 and suppose $\varepsilon > 0$. 2.1.1. a) By the Archimedean Principle, given $\varepsilon > 0$ there is an $N \in N$ such that $N > 1/\varepsilon$. Let $xn \to x0 \in (c, d)$. 10.1.6. By Theorem 10.14, if $xn \rightarrow a$ then $xnk \rightarrow a$. (bn - an) it follows from Theorem 12.4 that Vol (R \cap Q) = 0. k=1 $\sqrt[4]{\sqrt[4]{e}}$ v e) Since (k + 1/k k+1/2)/(1/k k) = k + 1/k $\rightarrow 1$ as $k \rightarrow \infty$, this series converges absolutely by the Limit Comparison Test. is open as Theorem 9.26 says it should, f -1 [0, π] = . b) Consider f (x) = ex - 2 cos x - 1. By Exercise 4.5.4b, it follows that f takes I onto some interval J, and f -1 is continuously differentiable on J. If y = 1 then f (x, y) = x3 + 3x - 1 which has no critical points. d) Integrating the binomial series term by term, $Z \propto x \sqrt{\arctan (x + 1)^2 - 1/2} x^2 + 1$ (-1)k k k 2k + 1 k=0 for all |x| < 1. b) The sequence (k hand, if $r \leq 1$, then (by Exercise 5.6.4) $\varphi(x) = x1/r$ is convex on [0, ∞). On the other hand, it is easy to check that $n^2 + 7n$ is not a perfect square for n = 1, 2, . Thus Z 2π (r cos θ)3 (r cos θ)3 (r cos θ)3 (r cos θ)2 (r sin θ) R := fxxx (c, d) cos(2\theta) d $\theta + fxxy$ $d\theta \ 6 \ 2 \ 0 \ 0 \ Z \ 2\pi \ (r \cos \theta)(r \sin \theta) 2 + fxyy \ (c, d) \cos(2\theta) \ d\theta \ 2 \ 0 \ Z \ 2\pi \ (r \sin \theta) 3 + fyyy \ (c, d) \cos(2\theta) \ d\theta.$ We obtain, from part a), y = xy, i.e., x = y. k! $(n + k)! \ 2 \ k \ 68 \ Copyright$ $(x, y) \ (x, y$ x < 0 and $\{(x, y) : x > 0\}$ separates the set. By the Density Theorem for Irrationals, there are infinitely many points in $(x - 2, x + 2) \cap (R \setminus Q)$. $g(0, 1) = \{0\} \cup (1, \infty)$ is connected as Theorem 9.30 says it should; $g[0, 1] = \{0\} \cup (1, \infty)$ is neither compact nor connected as Theorem 9.30 says it should; $g[0, 1] = \{0\} \cup (1, \infty)$ is neither compact nor connected as Theorem 9.30 says it should; $g[0, 1] = \{0\} \cup (1, \infty)$ is neither compact nor connected as Theorem 9.30 says it should; $g[0, 1] = \{0\} \cup (1, \infty)$ is neither compact nor connected as Theorem 9.30 says it should; $g[0, 1] = \{0\} \cup (1, \infty)$ is neither compact nor connected as Theorem 9.30 says it should; $g[0, 1] = \{0\} \cup (1, \infty)$ is neither compact nor connected as Theorem 9.30 says it should; $g[0, 1] = \{0\} \cup (1, \infty)$ is neither compact nor connected as Theorem 9.30 says it should; $g[0, 1] = \{0\} \cup (1, \infty)$ is neither compact nor connected as Theorem 9.30 says it should; $g[0, 1] = \{0\} \cup (1, \infty)$ is neither compact nor connected as Theorem 9.30 says it should; $g[0, 1] = \{0\} \cup (1, \infty)$ is neither compact nor connected as Theorem 9.30 says it should; $g[0, 1] = \{0\} \cup (1, \infty)$ is neither compact nor connected as Theorem 9.30 says it should; $g[0, 1] = \{0\} \cup (1, \infty)$ is neither compact nor connected as Theorem 9.30 says it should; $g[0, 1] = \{0\} \cup (1, \infty)$ is neither compact nor connected as Theorem 9.30 says it should; $g[0, 1] = \{0\} \cup (1, \infty)$ is neither compact nor connected as Theorem 9.30 says it should; $g[0, 1] = \{0\} \cup (1, \infty)$ is neither compact nor connected as Theorem 9.30 says it should; $g[0, 1] = \{0\} \cup (1, \infty)$ is neither compact nor connected as Theorem 9.30 says it should; $g[0, 1] = \{0\} \cup (1, \infty)$ is neither compact nor connected as Theorem 9.30 says it should (1, \infty) it should (1, \infty) is neither compact nor connected as Theorem 9.30 says it should (1, \infty) is neither compact nor connected as Theorem 9.30 says it should (1, \infty) is neither compact nor connected as Theorem 9.30 says it should (1, \infty) is neither compact nor connected as Theorem 9.30 sa $[1, \infty)$ is neither compact nor connected-note that Theorems 9.29 and 9.30 do not apply since g is not continuous. By Green's Theorem, $\partial Br(x0) = \lim (Qx - Py) dA = 0$ for all r > 0 sufficiently small, hence by Exercise 12.2.3, Z 1 (Qx - Py) dA = 0 for all r > 0 sufficiently small, hence by Exercise 12.2.3, Z 1 (Qx - Py) dA = 0 for all r > 0 sufficiently small, hence by Exercise 12.2.3, Z 1 (Qx - Py) dA = 0 for all r > 0 sufficiently small, hence by Exercise 12.2.3, Z 1 (Qx - Py) dA = 0 for all r > 0 sufficiently small, hence by Exercise 12.2.3, Z 1 (Qx - Py) dA = 0 for all r > 0 sufficiently small, hence by Exercise 12.2.3, Z 1 (Qx - Py) dA = 0 for all r > 0 sufficiently small, hence by Exercise 12.2.3, Z 1 (Qx - Py) dA = 0 for all r > 0 sufficiently small, hence by Exercise 12.2.3, Z 1 (Qx - Py) dA = 0 for all r > 0 sufficiently small, hence by Exercise 12.2.3, Z 1 (Qx - Py) dA = 0 for all r > 0 sufficiently small, hence by Exercise 12.2.3, Z 1 (Qx - Py) dA = 0 for all r > 0 sufficiently small, hence by Exercise 12.2.3, Z 1 (Qx - Py) dA = 0 for all r > 0 sufficiently small, hence by Exercise 12.2.3, Z 1 (Qx - Py) dA = 0 for all r > 0 sufficiently small, hence by Exercise 12.2.3, Z 1 (Qx - Py) dA = 0 for all r > 0 sufficiently small, hence by Exercise 12.2.3, Z 1 (Qx - Py) dA = 0 for all r > 0 sufficiently small, hence by Exercise 12.2.3, Z 1 (Qx - Py) dA = 0 for all r > 0 sufficiently small, hence by
Exercise 12.2.3, Z 1 (Qx - Py) dA = 0 for all r > 0 sufficiently small, hence by Exercise 12.2.3, Z 1 (Qx - Py) dA = 0 for all r > 0 sufficiently small, hence by Exercise 12.2.3, Z 1 (Qx - Py) dA = 0 for all r > 0 sufficiently small, hence by Exercise 12.2.3, Z 1 (Qx - Py) dA = 0 for A = 0 for A = 0 for A = 0. Pearson Education, Inc. Since F is C 1, it follows that f is C 2 on V. 0 c) If (φ , E) is the parameterization given in Example 13.32, then N φ = ($(a + b \cos v) \sin v$, so F · N φ = ($(a + b \cos v) \sin v$, so F · N φ = ($(a + b \cos v) \cos u \cos v$, b($a + b \cos v$) sin u cos v, b($a + b \cos v$) sin v), so F · N φ = ($(a + b \cos v) \sin v$, so F · N φ = ($(a + b \cos v) \sin v$), so F · N φ = ($(a + b \cos v) \sin v$, sin u cos v, b($a + b \cos v$) sin u cos v, b($a + b \cos v$) sin u cos v, b($a + b \cos v$) sin u cos v, b($a + b \cos v$) sin v). $= b(a + b \cos v)2 \sin u \cos v - b(a + b \cos v)2 \sin u \cos v + b2$ (a + b cos v) sin 2 v = ab sin 2 v + b3 sin 2 v cos v. Notice that $\varphi(t) = t(1 - x/t) - 1$ (x/t2) + log(1 - u) + log(1 third by z, adding and using both constraints, we see that $xy = \lambda$. Thus the points are (1, 5) and (-3, 29). But $E \subseteq A \subseteq E$ implies $A \setminus E \subseteq E \setminus E = \partial E$. Since (0, 0) lies outside of H, we can disregard it. b) If f(x) = |x|/x then $|f(x)| = 1 \rightarrow 1$ as $x \rightarrow 0$, but f(x) has no limit as $x \rightarrow 0$. 3.1.6. a) By Theorem 1.16, $0 \le ||f(x)| - |L|| \le |f(x) - L|$. 14.4.6. By Theorem 1.16, $0 \le ||f(x)| - |L|| \le |f(x)| - |L|| \le |L||| \le |L|| \le |L||| \le |L|| \le |L|| \le |L|| \le |L|| \le |L||| \le |$ 9.49, f is almost everywhere continuous on $[-\pi, \pi]$. Moreover, by the claim, $A N ! N X X (n+1) k+1 - 1/x^2 k+3 - 1/x^2 lim f (x) = 0 = f (n+1) (0)$. Suppose $\frac{1}{2} PN k - 1/x^2 x 6 = 0 = f (n+1) (0)$. Suppose $\frac{1}{2} PN k - 1/x^2 x 6 = 0 = f (n+1) (0)$. $\sin \theta \cos \theta + fyy \sin 2\theta$, and $u\theta\theta = -fx r \cos \theta + fxy r^2 \sin \theta \cos \theta - fyr sin \theta - fyr r^2 \sin \theta \cos \theta + fyy r^2 \cos 2\theta$. Thus by telescoping, n X (Mj (f) - mj (f))(xj - xj - 1) $\leq j=1$ n X (f (xj) - f (xj - 1))kP k = (f (b) - f (xj - 1))kP k = (f (b) - f (xj - 1))kP k = (f (b) - f (xj - 1))kP k = (f (b) - f (xj - 1))kP k Finally, $\sqrt{2}\sqrt{\sqrt{A(a, b)}} = G(a, b)$ if and only if a = b if and only if a = b if and only if a = b. By the Comparison Test and the p-Series Test, the original series diverges. Since L(1) = 0 by definition and L(x) is continuous at x = 1, this limit is of the form 0/0. If p > 1 is finite, let q = p. Hence this quotient is absolutely integrable by the Comparison Test. Since $\varphi(n + 1) = k0$, we conclude that φ takes {1, 2, . By assumption iv} and part c), $\mu \P \mu \P$ sin(x + h) - sin x cos h - 1 sin h = sin x + cos x \rightarrow sin x · 0 + cos x · 1 = cos x h h h as h \rightarrow 0. c) If $1/a \le 1/b$, then the Multiplicative Property implies b = ab(1/a) \le ab(by the Comparison Test. 166 Copyright © 2010 Pearson Education, Inc. By construction, V ($\partial E \cup E0$) < ². b) Since cos(nn/2) = 0 if n is odd, 1 if n = 4m and -1 if n = 4m + 2, lim suppose that each x₁ ≥ 0. b) To minimize the function F (a, b), as a and b vary, we first find the critical points by setting $\nabla F = 0$. Hence $\infty X (ak+1 - ak + ak-1) = k=1 \propto X$ $n \rightarrow \infty$ A similar argument establishes the second identity. 7.5.2. b) Let a = 3, b = 4, and $f(x) = \sin x$. Thus $\cap k \in N[(k-1)/k, (k+1)/k] = \{1\}$. Then $N\phi = (0, 0, 1)$ and $(\nabla \times F) \cdot N\phi = Qx - Py$. Similarly, if |p| > 1/e, this series diverges by the Ratio Test. 1.2.1. a) If a < b then a + c < b + c by the Additive Property., $xN \in E$ such that $N[E \subset E \subset Bg(x_j) (x_j)$. $k_0 - 1$, $k_0 + 1$, Choose (by Exercise 10.7.6d) a polynomial P such that $|f(x) - P(x)| < \epsilon/2$ for all $x \in [a, b]$, and (by part b) a polynomial Q, with rational coefficients, such that $|Q(x) - P(x)| < \epsilon/2$ for all $x \in [a, b]$, and (by part b) a polynomial Q, with rational coefficients, such that $|Q(x) - P(x)| < \epsilon/2$ for all $x \in [a, b]$, $a_1 = 1$ and $b_2 = 0$, $b_2 = 0$, Since $|H| = 2^2(b^2 - a^2)$. By Theorem 2.9i, $h(xn) \rightarrow L$ as $n \rightarrow \infty$. Since A and B are closed and bounded, use the Bolzano-Weierstrass Theorem to choose subsequences such that $xkj \rightarrow x0 \in A$ and $ykj \rightarrow y0 \in B$. π 12.3.2. a) $E = \{(x, y) : 0 \le x \le 1, x \le y \le x2 + 1\}$ and $Z \mid Z \mid Z \mid x \ge 1$, $x \le y \le x2 + 1\}$ and $Z \mid Z \mid Z \mid x \ge 1$. Conversely, if the integral on the left is zero, then $\Delta u \, dA = 0$ for all such regions $E \subset V \cdot c$) If $f(x) = x^3 - x + 5$ then f(0)(x) = 6x, f(3)(x) = 6, and f(k)(x) = 0 for all $k \ge 4$. Hence it has a convergent subsequence by the Bolzano-Weierstrass Theorem. Then $N\phi = (-2u, -2v, 1)$ points upward, i.e., the wrong way. j=1 9.2.3. For each $x \in E$, choose r = rx > 0 and $fx \ge 0$ such that f is C ∞ on R, f (t) = 1 for t \in Ir (x) := (x - r, x + r), and f (t) = 0 for t \in / Jr (x) := (x - 2r, x + 2r). 2N sin(u/2N) Fix u \in [0, π] and set g(x) = x sin(u/x) for x > 0. If x = y, then f (x, y) = sin2 x/(2x2) \rightarrow 1/2 as x \rightarrow 0. Conversely, if xnk \rightarrow a for every subsequence, then it converges for the "subsequence" xn . Since E is polygonally connected, there is no set g(x) = x sin(u/x) for x > 0. If x = y, then f (x, y) = sin2 x/(2x2) \rightarrow 1/2 as x \rightarrow 0. Conversely, if xnk \rightarrow a for every subsequence" xn . Since E is polygonally connected, there is no set g(x) = x sin(u/x) for x > 0. If x = y, then f (x, y) = sin2 x/(2x2) \rightarrow 1/2 as x \rightarrow 0. Conversely, if xnk \rightarrow a for every subsequence is no set g(x) = x sin(u/x) for x > 0. If x = y, then f (x, y) = sin2 x/(2x2) \rightarrow 1/2 as x \rightarrow 0. Conversely, if xnk \rightarrow a for every subsequence is no set g(x) = x sin(u/x) for x > 0. If x = y, then f (x, y) = sin2 x/(2x2) \rightarrow 1/2 as x \rightarrow 0. Conversely, if xnk \rightarrow a for every subsequence is no set g(x) = x sin(u/x) for x > 0. If x = y, then f (x, y) = sin2 x/(2x2) \rightarrow 1/2 as x \rightarrow 0. Conversely, if xnk \rightarrow a for every subsequence is no set g(x) = x sin(u/x) for x > 0. If x = y, then f (x, y) = sin2 x/(2x2) \rightarrow 1/2 as x \rightarrow 0. Conversely, if xnk \rightarrow a for every subsequence is no set g(x) = x sin(u/x) for x > 0. If x = y, then f (x, y) = sin2 x/(2x2) \rightarrow 1/2 as x \rightarrow 0. Conversely, if xnk \rightarrow a for every subsequence is no set g(x) = x sin(u/x) for x > 0. If x = y, then f (x, y) = sin2 x/(2x2) \rightarrow 1/2 as x \rightarrow 0. Conversely, if xnk \rightarrow 0. Conv is a continuous function $f:[0, 1] \rightarrow E$ with f(0) = x1 and f(1) = x2. b) Fix $x \in H$ o and choose a rectangle R such that $x \in Ro \subset H$. 10.3.4. If $A \subseteq B$ then Ao is an open set contained in B. It follows that $\sup A \leq \sup B/\epsilon$. Since f(x) is even, bk (f) = 0 for $k \in N$. Taking the supremum of this inequality over all kxk = 1, we obtain $M1 \leq kT$ k. Hence by b), f/gx = f(1/g)
is uniformly continuous on E. 4.2.7. a) It is well-known that if A, B \in R and m \in N, then Am – B m = (A – B)(Am-1 + Am-2 B + · · · + AB m-2 = B m-1). The condition 0 < y < x cannot be satisfied by the pair which eventuates when \sqrt{x} takes the minus sign and y the plus sign. Since f 0 (x) = x2-x (2 - x \log 2) < 0 for all x > 2/ log 2, f (x) is strictly decreasing for x large. Set k = n0 + 1. By part a, f(xn) is Cauchy, hence convergent since Y is complete. On the other hand, $\cup j=1$ Bj $\subseteq V$ since each Bj $\subseteq V$. Then supk $\geq n xk \leq -M$ for all $n \geq N$, i.e., lim supn $\rightarrow \infty xn = -\infty$. By the Extreme Value Theorem, there is an $x0 \in [a, b]$ such that f(x0) = M. If $x, y \in Q$, then (*) implies kT $(x, y_0) k \le M^2 \cdot kx - yk \le M^2 n |x_j - y_j|$ b) If (a, b, c) $\in K$ and (a/c, b/c, -1) = t(1, -1, 1), then t = -1, so a = -c and b = c. c) By Exercises 11.1.8 and 11.1.9, L{t2 cos t} = L{cos t}00 (s) = (s/(s_2 + 1)2)0 = 2(s_3 - 3s)/(s_2 + 1)2 (s_2 + the plane x = z out of the cylinder y + z = 1. Conversely, if $g \circ f$ is 1-1 (respectively, onto), then by parts a) and b), $f \equiv g - 1 \circ g \circ f$ is 1-1 (respectively, onto). c) Since $x \circ f = 1$. Since $U \cap E \circ G$ is open (see Remark 8.27ii), there is an r > 0 such that Br (x) $\subset U \cap E \circ G$ A. Thus by the Mean Value Theorem, there is a $c \in (a, b)$ such that f(x) - f(a+) = g(x) - g(a) = g 0 (c)(x - a) = f 0 (c) part b) and Theorem 8.17, kT (x)k \leq M1 kxk. 3.3.4. Let g(x) = f (x) - x. k ak 2n-k an-k k an 2n-k k=0 k=0 1.4.1. a) By hypothesis, x1 > 2. Q(x) bm + \cdots + b0 x - m If m = n, then xn-m = 1 for x 6 = 0 and xn-m-k $\rightarrow 0$ as x $\rightarrow \pm \infty$ for all k > 0. n $\rightarrow \infty$ n $\rightarrow \infty$ even if f were positive on [a, b]. 14.3 Growth of Fourier Coefficients. A p smooth C ∞ parameterization, i.e., $\psi(u, v) = (u, v, \pm c \ 1 - u2 \ /a2 - v \ 2 \ /b2 \)$ and B = {(u, v) : u2 \ /a2 + v \ 2 \ /b2 \) and B = {(u, v) : u2 \ /a2 \) and B = {(u, v) : u2 \ /a2 \) and B = {(u, v) : u2 \ /a2 \) and B = {(u, v) : u2 \ /a2 \) and B = {(u2 \ in some set B if and only if its complement B \ E is relatively closed in B. t t u 1 1 1 e) $L(e) = \lim_{n \to \infty} L(1 + 1/n)/(1/n)$. c) If E is compact. Notice by hypothesis that $q \le p0$ implies $aq \le ap0$. 13.2.5. The easy way is to apply Theorem 12.65 directly. Since $\varphi(x) := 1 + x^2$ is convex on [0, 1], it follows from the Fundamental Theorem of Calculus, Jensen's Inequality, Definition 13.6, and the trivial $\sqrt{1 + |f 0(x)|} dx = a + b$. 9 Copyright © 2010 Pearson Education, Inc. Let curl (P, Q, R) = (x - 2z, -y, 0). If (a, b, c) belongs to the cone, so the cone, belongs to the c then $c^2 = a^2 + b^2 = c^2 + b^2$, i.e., b = 0. Thus the expressions on the right side of statement b) are both well defined. c) We may suppose that $E = \{(x, c) : a \le x \le b\}$. Thus P is uniformly continuous on R. Since $1 - \cos(M/k)$ for $(x, y) \in E$ and k sufficiently large. If f is NOT integrable, the symbol a f (x) dx makes no sense. 10.1.7. If xn is Cauchy, then there is an N such that $n \ge N$ implies $\sigma(xN, xn) < 1$. d) It is clear by construction that f is increasing on [0, 1]. c) Let $L = -\infty$ and suppose without loss of generality that M < 0. Then there exist points a < c < x < d < b such that f (x) lies above the chord through (c, F (c)) and (d, F (d)). c) It converges by the Root Test, since \sqrt{k} ak $\equiv k+1$ 1 $\rightarrow < 1$. By the Extreme Value Theorem, g is bounded on [a, b]. Hence by assumption iii), cos $x = 1 - 2 \sin 2 (x/2) \rightarrow 1 - 2 \sin 2 (x/2) \rightarrow$ Characterization of Continuity, $(f - 1)\sqrt{0}$ is continuous at x0. Given $\epsilon > 0$, choose $N \in N$ such that $n \ge N$ implies $\rho(xn, a)$, $\rho(yn, a) < \epsilon/2$. c) Since $E \subset Z$, given any $x \in R$, $(x - 1/2, x + 1/2) \cap E$ contains at most one point. 8M Since f = g on [t1, tN - 1], it follows from (*), $U(f, P) - L(f, P) = N X (Mj(f) - mj(f))(tj - tj - 1) j = 1 \le 2M (t1 - t0) + M$ Therefore, $\tilde{A} n \mid \mu \P Z X 1 \mid \infty n - x \mid 1 - 1 - 1 x e dx = e + + \dots + 1 + 1 = e . 2 k \rightarrow \infty e lim P \infty - k P \infty 4 k But is a geometric series which converges. - 3 b) Let T (1, 0, 0, 0) = (a, b, c). By definition, this means <math>x = f(a)$ and $f(a) \in E$. Then supk $\geq n xk = \infty$ for all $n \in N$ so by definition, lim supk $\geq n xk = \infty$ for all $n \in N$ so by definition, lim supk $\geq n xk = \infty$ for all $n \in N$ so by definition, lim supk $\geq n xk = \infty$ for all $n \in N$ so by definition, lim supk $\geq n xk = \infty$ for all $n \in N$ so by definition, lim supk $\geq n xk = \infty$ for all $n \in N$ so by definition, lim supk $\geq n xk = \infty$ for all $n \in N$ so by definition, lim supk $\geq n xk = \infty$ for all $n \in N$ so by definition, lim supk $\geq n xk = \infty$ for all $n \in N$ so by definition, lim supk $\geq n xk = \infty$ for all $n \in N$ so by definition, lim supk $\geq n xk = \infty$ for all $n \in N$ so by definition, lim supk $\geq n xk = \infty$ for all $n \in N$ so by definition, lim supk $\geq n xk = \infty$ for all $n \in N$ so by definition, lim supk $\geq n xk = \infty$ for all $n \in N$ so by definition, lim supk $\geq n xk = \infty$ for all $n \in N$ so by definition, lim supk $\geq n xk = \infty$ for all $n \in N$ so by definition, lim supk $\geq n xk = \infty$ for all $n \in N$ so by definition, lim supk $\geq n xk = \infty$ for all $n \in N$ so by definition, lim supk $\geq n xk = \infty$ for all $n \in N$ so by definition, lim supk $\geq n xk = \infty$ for all $n \in N$ so by definition, lim supk $\geq n xk = \infty$ for all $n \in N$ so by definition, lim supk $\geq n xk = \infty$ for all $n \in N$ so by definition, lim supk $\geq n xk = \infty$ for n = 0. $xn = \lim (\sup xk) = \infty = s. (1 - 2k)(-1 - 2k)(-1 - 2k)/(1 \cdot 4 \cdot 8.4.10)$. Hence by the Chain Rule and a one-dimensional change of variables, we have Z g($\psi(u)$) $k\psi 0$ (u)k du = J N Z X k=1 = g \circ \varphi \circ \tau (u) $|\tau 0 (u)| k\phi 0 \circ \tau$ (u)k du = J N Z X k=1 = g \circ \varphi \circ \tau (u) $|\tau 0 (u)| k\phi 0 \circ \tau$ (u)k du Jk0 N Z X k=1 Z 0 g $\circ \phi(t) k\phi 0$ (t)k dt. $-- 0^{-4} 0 \pi/4 0$ Using the substitution t = $2ax/\pi$, dt = $2a/\pi dx$, this last integral can be estimated by Z $\pi/4 = 0 \sqrt{-a} 2/2 - 2ax/\pi \pi dx = 2a Z a/2 - t e 0 \pi dt \le 2a Z 0 \infty e^{-t} dt \le \pi$. Since f is continuous, we can choose a $\delta \in (0, \delta 0)$ such that $|t - t0| < \delta$ implies $|f(t) - f(t0)| < \epsilon 0/2$. k=0 f Pn-1 2 7.4.7. a) Fix $x \in [-1, 1]$. It almost looks connected, except that (-1, 0) and (1, 0) do not belong to the set. If f(x) = -x2 and g(x) = 1, then $f(x) \to 0$ as $x \to 0$. $0+ \text{ and } g(x) \ge 1$, but $g(x)/f(x) = -1/x2 \rightarrow -\infty$ as $x \rightarrow 0+$. Repeating the argument in Example 5.12, we can prove that f is not integrable on [0, 1]. Therefore, $\lim \text{ sups} \rightarrow \infty L\{f\}(s) \ge 0$ as $s \rightarrow \infty$. Notice that $kN\phi k = k(-x, -y, 1)k = 1 + x2 + y2$. \sqrt{d} Since $\log k < k$ for k large, k 3 $\log 2 k/ek < k 4$ /ek for k large. Its speed is $k\psi 0$ (t)k = k(a $\cos t_{x} - a \sin t_{x} = a \cdot b$ If x and y are finite, then the result follows from Theorem 2.17. 7.5.1. Let $f(x) = x^{3} + 3x^{2} + 4x + 1$. Since Y is complete, it follows that $f(xn) \rightarrow y$ for some $y \in Y$. Hence we see by induction that $f(xn) = f(x + \cdots + x) = nf(x)$ holds for all $n \in Z$. Thus the integral over C is zero. $h \rightarrow 0 + h \rightarrow 0 + h - h + a$ Suppose that f is odd. a b) By Exercise 10.7.6d, choose polynomials PN which converge to f uniformly on [a, b] as $N \rightarrow \infty$. Since f and ϕ are 1-1, $\psi(x) = \psi(y)$ implies $\varphi(x) = \varphi(y)$ implies x = y. 7.4.5. We begin with a general observation. If the second case holds, then [H \subseteq Bra (a). 3.1.9. Let $\frac{1}{2} \epsilon = \min M - f(a) f(a) - m$, 2.2 $\frac{3}{4}$. Since $y = 1 \pm x$ are lines with y-intercept 1, it
is easy to see that this ball is a diamond with vertices (1, 0), (0, 1), (-1, 0), and (0, -1). d) $\lim x \to 0 + \log(1 - x^2)/1 = \lim x \to 0 + (1 - x^2)/1 = \lim x \to 0 +$ ∞ . n $\alpha \alpha$ If $\alpha \leq 0$ and $xn = 2/((2n + 1)\pi)$ then $x\alpha$ n sin(1/xn) = (-1) xn does not converge as $n \rightarrow \infty$, i.e., x sin(1/x) α has no limit as $x \rightarrow 0+$. 4sx 4ty 2sx -2ty 112 Copyright (0, 0), since $\partial f(h, 0)$ -f(0, 0) h3 (0, 0) = lim = lim 3 = 1, h $\rightarrow 0$ h $\rightarrow 0$ h ∂x h and $\partial f(0, h) - f(0, 0)$ 0 (0, 0) = lim = lim 3 = 0. By hypothesis, ak+1 \leq a2k for all k \in N. Moreover, by the Borel Covering Lemma, there exist relative closed balls Exj such that E = \cup Exj . Then by Exercise 1.2.5d, 3 < xn+1 < xn . The inverse is f -1 (x) = arctan(x - \pi). Hence by the Generalized Mean Value Theorem, for each $x \in [a, b]$ there exists a c between a and x such that f 0 (c)(f - 1 (x) - f - 1 (a)) = (f - 1)0 (c)(f (x) - f (a)). Since $sin(x + \pi) = -sin x$, we have by Taylor's formula that $3^{--}\delta^{--}\delta 3 | sin(x + \pi) + \delta | = |\delta - sin \delta| = |-cos c|^{----} \le .3 (2k - 1) (2k + 1)3 0 k = 1 k = 0 7.2.5$. Since $| sin(x/(k + 1))/k | \le |x|/(k(k + 1))$, the series converges uniformly on any closed bounded interval [a, b] by the Weierstrass M-Test. Since $|xN - yN| = \delta/2 < \delta$, we have $1 = 2 |xN - yN| < |xN - yN| = |P(xN) - P(yN)| < 2 \delta$ a contradiction. -2/17 3/17 b) Since f (u, v) = (0, 1) implies u = $(2k + 1)\pi/2$ or u = $2k\pi$, $k \in Z$ and \cdot Df (u, v) = we have $1 = 2 |xN - yN| = |P(xN) - P(yN)| < 2 \delta$ a contradiction. -2/17 3/17 b) Since f (u, v) = (0, 1) implies u = $(2k + 1)\pi/2$ or u = $2k\pi$, $k \in Z$ and \cdot Df (u, v) = we have $1 = 2 |xN - yN| = |P(xN) - P(yN)| < 2 \delta$ a contradiction. -2/17 3/17 b) Since f (u, v) = (0, 1) implies u = $(2k + 1)\pi/2$ or u = $2k\pi$, $k \in Z$ and \cdot Df (u, v) = we have $1 = 2 |xN - yN| = |P(xN) - P(yN)| < 2 \delta$ a contradiction. -2/17 3/17 b) Since f (u, v) = (0, 1) implies u = $(2k + 1)\pi/2$ or u = $2k\pi$, $k \in Z$ and \cdot Df (u, v) = we have $1 = 2 |xN - yN| = |P(xN) - P(yN)| < 2 \delta$ a contradiction. -2/17 3/17 b) Since f (u, v) = (0, 1) implies u = (2k + 1)\pi/2 or u = $2k\pi$, $k \in Z$ and \cdot Df (u, v) = we have $1 = 2 |xN - yN| = |P(xN) - P(yN)| < 2 \delta$ a contradiction. -2/17 3/17 b) Since f (u, v) = (0, 1) implies u = (2k + 1)\pi/2 or u = $2k\pi$, $k \in Z$ and \cdot Df (u, v) = we have $1 = 2 |xN - yN| = |P(xN) - P(yN)| < 2 \delta$ a contradiction. -2/17 3/17 b) Since f (u, v) = (0, 1) implies u = (2k + 1)\pi/2 or u = $2k\pi$, $k \in Z$ and \cdot Df (u, v) = we have $1 = 2 |xN - yN| = |P(xN) - P(yN)| < 2 \delta$ a contradiction. -2/17 3/17 b) Since f (u, v) = (0, 1) implies u = (2k + 1)\pi/2 or u = $2k\pi$. 1 or D-1 f(0, 1) = 1 1 0 1 - 1 0 1 = 1 1 - 1 -1 0 1 = 1 1 - 1 1 1 1 2 a - x1 a + x1 x1 $\partial D(\hat{-1}) f(a, b)h + (a, b)k \partial x \partial y$ $\hat{-1} \mu X - 1 \partial f = (a, b)hj + 1 k - 1 - j j \partial x \partial y = 0$ $D(\hat{)} f((a, b); (h, k)) = + \hat{-1} \mu X - 1 j j = 0 = \hat{\mu} \hat{A} \hat{X} j = 0 j \partial f \partial x j \partial y - j \partial f (a, b)hj k - j \partial x \partial y = 0$ $D(\hat{)} f((a, b); (h, k)) = + \hat{-1} \mu X - 1 j j = 0 = \hat{\mu} \hat{A} \hat{X} j = 0 j \partial f \partial x j \partial y - j \partial f (a, b)hj k - j \partial x j \partial y = 0$ $D(\hat{)} f((a, b); (h, k)) = + \hat{-1} \mu X - 1 j j = 0 = \hat{\mu} \hat{A} \hat{X} j = 0 j \partial f \partial x j \partial y - j \partial f (a, b)hj k - j \partial x j \partial y - j \partial f (a, b)hj k - j \partial x j \partial y = 0$ < 1 $(k + 1)kk^{-}(pk)k/k!k63$ Copyright © 2010 Pearson Education, Inc. 3 b) Let $\varepsilon > 0$ and let $\delta = \varepsilon/4$. If A = 0, then f(x) = f(x) - f(a) = f(x) - f(x) - f(a) = f(x)is not connected. By i), either $a - b \in P$, $b - a = -(a - b) \in P$, or a - b = 0. $h \rightarrow 0$ h h Thus F is a differentiable real function on [0, 1], and it follows from the one-dimensional Mean Value Theorem that ZZ $2\pi F \cdot n \, d\sigma = (16 - 2) \, dt = 28\pi$., n} into {1, 2, . Finally, it is easy to verify that if x0 = 6n by induction, 2 < xn+1 < xn for all $n \in N$. $-\pi d$ By the given inequality and part c), $\infty X n (|ak(f)| + |bk(f)|) = \infty$ $2X - 1X(|ak(f)| + |bk(f)|) n = 1 k = 2n - 1 k = 1 \le \infty X \hat{A} n/2 2 n = 1 \le n 2X - 1 ! 1/2 (a2k(f) + b2k(f)) k = 2n - 1 \infty M \pi \alpha X n(1/2 - \alpha) 2$. Let $^2 > 0$ be so small that $[x0 - ^2, x0 + ^2] \subset (c, d)$. b) Let $x0 \in \mathbb{R}$. It's true for n = 1. 0 Therefore, $L\{f\}0(s) = -L\{tf(t)\}(s)$. 2Vol (E) $R_j \cap E6 = \emptyset R_j \subseteq E\setminus E0$ Therefore, f is integrable on E. Let $\delta > 0$. Using f (a)f (a + h) as a common denominator and the definition of T, we have 1 1 f(a) - f(a + h) - f(a)f(a + h) + Df(a)(h) = f(a + h) $(nx) = f(x + \dots + x) = f(x)$ for $n \in N$. Indeed, let f(x) = x + 2, g(x) = x, and [a, b] = [-1, 1]. Let $^2 > 0$ and set $\delta = ^2/(|m| + 1)$. c) By definition, $(x) \to \infty$ as $x \to a$. 2 2 2 b) The formula holds for n = 1. Then $f(x) \in f(A)$ so f(x) = f(a) for some $a \in A$. Thus $|a \cdot (b \times c)| + |a \cdot b| \le 1$. c) Suppose $f(x) \to \infty$ as $x \to a$. 2 2 2 b) The formula holds for n = 1. Then $f(x) \in f(A)$ so f(x) = f(a) for some $a \in A$. Thus $|a \cdot (b \times c)| + |a \cdot b| \le 1$. c) Suppose $f(x) \to \infty$ as $x \to a$. 2 2 2 b) The formula holds for n = 1. Then $f(x) \in f(A)$ so f(x) = f(a) for some $a \in A$. Thus $|a \cdot (b \times c)| + |a \cdot b| \le 1$. c) Suppose f(x) = (a + b) = (is convex. Hence f(x) cannot be continuous at x = 1. Hence by the Intermediate Value Theorem, there is a $c \in [a, b]$ such that f(c) = q(c). Thus f is uniformly continuous on (a, b) by Theorem 3.40. Hence by Taylor's Formula, h2 h2 w(x2 + h, t2) = w(x2, t2) + wxx (x2, t2) + wxx (x2, t2) + (wxx (c, t2) - wxx (x2, t2)) 2 2 for some c between x2 and x2 + h. a) By Exercise 11.6.9, the normal of H at (a, b, c) is parallel to $\nabla F = (2x, 2y, -2z)$. A similar estimate holds for the sine terms. c) To show f (f -1 (E)) = E, let x \in E. Therefore, Br (x) \subseteq U and U is open. Let $\varphi(t) = (t, f(t))$ and $\psi(u) = (f - 1, (u), u)$. b) Let $x \in X$. n Thus if N is so big that $n \ge N$ implies |NOL + f(t)| = (t, f(t)) = E. $(N0)|/n < \epsilon/2$, then the estimate above can be continued as $|f(n)/n - L| < \epsilon/2 + \epsilon/2 = \epsilon$. In particular, the pair U, V separates E, a contradiction. b) By the definition of B0, it is clear that f takes A onto B0. Similarly, div curl F = (F3)yx - (F1)yz = 0. c) Since ((k + 2)k+1/(pk+1 (k + 2)!)/((k + 1)k/(pk k!)) = ((k + 2)k+1/(pk + 1 (k + 2)!)/((k + 1)k/(pk k!)) = (k + 2)k+1/(pk + 1 (k + 2)!)/((k + 1)k/(pk k!)) = (k + 2)k+1/(pk + 1 (k + 2)!)/((k + 1)k/(pk k!)) = (k + 2)k+1/(pk + 1 (k + 2)!)/((k + 1)k/(pk k!)) = (k + 2)k+1/(pk + 1 (k + 2)!)/((k + 1)k/(pk k!)) = (k + 2)k+1/(pk + 1 (k + 2)!)/((k + 1)k/(pk k!)) = (k + 2)k+1/(pk + 1 (k + 2)!)/((k + 1)k/(pk k!)) = (k + 2)k+1/(pk + 1 (k + 2)!)/((k + 1)k/(pk k!)) = (k + 2)k+1/(pk + 1 (k + 2)!)/((k + 1)k/(pk k!)) = (k + 2)k+1/(pk + 1 (k + 2)!)/((k + 1)k/(pk k!)) = (k + 2)k+1/(pk + 1 (k + 2)!)/((k + 1)k/(pk k!)) = (k + 2)k+1/(pk + 1 (k + 2)!)/((k + 1)k/(pk k!)) = (k + 2)k+1/(pk + 1 (k + 2)!)/((k + 1)k/(pk k!)) = (k +
2)k+1/(pk + 1 (k + 2)!)/((k + 1)k/(pk k!)) = (k + 2)k+1/(pk + 1 (k + 2)!)/((k + 1)k/(pk k!)) = (k + 2)k+1/(pk + 1 (k + 2)!)/((k + 1)k/(pk k!)) = (k + 2)k+1/(pk + 1 (k + 2)!)/((k + 1)k/(pk k!)) = (k + 2)k+1/(pk + 1 (k + 2)!)/((k + 1)k/(pk k!)) = (k + 2)k+1/(pk + 1 (k + 2)!)/((k + 1)k/(pk k!)) = (k + 2)k+1/(pk + 1 (k + 2)!)/((k + 1)k/(pk k!)) = (k + 2)k+1/(pk + 1 (k + 2)!)/((k + 1)k/(pk k!)) = (k + 2)k+1/(pk + 1 (k + 2)!)/((k + 1)k/(pk k!)) = (k + 2)k+1/(pk + 1 (k + 2)!)/((k + 1)k/(pk k!)) = (k + 2)k+1/(pk + 1 (k + 2)!)/((k + 1)k/(pk k!)) = (k + 2)k+1/(pk + 1 (k + 2)!)/((k + 1)k/(pk k!)) = (k + 2)k+1/(pk + 1 (k + 2)!)/((k + 1)k/(pk k!)) = (k + 2)k+1/(pk + 1 (k + 2)!)/((k + 1)k/(pk k!)) = (k + 2)k+1/(pk + 1 (k + 2)!)/((k + 1)k/(pk k!)) = (k + 2)k+1/(pk + 1 (k + 2)!)/((k + 1)k/(pk k!)) = (k + 2)k+1/(pk + 1 (k + 2)!)/((k + 1)k/(pk k!)) = (k + 2)k+1/(pk + 1 (k + 2)!)/((k + 1)k/(pk k!)) = (k + 2)k+1/(pk k!)) $+ 2)/(k + 1)k+1 \cdot (1/p) \rightarrow e/p$ as $k \rightarrow \infty$ and e/p < 1, this series $\sqrt{converges}$ absolutely for all $x \in \mathbb{R}$. Moreover, if k=1 ak converges, $P \propto then$ it surely has bounded partial ∞ sums. Hence by Stokes's Theorem, $\neg Z \neg 1 \neg \neg F \cdot T ds - (\nabla \times F)(x) \cdot n d\sigma(x) - (\nabla \times F)(x) - (\nabla$ above (by x0), we conclude that $yn \rightarrow y$ as $n \rightarrow \infty$ for some $y \in \mathbb{R}$. $x \in \mathbb{E}$ Since E is closed and bounded, we conclude by the Heine-Borel Theorem that there exist x1, Therefore, f/g is uniformly continuous on [a, b] by Exercise 3.4.5d. c) By part b) and hypothesis, if f(x) = ax, then ax+h - ax and -1 = ax lim $= ax \cdot 1 = ax$. Thus dL/L = 2dT/T + dg/g. Hence an equation of the plane is given by $((b - a) \times (c - a)) \cdot (x, y, z) = ((b - a) \times (c - a)) \cdot (a, b)/2$. Thus f belongs to Lip 1. Hence the series converges by the Comparison Test and the p-Series test. For example, if k=1 (ak + bk) and k=1 ak converges, then k=1 (ak + bk - ak) = Pc) P $\infty \infty$ (a + b) – a converges by Theorem $6.10. \partial E E 13.5.10. 9.3.4.$ Let $a \in Rn$. $an \cdot a2j = 1$ is 1 if n is odd, and 0 if n is even. By L'H^opital's Rule, this sequence converges to e-x as $k \to \infty$ for all $x \in R$. Since it is nonempty, it follows from the Completeness Axiom that E has a finite sum of a $C y \in Br(xj)(xj)$ for some j. Hence E is an interval, so connected by Theorem 10.56. Combining these statements, if $q \in Q$ then q = n/m so 3n' 3x' n f x = nf = f(x) = qf(x) m m m for $x \in R$. hence convergent. 4.4.0. a) False. b) The statement is: If f is differentiable on (a, b) and has a proper local minimum at x0, then f 0 (x0) = 0 and given $\delta > 0$ there exist x1 < x0 < x2 such that f 0 (x1) < 0, f 0 (x2) > 0, and |x| = -ar1 - k1, br1 + 1 = -ar $x_0 | < \delta$ for j = 1, 2. Thus the desired identity follows from setting A = xq and B = aq, where q = n/m. e) This is the set of points on the circle $(x - 1)^2 + y^2 = 1$ or on the x axis between x = 2 and x = 3. Hence f is uniformly continuous on E. It is evidently connected. By Theorem 3.40, f is uniformly continuous on [0, N]. It also is connected because it cannot be broken into disjoint open pieces. Since $A \cap B = \emptyset$, $x0 \ 6 = y0$. Also, $f \ 0 \ (x) = x2 \ (2xex) + 2xex = 2 \ 2xex \ (x2 + 1) > 0$ when x > 0, so f is strictly increasing, hence 1-1 on $(0, \infty)$. A similar argument works for b < 0. In particular, |an + bn| diverges to ∞ . Since $(x, y) \in R$ implies $k(x, y)k \le |x| + |y| \le max\{|a|, |b|\} + max\{|c|, |d|\}$, R is bounded. Since these functions have no common zero, this function has no local extrema. Thus by Taylor's Formula, there is an x1 between b and c such that f(c) = f(n)(x2)(c - b)n. For the boundary, let $x = 2 \cos \theta$ and $y = \sin \theta$. Suppose $k\nu \in N$ have been chosen so that $k1 < k2 < \cdots < kj$ and $xk\nu > r$ for $\nu = 1, 2, .6.4.2$. a) By the Ratio Test, this series converges for all |x| < 1 and diverges for all |x| > 1.9.5.1. a) Since $1/k \rightarrow 0$ as $k \rightarrow \infty$, this set is closed and bounded, hence compact. Let M be the maximum of f on [a, b]. Applying the Mean Value Theorem to f 0, there is a $c \in (c1, c2)$ such that 0 < f 0 (c2) - f 0 (c1) = (c2 - c1) f 00 (c)Then $\{V\alpha\}\alpha\in A \text{ is an open covering of } H. c\}$ Let θ s represent the angle between $\nu 0$ (s) and suppose for simplicity that s > s0. $k \to \infty E E 12.2.5$. Let $^2 > 0$ and choose M > 0 such that $|f(x)| \le M$ for all $x \in E$. 84 Copyright (0, 0) = 0. xn = 1/n converges to 0 and yn = n2 > 0, but xn yn = n does not converge. c) Since $(\pi k+1/(k+1)!)/(\pi k/k!) p = \pi/(k+1) \rightarrow 0$ as $k \rightarrow \infty$, this series converges by the Ratio Test. } then let Rj be a rectangle which contains xj such that |Rj| < 2/2j. By Green's Theorem and hypothesis, $Z Z Z F \cdot T ds = C1 (Qx - Py) dA = 0$. Hence by the Transitive Property, ac < bd. Let (x, y) be a point on the line segment between (x1, y1) and (x2, y2), and (x, y*) be a point on the chord from (x1, f(x1)) to (x2, f(x2)). Conversely, suppose E $o = \emptyset$. Since tan $3\theta = \tan \theta(\sec 2\theta - 1)$, it follows that Z $\pi/4 0 1 \tan \theta \sec \theta f(\sec \theta) d\theta = 2 3 2 Z 2 2 (u - 1) f(u) du = 1 1 1 (2 - 3) = -.48$ Copyright © 2010 Pearson Education, Inc. If it holds for some $n \in N$ then by Taylor's Formula, |f(xn)| = |f(xn) - f(xn-1) + f(xn-1)| = |f(xn) - f(xn-1 $\circ \tau$ on ((j - 1)/N, j/N). 2.5 Limits supremum and infimum. Hence $|xn yn - (xm ym)| \le |xn - xm| |ym| + |xn| |yn - ym| < \varepsilon$ for n, $m \ge N$. Therefore, there exists a finite subset A0 of A such that $\{H \cap V\alpha\} \alpha \in A0$ covers H. 9.3.6. a) We begin by proving that if g(x, y) := f(x) and $f(x) \rightarrow f(a)$ as $x \rightarrow a$, then $g(x, y) \rightarrow f(a)$ as $(x, y) \rightarrow (a, b)$ no matter what b is. 155 Copyright © 2010 Pearson Education, Inc. Hence $\Gamma 00 \ge 0$, and we conclude by Theorem 3.8. Conversely, if f and f + g are continuous at a then so is f + g by Theorem 2.9 we may suppose that $|x| = \infty$. The converse is trivial. By definition, n X n X $\partial 2f D(2) f(c; x - a) = (c)(xi - ai)(xj - aj)$. By Theorem 4.18, $0 \le f(x0 -), f(x0 +)) \cap f([0, 1]) = \emptyset$. Set $M := \max\{\sup A, \sup B\}$ and observe that M is an upper bound of both A and B. Solving for the integral, we obtain 0 = -st sin bt dt = $b/(s_2 + b_2)$. To evaluate the integral over T1, let $\varphi(y, z) = (0, y, z)$ and E be the triangle with vertices (0, 0, 0), (0, 1, 0), and (0, 0, 1). Let $^2 > 0$. By repeating the argument in part c), but looking for lower estimates $\sqrt{e^2 |Q|}$ for this time, we can show that $S \circ \varphi(Q)$ contains a cube with sides $s(1 - ^2 nM)$, so Vol $(S \circ \varphi(Q)) \ge C$ e some constants $C^2 \rightarrow 1$ as $^2 \rightarrow 0$. Since kN φ k = k(f (u)f 0 (u), -f (u) cos v, -f (u) sin v, k = |f (u)| p 1 + (f 0 (u))2, 141 Copyright © 2010 Pearson Education, Inc. 8.1.6. a) (4, 5, 6) - (1, 2, 3) = (-1, 2, -1) = 0, so the sides of this triangle emanating from (1, 2, 3) are orthogonal. 5 1+x b) By Theorem 7.33 and Example 7.45, ex = $1 + x \tilde{A} \propto X xk k = 0$ [$\tilde{A} k! \propto X ! kk(-1)x = k = 0 \propto X k = 0$ (k X (-1)k - j = 0 j!) x for |x| < 1. P \propto Set f (x, y) = k = 1 ($\varphi k(x) - \varphi k + 1$ (x)) $\varphi k(y)$, and note that f is continuous in each variable. Case 1. (Note: Because the boundary of [a, b] contained only isolated points, this and the Sequential Characterization of Limits was enough to conclude that g was continuous on X. Let P (x) = an xn + \cdots + a0 and Q(x) = bm xm + \cdots + b0, where m \geq n. 4.5.0. a) False because I need not be an interval. Hence the claim holds for n = 1. Given $^2 > 0$ choose $\delta > 0$ such that x, y \in E and $|x - y| < \delta$ then $|(f + g)(x) - (f + g)(y)| \leq |f(x) - f(y)|$ and $|g(x) - g(y)| < ^2/(2M)$. If x, y \in E and $|x - y| < \delta$ then $|(f + g)(x) - (f + g)(y)| \leq |f(x) - f(y)|$ and $|g(x) - g(y)| < ^2/(2M)$. -f(y)| + |g(x) - g(y)| 0 and both |f(x)| and |g(x)| are less than M for $x \in E$. Suppose it holds for some all $k \in [0, j]$ for some $j \ge 0$. Since |x - rn| < 1/n, it follows from the Squeeze Theorem that $rn \to x$ as $n \to \infty$. e) Since |x - rn| < 1/n, it follows from the Squeeze Theorem that $rn \to x$ as $n \to \infty$. e) Since |x - rn| < 1/n, it follows from the Squeeze Theorem that $rn \to x$ as $n \to \infty$. e) Since |x - rn| < 1/n, it follows from the Squeeze Theorem that $rn \to x$ as $n \to \infty$. x)2 and g 0 (t) = -x/t2 - 1/t. Hence {xn} is Cauchy and must converge by Theorem 2.29. 4.2.8. Clearly, f 0 (x) exists when x 6 = 0., 13.3.9. Let (x, y, z) = $\varphi(u, v)$ and (u, v) = $\psi(t)$. Then Z Z F · T ds = C(x) Z x P dx + Q dy = C(x) P (u, y) du + 0. x2 + y 2 - 2x - 2y + 2 (x,y) \rightarrow (1,1) y 2 + 1 9.3.2. a) The iterated limits are 0. P2k 6.1.8. a) Since the ak 's are decreasing, $ka2k = a2k + \cdots + a2k \le ak+1 + ak+2 + \cdots + a2k \le i = inf n \in N$ (supk $\ge n \times k = 0$ af 2 (x) dx = 0 a f 2 (x) dx Education, Inc. On the other hand, it is easy to see by parts that Z 1 π 2 bk (f) = x sin kx dx = $-\pi\pi$ k for k \in N. c) False. 10.6.5. By Theorem 10.62, f (E) is connected in R. On the other hand, $\sqrt{(k/(k+1))/(1/k)} = k/(k+1) \rightarrow 1$ as $k \rightarrow \infty$. b) Let f = 1, n = 1, and [a, b] = [-1, 1]. $\leq k + 1 \rightarrow 1$ as $k \rightarrow \infty$. b) Let f = 1, n = 1, and [a, b] = [-1, 1]. $\leq k + 1 \rightarrow 1$ as $k \rightarrow \infty$. b) Let f = 1, n = 1, and [a, b] =
[-1, 1]. $\leq k + 1 \rightarrow 1$ as $k \rightarrow \infty$. b) Let f = 1, n = 1, and [a, b] = [-1, 1]. $\leq k + 1 \rightarrow 1$ as $k \rightarrow \infty$. b) Let f = 1, n = 1, and [a, b] = [-1, 1]. converges uniformly on [0, 1] by the Weierstrass M-Test. c) Since $(2k+1/(k+1)!)/(2k/k!) = 2/(k+1) \rightarrow 0$ as $k \rightarrow \infty$, this series converges by the Ratio Test. 17 Copyright © 2010 Pearson Education, Inc. Thus the graph of $y = a^2 - x^2$ is a semicircle centered at the origin of radius a. x b) Let u = ex and dv = f 0 (x) dx. But xnk $\rightarrow 0$ when $x \in [0, 1)$ and to 1 when x = 1. It follows from Remark 9.38 that $A \cap B$ is compact. 4.5.3. Let $f(x) = \sin x$. The longest diagonal of this cube is x := (b, b, ... 64 Copyright © 2010 Pearson Education, Inc. c) False, but it's not a l'H^oopital problem. P ∞ 9.6.2. By the Extreme Value P ∞ Theorem, f1 is bounded on E and by Dini's Theorem, k=1 gk = g uniformly on E. Hence by the Comparison Test, $\infty X N X akj \le k=1 j=1 \infty X \alpha k = k=1 \infty X \infty X akj$. In particular, f is differentiable at (0, 0). Hence by part b), DR f (x) ≤ 0 for uncountably many $x \in (c, d) \subset (a, b)$, a contradiction. 9.2.7. a) Since both sets are nonempty and kx - yk is bounded below by 0, the dist (A, B) exists and is finite. Hence by the Heine- Borel Theorem, E is compact. k Let a, b \in R. d) By l'H^oopital's Rule, $(1 - x2) l/x \rightarrow 1$ as $x \rightarrow 0+$. Therefore, does not converge, then k=0 ak $=\infty$. 5.1.2. a) The points are obviously increasing, beginning with 0/n = 0 and ending with n/n = 1. 4.2.0. a) True., hence $|ak| \leq |aN|$ (x) = xi11. Then $y_2 + z_2 = sin2t + cos2t = 1$ and x = z. c) $1/(2n + 1)! \ge 0.00000005$ implies $(2n + 1)! \ge 20,000,0000.$ Thus we can parameterize ∂S by $\varphi(t) = (cost, sint/2), t \in [0, 2\pi], d$) Let G(f) represent the graph of y = f(x) as x varies over [a, b]. Conversely, if xk belongs to any relatively open set for large k then, since $U = E \cap B\epsilon$ (a) is relatively open in E and contains a, $xk \in U \subseteq B\epsilon$ (a) for large k, i.e., $xk \rightarrow a$ as $k \rightarrow \infty$. P ∞ k=0 xk /k! = E(x). Since f 0 (x) > 0 when x > e1/\alpha and f (sup E). Thus P satisfies ii). c) If p = 0, the series obviously doesn't make sense, so we can suppose that p = 0. If $n \ge N$, then $|x + yn - x| = |yn| < \delta$, so $|fn(x) - f(x)| = |f(x + yn) - f(x)| < \epsilon$. Thus the series ak /(k + 1)p has nonnegative terms and is dominated by M/k p. A direct calculation yields y6 > 3.141557494 and x7 < 3.14161012. 0 = lim 11.2.8. Since T is linear, kT (a + h) - T (a) - T (h)k kT (a) + T (h) - T (a) - T (h)k 0 = = 0.27 p 0.2 √ c) Since $\varphi(t) = \sqrt{t} (1, 1, 1)$, this curve is the straight line from (0, 0, 0) to (4, 4, 4). b > x > b - \delta implies $|g(x)| < \sqrt{t} (1, 1, 1)$, this curve is the straight line from (0, 0, 0) to (4, 4, 4). b > x > b - \delta implies $|g(x)| < \sqrt{t} (1, 1, 1)$, this curve is the straight line from (0, 0, 0) to (4, 4, 4). b > x > b - \delta implies $|g(x)| < \sqrt{t} (1, 1, 1)$, this curve is the straight line from (0, 0, 0) to (4, 4, 4). b > x > b - \delta implies $|g(x)| < \sqrt{t} (1, 1, 1)$, this curve is the straight line from (0, 0, 0) to (4, 4, 4). b > x > b - \delta implies |g(x)| < \sqrt{t} (1, 1, 1), this curve is the straight line from (0, 0, 0) to (4, 4, 4). b > x > b - \delta implies |g(x)| < \sqrt{t} (1, 1, 1), this curve is the straight line from (0, 0, 0) to (4, 4, 4). b > x > b - \delta implies |g(x)| < \sqrt{t} (1, 1, 1), this curve is the straight line from (0, 0, 0) to (4, 4, 4). b > x > b - \delta implies |g(x)| < \sqrt{t} (1, 1, 1), this curve is the straight line from (0, 0, 0) to (4, 4, 4). b > x > b - \delta implies |g(x)| < \sqrt{t} (1, 1, 1), this curve is the straight line from (0, 0, 0) to (4, 4, 4). b > x > b - \delta implies |g(x)| < \sqrt{t} (1, 1, 1), this curve is the straight line from (0, 0, 0) to (4, 4, 4). b > x > b - \delta implies |g(x)| < \sqrt{t} (1, 1, 1), the straight line from (0, 0, 0) to (4, 4, 4). b > x > b - \delta implies |g(x)| < \sqrt{t} (1, 1, 1), the straight line from (0, 0, 0) to (4, 4, 4). b > x > b - \delta implies |g(x)| < \sqrt{t} (1, 1, 1), the straight line from (0, 0, 0) to (4, 4, 4). b > x > b - \delta implies |g(x)| < \sqrt{t} (1, 1, 1), the straight line from (0, 0, 0) to (4, 4, 4). b > x > b - \delta implies |g(x)| < \sqrt{t} (1, 1, 1), the straight line from (0, 0, 0) to (4, 4, 4). b = x > b - \delta implies |g(x)| < \sqrt{t} (1, 1, 1), the straight line from (0, 0, 0) to (4, 4, 4). b = x > b - \delta implies |g(x)| < \sqrt{t} (1, 1, 1), the straight line from (0, 0) to (1, 1, 1). The straight line from (0, 0) to (1, 1, 1). The straight line from (0, 0) to (1, 1, 1). converges, then there is an M > 0 such that $|xn| \le M$. 0 12.1.5. a) Notice by definition that E = E 0 and E = E. If > M. $\log(\log k)$ plog k e k e Thus the original series converges by the Comparison Test. E b) If x = 3t/(1 + t3), then xdy - ydx = 3t 3(1 - 2t3) dt 3t2 3(2t - t4) dt 9t2 dt - = .79 Copyright © 2010 Pearson Education, Inc. $\sqrt[4]{v}$ h) Multiplying top and bottom by (x + 4 + x + 1)(x + 3 + x + 1) we obtain p p $\sqrt[4]{v}$ 3 x+3+ x+1 3 1 + 3/x + 1 + 1/x 3 $\sqrt[4]{p}$ = p \rightarrow 2 x+4+ x+1 2 1 + 4/x + 1 + 1/x 3 $\sqrt[4]{p}$ = p \rightarrow 2 x+4+ x+1 2 1 + 4/x + 1 + 1/x 3 $\sqrt[4]{p}$ = p \rightarrow 2 x+4+ x+1 2 1 + 4/x + 1 + 1/x 3 $\sqrt[4]{p}$ = p \rightarrow 2 x+4+ x+1 2 1 + 4/x + 1 + 1/x 3 $\sqrt[4]{p}$ = p \rightarrow 2 x+4+ x+1 2 1 + 4/x + 1 + 1/x 3 $\sqrt[4]{p}$ = p \rightarrow 2 x+4+ x+1 2 1 + 4/x + 1 + 1/x 3 $\sqrt[4]{p}$ = p \rightarrow 2 x+4+ x+1 2 1 + 4/x + 1 + 1/x 3 $\sqrt[4]{p}$ = p \rightarrow 2 x+4+ x+1 2 1 + 4/x + 1 + 1/x 3 $\sqrt[4]{p}$ = p \rightarrow 2 x+4+ x+1 2 1 + 4/x + 1 + 1/x 3 $\sqrt[4]{p}$ = p \rightarrow 2 x+4+ x+1 2 1 + 4/x + 1 + 1/x 3 $\sqrt[4]{p}$ = p \rightarrow 2 x+4+ x+1 2 1 + 4/x + 1 + 1/x 3 $\sqrt[4]{p}$ = p \rightarrow 2 x+4+ x+1 2 1 + 4/x + 1 + 1/x 3 $\sqrt[4]{p}$ = p \rightarrow 2 x+4+ x+1 2 1 + 4/x + 1 + 1/x 3 $\sqrt[4]{p}$ = p \rightarrow 2 x+4+ x+1 2 1 + 4/x + 1 + 1/x 3 $\sqrt[4]{p}$ = p \rightarrow 2 x+4+ x+1 2 1 + 4/x + 1 + 1/x 3 $\sqrt[4]{p}$ = p \rightarrow 2 x+4+ x+1 2 1 + 4/x + 1 + 1/x 3 \sqrt[4]{p} = p \rightarrow 2 x+4+ x+1 2 1 + 4/x + 1 + 1/x 3 \sqrt[4]{p} = p \rightarrow 2 x+4+ x+1 2 1 + 4/x + 1 + 1/x 3 \sqrt[4]{p} = p \rightarrow 2 x+4+ x+1 2 1 + 4/x + 1 + 1/x 3 \sqrt[4]{p} = p \rightarrow 2 x+4+ x+1 2 1 + 4/x + 1 + 1/x 3 \sqrt[4]{p} = p \rightarrow 2 x+4+ x+1 2 1 + 4/x + 1 + 1/x 3 \sqrt[4]{p} = p \rightarrow 2 x+4+ x+1 2 1 + 4/x + 1 + 1/x 3 \sqrt[4]{p} = p \rightarrow 2 x+4+ x+1 2 1 + 4/x + 1 + 1/x 3 \sqrt[4]{p} = p \rightarrow 2 x+4+ x+1 2 1 + 4/x + 1 + 1/x 3 \sqrt[4]{p} = p \rightarrow 2 x+4+ x+1 2 1 + 4/x + 1 + 1/x 3 \sqrt[4]{p} = p \rightarrow 2 x+4+ x+1 2 1 + 4/x + 1 + 1/x 3 \sqrt[4]{p} = p \rightarrow 2 x+4+ x+1 3 1 + 3/x + 1 + 1/x 3 \sqrt[4]{p} = p \rightarrow 2 x+4+ x+1 3 1 + 3/x + 1 + 1/x 3 \sqrt[4]{p} = p \rightarrow 2 x+4+ x+1 3 1 + 3/x + 1 + 1/x 3 \sqrt[4]{p} = p \rightarrow 2 x+4+ x+1 3 1 + 3/x + 1 + 1/x 3 \sqrt[4]{p} = p \rightarrow 2 x+4+ x+1 3 1 + 3/x + 1 + 1/x 3 \sqrt[4]{p} = p \rightarrow 2 x+4+ x+1 3 1 + 3/x + 1 + 1/x 3 \sqrt[4]{p} = p \rightarrow 2 x+4+ x+1 3 1 + 3/x + 1 + 1/x 3 \sqrt[4]{p} = p \rightarrow 2 x+4+ x+1 3 1 + 3/x + 1 + 1/x 3 \sqrt[4]{p} = p \rightarrow 2 x+4+ x+1 3 1 + 3/x + 1 + 1/x 3 \sqrt[4]{p} = p \rightarrow 2 x+4+ x+1 3
1 + 3/x + 1 + 1/x 3 \sqrt[4]{p} = p \rightarrow 2 x+4+ x+1 3 1 + 3/x + 1 + 1/x 3 \sqrt[4]{p} = p \rightarrow 2 x+4+ x+1 3 1 N, choose j > N such that inf $k \ge n xk + 2 > xj$, i.e., $1/(\inf k \ge n xk + 2) < 1/xj \le \sup k \ge n (1/xk)$. If $|ak+1|/|ak| \le x$ for k > N, then $|aN+1| \le x|aN|$, $|aN+2| \le x2|aN|$, |a| = x2|aN|, |balls, then $\rho(a, b) \leq \rho(x, a) + \rho(x, b) < 2r = \rho(a, b)$, a contradiction. Let $x \in E \cap U$ and $y \in E \cap V$. Since f is continuous, and both B and E are closed, it follows that $a \in B$ and $f(a) \in E$. By (2), this occurs if and only if the angle between them is $\pi/2$. $3\pi/2 \pi/2 - \pi \sin^2 t$ dt = $\sqrt{2}$. Note: $dL/L = \pm 0.05$ does not work because then dT/T = (dL/L - dq/q)/2 might equal (0.05 + 0.01)/2 = 0.03, outside the 2% error allowed for T . 8.4.5. Suppose $x \in /E$ o but Br $(x) \subset E$. If f(q) = 0 for all $q \in Q \cap [0, 1]$, it follows that $f(x_0) = 0$. Since E is a nonempty subset of [a, b], c) Since F is a nonempty subset of [a, b], sup E is a finite real number which belongs to [a, b]. if f is differentiable at x0 and f has a local minimum at x0 then f 0 (x0) = 0. $x \in [a, b]$, we have that fn \rightarrow f in C[a, b], we have that fn \rightarrow f in C[a, b], we have that fn \rightarrow f in C[a, b], we have that fn \rightarrow f in C[a, b], we have that fn \rightarrow f in C[a, b], we have that fn \rightarrow f in C[a, b] if and only if fn \rightarrow f in C[a, b]. -f(x). Plugging this into the constraint, we obtain z 3/4 = 16, i.e., z = 4, x = y = 2. $2uy 2vx \partial(u, v)$ Thus by the Implicit Function Theorem, if F(x0, y0, u0, v0) = (0, 0), x206 = y02, and u06 = 06 = v02, then such solutions u, v = 16, i.e., z = 4, x = y = 2. $2uy 2vx \partial(u, v)$ Thus by the Implicit Function Theorem, if F(x0, y0, u0, v0) = (0, 0), x206 = y02, and u06 = 06 = v02, then such solutions u, v = 16, i.e., z = 4, x = y = 2. $2uy 2vx \partial(u, v)$ Thus by the Implicit Function Theorem, if F(x0, y0, u0, v0) = (0, 0), x206 = y02, and u06 = 06 = v0, then such solutions u, v = 16. $|f(x) - f(y)| < ^2$. 10.2 Limits of Functions. Then f(0) = 0, and $f(3) = \log(5)$, it follows that $f(E) = [0, \log(5)]$. d) By hypothesis, $3 < x_1 < 5$. b) If $\{V\alpha\}$ is an open covering of f(H), then $\{f - 1, (V\alpha)\}$ is a relatively open covering of f(H), then $\{f - 1, (V\alpha)\}$ is a relatively open covering of f(H), then $\{f - 1, (V\alpha)\}$ is a relatively open covering of f(H), then $\{f - 1, (V\alpha)\}$ is a relatively open covering of f(H), then $\{f - 1, (V\alpha)\}$ is a relatively open covering of f(H), then $\{f - 1, (V\alpha)\}$ is a relatively open covering of f(H), then $\{f - 1, (V\alpha)\}$ is a relatively open covering of f(H), then $\{f - 1, (V\alpha)\}$ is a relatively open covering of f(H). is a constant B α such that $x\alpha \leq B\alpha$ ex for all $x \in (0, \infty)$, $B\alpha \rightarrow 1$ as $\alpha \rightarrow 0+$, and $B\alpha \rightarrow 0$ as $\alpha \rightarrow \infty$. b) The correct statement is: If x is a lower bound of E and $x \in E$ then $x = \inf E$. 8.4.1. a) The closure is $\mathcal{O} \setminus \{0\}$, the interior is \mathcal{O} , the boundary is $E \cup \{0\}$, the interior is \mathcal{O} , the boundary is $E \cup \{0\}$, the interior is \mathcal{O} , the boundary is $E \cup \{0\}$, the interior is \mathcal{O} , the boundary is $E \cup \{0\}$, the interior is \mathcal{O} , the boundary is $E \cup \{0\}$, the interior is \mathcal{O} , the boundary is $E \cup \{0\}$. x, it follows from factoring that $\sqrt{1 - \cos x} = \sqrt{1 + \cos x}$ bounded interval I, we know from the Extreme Value Theorem that there is a $c \in I$ such that $|f 0(x)| \ge |f 0(c)| > 0$ for all $x \in I$. It follows that $x \in E \setminus E$ o $\subseteq \partial E$. $n \to \infty$ $n \to \infty$ 2.5.7. It suffices to prove the first identity. Thus 0 = -xy dx converges uniformly on $[1, \infty)$ by definition. Since it also contains $\psi(0) = (1, 0, 1, 0)$ and $\psi(1) = (1, 1, 2, 1)$, it follows that a+b+c=0 a+c=1 a + b + 2c + d = 1. Therefore, E has no cluster points. First observe by a one- dimensional result that wt (x2, t2) = 0. The ψ is 1-1 from $\{1, 2, . \pi 2k + 1 \pi 0 k = 1 \text{ Since by a sum angle}$ formula and telescoping we have 2 sin t N -1 X cos(2k + 1)t = N -1 X k=1 it follows that (sin 2kt - sin(2k - 2)t) = sin 2N t, k=1 4 (S2N f)(x) = $\pi Z x 0 \sin 2N t dt$. x = ∞ . Then Z $\delta Z \propto L{f}(s) \leq 2(s - a) e^{-\delta(s-a-1)} e^{-t} |\varphi(t)| dt =: I1 + I2$. The set A in Example 12.2 is countable but not a Jordan region. 2 2 b) By part a) (SN f)(0) + \cdots + (S2N f)(0) N + 1 (S0 f)(0) + \cdots + (S2N f)(0). But the intersection of two open sets is an open set. c) Let C1 represent the piece in x = b, C3 and choose a partition $P = \{x0, x1, .54 \text{ Copyright } \mathbb{O} \ 2010 \text{ Pearson Education, Inc. } 6.5.4. \text{ Fix } n \ge N \ . P \text{ ak } cos(kx) \text{ converges for each } x \in (0, 2\pi). b)$ By Remark 8.10 and the Squeeze Theorem, kxk × yk k ≤ kxk k kyk k ≤ M kxk k → 0 as k → ∞. If ak = 1/k 2, then k=1 ak is absolutely convergent, but |ak |1/k → P1 as k → ∞. Therefore, Z $\pi/2 L(C) = 4 0$ y): $x^2 + 4y^2 > 1$ is open. Thus E1 \cap E2 is a Jordan region by Theorem 12.4. Since E1 \setminus E2 = E1 \cap E2c and $\partial(E2c) = \partial E1 \cup \partial E2$, the set E1 \setminus E2 is also a Jordan region. b) Clearly, $s^{2n+2} = s^{2n} + 1/(2n + 1) - 1/(2n + 2) > s^{2n}$ and $s^{2n+1} = s^{2n-1} - 1/(2n + 1) < s^{2n-1}$. In particular, Z lim $n \rightarrow \infty$ a b Z b ³ x'n - x 1 + e dx = dx = b - a. Since L is 1-1, we have y = tq, i.e., E(xq) = (E(x))q. Hence by induction, xn is decreasing 13 Copyright © 2010 Pearson Education, Inc. Since N\u03c6 x is continuous, it follows from the sign preserving property that there is an r(x) > 0 such that N\u03c6 x is nonzero on a relative closed ball Ex := E \cap Br(x) (x). Then xk vk $\geq (x - b - a)$. ²)yk for each $k \ge n$, i.e., supk $\ge n$ (x yk) $\ge (x - 2)$ supk $\ge n$ yk. If xk $\in E$ satisfies xk $\rightarrow 0$ as $k \rightarrow \infty$, then by the continuity of f, $|f(0)| = \lim |f(xk)| \le \lim kxk k\alpha = 0$. By hypothesis, nan = so an = n+1 n 1 - = , n+2 n+1 (n + 1)(n + 2) 1 1 = n(n + 1)(n + 2) 1 = n(n + 1)(n + 2)(n Boundary. Finally, if p < 1 then $Z \propto dx \ge 1 + xp \ 0 \ Z \propto 1 dx \ 1 \ge 1 + xp \ 0 \ Z \propto 1 dx$ and f 0 (x0) = nxn-1. Now ∂S has two pieces: C1 given by $\varphi(t) = (\sin t, 2, \cos t), t \in [0, 2\pi]$, and C2 given by $\psi(t) = (2 \cos t, 4, 2 \sin t), t \in [0, 2\pi]$. Hence by Theorem 3.8 and part a), $f + (x) \rightarrow L - as x \rightarrow x0$ through E. 2 ζ) Since f 0 (x) = (1 - xp)/(x2 + 1)2 is never zero on (-1, 1), f is 1-1 on [-1, 1]. Since E is bounded, the Bolzano-Weierstrass Theorem implies that some subsequence xkj converges to a point a. 10.4.10. Hence it follows from Gauss' Theorem that ZZ ZZ (f F) \cdot n d σ = F \cdot F dV. Let x $\in \cap \alpha \in A \to \alpha$. b), c) Repeat the proof of Exercise 9.4.7. 10.6.7. a) Repeat the proof of Exercise 9.4.7. a) Repea The ratio is less than or equal to 1/2 for k > N = 2. Since $2^{-2} 2^{-1} xn^{-2} xn^{-2} y2^{-1} y2^{-1}$ if x > x0, choose r < x0 < x < q such that q - r < 1/N. 4.2.2. a) By the Product and Chain Rules, g 0 (x) = 2x2 f 0 (x2) + f (x2), so $g 0 (2) = 8f 0 (4) + f (4) \sqrt{= 8e + 3}$. Notice once and for all that $\mu \P 3(1 - 2t3) 3(2t - t4) 0 \varphi(t) = r$. Since E is not polygonally connected, there exist points x0 6 = y0 in E such that $Ux0 \cap Uy0 = \emptyset$. If p > 0, then $\log(\log(\log k)) > 2/p$ for large k implies that p $\log(\log(\log k)) > 2$ for large k. Since 1 + n > 0 for all n \in N, it follows that n + 1 > 100, i.e., n > 99. Let Ba be relatively open in E, i.e., Ba = E \cap Va for open sets Va in Rn . 0 2 x(ex - 1) dx = 0 Z x2 + z 2 Z 1 8 (x + y) dy dx = Z e - 2 . 4.1.7. a) Let yn $\rightarrow x0 \in (0, \infty)$. 0 0 b) Since f is bounded, R $|\varphi(t)| \leq M < \infty$ for all $t \in (0, \infty)$. Since $\sqrt{\sqrt{0}} \varphi(t) = (-z0 \sin t, z0 \cos t, 0)$, we have $\sqrt{\sqrt{\sqrt{0}}} \varphi(t) = (-z0 \sin t, z0 \cos t, 0)$, we have $\sqrt{\sqrt{\sqrt{0}}} \varphi(t) = (-z0 \sin t, z0 \cos t, 0)$, we have $\sqrt{\sqrt{\sqrt{0}}} \varphi(t) = (-z0 \sin t, z0 \cos t, 0)$, we have $\sqrt{\sqrt{\sqrt{0}}} \varphi(t) = (-z0 \sin t, z0 \cos t, 0)$, we have $\sqrt{\sqrt{\sqrt{0}}} \varphi(t) = (-z0 \sin t, z0 \cos t, 0)$, we have $\sqrt{\sqrt{\sqrt{0}}} \varphi(t) = (-z0 \sin t, z0 \cos t, 0)$, we have $\sqrt{\sqrt{\sqrt{0}}} \varphi(t) = (-z0 \sin t, z0 \cos t, 0)$, we have $\sqrt{\sqrt{\sqrt{0}}} \varphi(t) = (-z0 \sin t, z0 \cos t, 0)$, we have $\sqrt{\sqrt{\sqrt{0}}} \varphi(t) = (-z0 \sin t, z0 \cos t, 0)$, we have $\sqrt{\sqrt{0}} \varphi(t) = (-z0 \sin t, z0 \cos t, 0)$, we have $\sqrt{\sqrt{0}} \varphi(t) = (-z0 \sin t, z0 \cos t, 0)$, we have $\sqrt{\sqrt{0}} \varphi(t) = (-z0 \sin t, z0 \cos t, 0)$, we have $\sqrt{\sqrt{0}} \varphi(t) = (-z0 \sin t, z0 \cos t, 0)$, we have $\sqrt{\sqrt{0}} \varphi(t) = (-z0 \sin t, z0 \cos t, 0)$. exists nowhere on R., xk+1 are distinct points in E (a - r, a + r). By b), the former set is countable. But $|\log x| \leq Cx\alpha$, so $\log k/k p \leq C/k p - \alpha$. 3.4.8. a) By symmetry, we need only show f (b-) exists., tN } be a partition of I which satisfies $^2 kP k < \min\{\delta, \}$. Hence by Exercise 7.1.5c, fn /gn = fn (1/g) = f/g uniformly on [a, b] as $n \to \infty$. g g g 2 4.2.6. By the Product Rule, this formula holds for n = 1. By iterating
Exercise 4.1.9, we see that all derivatives of f of even order are even functions. h |h| Since $\alpha > 1$, this last number converges to zero as $h \rightarrow 0$ in I. Parameterize C1 by $\varphi(t) = (8 \cos t, 1, 8 \sin t), t \in [0, 2\pi]$. Thus Z Z F \cdot T ds = $\partial S 2\pi (3 \sin t + 3 \cos t)(-3 \sin t) dt = -9\pi$. Then x = 1 + 20 and bottom by 1/n.) d) Multiply top and bottom by 1/n. of (x) = ex - 1 + 20 and bottom by 1/n.) d) Multiply top and bottom by 1/n. d) Multiply top and bottom by 1/n. do (x) = ex - 1 + 20 and b obtain p p $\sqrt{4} + 1/n - 1 - 1/n 4n + 1 - n 2 - 11 \sqrt{p} = p \rightarrow =$. Thus E := U = X \ V is clopen and $\emptyset \subset E \subset X$. Conversely, if E \cap (a - r, a + r). Using a calculator, we see that x1 = 3.142546543, x2 = 3.141592654, and x4 = 3.141592654, and x4 = 3.141592654, 11.4.3. If f is homogeneous of order k, then f (0) = f (0 · x) = 0k f (x) = 0. e) Suppose $\alpha > 0$ and x < y. 105 \sqrt{d} $E = \{(x, y, z) : 0 \le y \le 1, y \le x \le 1, x_3 \le z \le 1\}$, hence by Fubini's Theorem and the substitution 3 u = x + 1, we have Z 1 Z 0 Z 1 $\sqrt{1}$ p 2 3 x3 + z dz dx dy = x3 y 2 3 Z Z 1 0 Z x2 ((x3 + 1)3/2 - (2x3)3/2) dy dx 0 1 (x2 (x3 + 1)3/2 - x2 (2x3)3/2) dx \sqrt{Z} 2 2 3/2 27/2 4(2 2 - 1) = u du - e. It follows that $0 < 2x^2 + x - 3 = (x - 1)(2x + 3) < 7\delta \le 1/M$. 11.2.3. By definition, f (h, 0) - f (0, 0) fx (0, 0) = lim = lim h \rightarrow 0 h \rightarrow 0 h p |h \cdot 0| = 0. If $\{V\alpha\}\alpha \in A$ is a relatively open covering of H then $\{H \cap V\alpha\}\alpha \in A$ is a relatively open covering of H. By the Inverse Function Theorem, 0 = (f - 1) 0 (b) = 1 f 0 (a) implies 0 = 1, a contradiction. 6.1.7. a) Let x, y \in I. Since F (0, 0, 0) = 0 and ∂ F = xy(2 cos y - cos z) + xyz sin z + cos x ∂ z equals 1 6 = 0 at (0, 0, 0), the expression has a differentiable at (0, 0, 0) by the Implicit Function Theorem. Thus f is not differentiable at (0, 0, 0) = 0 and ∂ F = xy(2 cos y - cos z) + xyz sin z + cos x ∂ z equals 1 6 = 0 at (0, 0, 0), the expression has a differentiable at (0, 0, 0) by the Implicit Function Theorem. Weierstrass Theorem, A is finite. Similarly, (arctan x)0 = $1/\sec 2 y = 1/(1 + x^2)$ for x = tan y $\in (-\infty, \infty)$. Let $^2 > 0$ and set $\delta = ^2$. $\sqrt{}$ hypothesis $\sqrt{}$ b) Every convergent sequence in E := $(2, 3) \cap Q$ must have a limit in Q and cannot converge to the irrational endpoints, so by Theorem 10.16, E is closed. Then Q(x) := bn xn + \cdots + b1 x + b0 is a polynomial with rational coefficients which satisfies $|P(x) - Q(x)| \le |an - bn| |x|n + \cdots + |a0 - b0| < \varepsilon$ for all $x \in [a, b]$. Since $\{Ir(x)\}x \in E$ covers the compact set E, there exist finitely many xj 's in E such that N [$E \subset Irj(xj)$ | j=1 PN SN for rj = r(xj). e) By the Product and Reciprocal Rules, $D(f/g)(a) = D(f \cdot (1/g))(a) = (1/g(a))Df(a) + f(a)(-Dg(a)/g 2 \cdot (a) = (g(a))Df(a) + f(a)(-Dg(a)/g 2 \cdot (a) =$ (a) $-f(a)Dg(a))/g^2(a)$. 9.1.4. Let $\varepsilon > 0$ and choose N so that $k \ge N$ implies $kxk - ak < \varepsilon/2$ and $kxk - yk k < \varepsilon/2$. Hence by Jensen's inequality, $\mu Z \ 1 \ q \ 1 \ r \ r \ |f(x)| \ dx = \varphi(|f(x)|) \ dx = |f(x)| \ dx$. For n = 3, this is about 54.74 degrees. c) If xn = 1 + 1/n, then $xn \rightarrow 1$ and $1/\log xn \rightarrow +\infty$ as $n \rightarrow \infty$. 2.4.2. If xn is Cauchy, then there is an $N \in N$ such that $n \ge N$ implies |xn - xN| < 1. d) Since 1 + (-1)n/n = 1 - 1/n when n is odd and 1 + 1/n when n is odd and 1 + 1/n when n is even, inf E = 0 and sup E = 3/2. Indeed, if M > 1, then set $r = \min\{kyj - ykk : j, k \in [1, M]\}$ and notice that the Br (yj)'s are open, nonempty, and disjoint, hence separate f (K). Hence Z Z F · T ds = C 3 (3t3 - 2t2) dt = 1 128. Taking the limit of this inequality as $n \rightarrow \infty$ and as $^2 \rightarrow 0$, we conclude that $1/s \leq \lim \text{ supn} \rightarrow \infty$ (1/xn). d) Repeat the proof of Theorem 3.9, replacing the absolute value by the norm sign. c) dz = (1 - x2 + y 2) x dx + dy. Moreover, by the Weierstrass M-Test, Bn (x) converges uniformly on each closed bounded interval [a, b]., 1) belongs to Rn but has no convergent subsequence. If one of these partial derivatives is nonzero, then by Lagrange's Theorem there is a scalar λ such that $\nabla f(a, b, c) = \lambda \nabla g(a, b, c)$. Since f is differentiable at a, 13 /khk $\rightarrow 0$ as h $\rightarrow 0$. Similarly, the other three entries also coincide. By iterating Exercise 4.1.9, we see that all derivatives of f of odd order are even functions. Thus $|f(x) - L| = |x^2 + 2x - 8| = |x + 4| |x - 2| < 7\delta \le \varepsilon$ for every x which satisfies $0 < |x - 2| < \delta$. 11.6.5. Let $F(x, y, u, v, s, t) = (u^2 + sx + ty, v^2 + tx + sy, 2s^2 x + 2t^2 y - 1, s^2 x - t^2 y)$ and observe that $(2u \ 0 \ \partial(F1, F2, F3, F4) | 0 \ 2v = \det (0 \ 0 \ \partial(u, v, s, t) \ 0 \ x \ y \ x \ |) = -64uvsxty$. b) Let $x \in U$. Let $f(x) = x^3 + x$. 7.5.7. a) Since $f(\beta n = x^3 + x^2 + y^2 + y^2$ $f(\alpha n) = f(\beta n) - f(\alpha n)$, and $1 = \beta n - xx - \alpha n + \beta n - \alpha n \psi$ we have $f(\beta n) - f(\alpha n) - \gamma = \beta n - \alpha n \psi$ we have $f(\beta n) - f(\alpha n) - \gamma = \beta n - \alpha n \psi$ we have $f(\beta n) - f(\alpha n) - \gamma = \beta n - \alpha n \psi$ we have $f(\beta n) - f(\alpha n) - \gamma = \beta n - \alpha n \psi$ and $\eta = \beta n - \alpha n
\psi$ and $\eta = \beta n - \alpha n \psi$ and $\eta = \beta n - \alpha n \psi$ and $\eta = \beta n - \alpha n \psi$ and $\eta = \beta n - \alpha n \psi$ and $\eta = \beta n - \alpha n \psi$ and $\eta = \beta n - \alpha n \psi$ and $\eta = \beta n - \alpha n \psi$ and $\eta = \beta n - \alpha n \psi$ and $\eta = \beta n - \alpha n \psi$ and $\eta = \beta n - \alpha n \psi$ and $\eta = \beta n - \alpha n \psi$ and $\eta = \beta n - \alpha n \psi$ and $\eta = \beta n - \alpha n \psi$ and $\eta = \beta n - \alpha n \psi$ and $\eta = \beta n - \alpha n \psi$ and $\eta = \beta n - \alpha n \psi$ and $\eta = \beta n$ $-4 - 4 = \rightarrow 2 \ 2 \ 1 - 2n + 3n (1/n) - (2/n) + 3 \ 3 \ as \ n \rightarrow \infty$. Since $E \subseteq U \cup V$ we may suppose $x \in U$. However, the series converges by the Logarithmic Test since $\log((\log k)\log k) / \log k = \log \log k \rightarrow \infty$ as $k \rightarrow \infty$. Thus fn is uniformly Cauchy. Therefore, $\sqrt{x} = 2$, 3. Then $\varphi = \psi \circ \tau$, hence $(\psi, [0, 1])$ and (φ, I) are orientation equivalent. b) If C(x, y) and D(x, y) = 0. y) are piecewise smooth curves from (x0, y0) to (x, y) and let C represent the curve C(x, y) followed by -D(x, y), i.e., D(x, y) in reverse orientation. By definition, $x \in X$ and $x \in /E\alpha$ for some $\alpha \in A$. f(x) = g(x) = x are increasing on [-1, 0] but $f(x)g(x) = x^2$ is decreasing on [-1, 0]. 10.2.1. a) Let $^2 > 0$ and $x \in R$. b) Since $k \ 1/k \ k = 1/k \rightarrow 0$ as $k \rightarrow \infty$, this series converges by the Root Test. 11.4.10. Hence by the Squeeze Theorem, xn /yn \rightarrow x/y as n $\rightarrow \infty$. b) By part a), if f (x, y, z) is an extremum then 4y 2 x - 2y 2 z = 0 and 4x2 y - 2x2 z = 0, i.e., y 2 (4x - 2z) = 0 = x2 (4y - 2z). Then x \geq C implies logc xq 1 \leq p = p-q. (Such a function can be constructed by making f piecewise linear on each [k, k + 1],

its graph forming Ra triangle whose peak occurs at the midpoint of [k, k + 1] with height P $\infty \propto 2/k$.) Then k=1 f (k) = 1 converges but 1 f (x) dx = ∞ . Hence by the Implicit Function Theorem such functions u, v, s, t exist. Finally, if θ is the angle between $\varphi(t1) - \varphi(0)$ and $\varphi(t2) - \varphi(0)$, then by (3) in 8.1 we have t1 a \cdot (t2 a) t1 t2 cos $\theta = = \pm 1$. But by Definition 2.14 (with M = 0), xn > 0 > yn for n sufficiently large, which contradicts the hypothesis $xn \le yn \cdot 0$. If $u = \sec \theta \cdot \sec \theta \tan \theta \, d\theta$. As $t \to -1+$, $x \to -\infty$, $y \to \infty$, $y/x = t \to -1$, and $dy/dx \to -1$. We may suppose g(x) > 0. d) By b), there exist $x, y \in \mathbb{R}$ such that $xn \downarrow x$ and $yn \uparrow y$ as $n \to \infty$. Thus by part a), this integral converges if and only if p > 1. Since F (0, 0, 0) = 0 and ∂ F (0, 0, 0) = 1 + gz (0, 0, 0) > 1 ∂ z is nonzero, the expression has a differentiable solution near (0, 0, 0) by the Implicit Function Theorem. b) By Example 14.9, this is the Fourier series of |x|. Since f is integrable, there is a partition P of [a, b] such that kP k < $^2/(8mC)$, Z b Z b 2 U (f, P) < f (x) dx + and L(f, P) .53 Series with Nonnegative Terms...... .55 Absolute Convergence..... .57 Alternating ...63 Chapter 7: Infinite Series of Functions 7.1 7.2 7.3 7.4 7.5 Uniform Convergence of Sequences...60 Estimation of Series.. ..62 Additional Tests. ..65 Uniform Convergence of Series.. .67 Power ..74 Chapter 8: Euclidean Spaces 8.1 8.2 8.3 8.4 Algebraic Structure. 76 Planes and Linear Transformations ...69 Analytic Functions. ...72 Applications... .80 Chapter 9: Convergence in Rn 9.1 9.2 9.3 9.4 9.5 9.6 Limits of Sequences... .82 Heine-Borel Theorem .79 Interior, Closure, and Boundary. .83 Limits of Functions .86 Compact Sets .87 Applications .88 Chapter 10: Metric Spaces 10.1 10.2 10.3 10.4 10.5 10.6 10.7 Introduction. Function 93 Connected Sets .94 Continuous Functions..... Closure, and Boundary, .92 Compact Sets 96 Stone-Weierstrass Theorem... .97 Copyright © 2010 Pearson Education. Inc. Hence by the Comparison Theorem and Exercise 5.4.2a the integral converges: 1 (1/x) sin(1/x) dx \leq 1 (1/x) in(1/x) dx \leq 0. But U is open in X by Theorem 10.31. Thus B is orientation compatible with A. Thus by the continuity of ax, Theorem 3.8, and the fact that the laws of exponents hold for rational powers, we conclude that $ax+y = \lim atn +qn = \lim atn +qn = \lim atn aqn = ax ay$. Let $^2 > 0$ and choose a grid G such that Vol (E1 \cup E2; G) and V (E1 \cap E2; G) $<^2$. 9.4.8. By the proof of Lemma 3.38, if f is uniformly continuous, then f takes a Cauchy sequence in E to a Cauchy sequence in Rm. Since $x \in [1, 2]$, we have $|x - 1| = x - 1 \le 1$ and $c \ge 1$. It follows that $1 \ 1 \ 1 = p \log k \log(\log(\log k)) < 2 \log k = 2 \cdot 4 \cdot 1 \cdot 9$. a) Suppose that f is odd and differentiable on I and $x \in I$. By the Bolzano-Weierstrass Theorem, xn has a convergent subsequence, i.e., there is an $x \in R$ and integers nk such that xnk $\rightarrow \infty$. Then $\tau 0$ (u) > 0 and $\varphi \circ \tau (u) = \varphi(f - 1 (u)) = (f -$ -1 (-u), -u) = $\psi(u)$. Suppose f has a point of discontinuity $x_0 \in [0, 1]$. Then f has no limit as $x \to 0$, but the limit must be zero. Since -E is a bounded, nonempty subset of Z, it has a supremum by the Completeness Axiom, and that supremum belongs to -E by Theorem 1.15. Then 0 < a - 2 < 1 so $0 < a - 2 < 1 \sqrt{2}$. by (6). 0 151 Copyright © 2010 Pearson Education, Inc. Since tn $\rightarrow \infty$, it follows from the Squeeze Theorem that sn $\rightarrow \infty$ as n $\rightarrow \infty$. c) Let (a, b, c) there is an N \in N such that n \geq N implies $|xn - 1| < \epsilon/2$. dxn if n is odd 0 if n is even we have by Stokes's Theorem that $Z \omega = \partial E$ n $Z X_j = 1$ (-1)j-1 dx1. b) To prove the Trichotomy Property, suppose a, b $\in \mathbb{R}$. In particular, f (a) $\in /Be0$ (a), a contradiction. Then f is differentiable on (0, 1) and f (0) = f (1) = 0, but f is not even continuous on [0, 1]. Given $\varepsilon > 0$, choose $N \in \mathbb{N}$ so that $N \geq 3$ and $2/N < \varepsilon$. 7.5.6. Let $|f 00(x)| \leq M$ and choose r0 < 20 /M. Taking the infimum of this inequality over all $x \in E$, we conclude that inf $x \in E$ (x, a) $\geq 2 > 0$. $\pi - \pi \pi - \pi 14.2.3$. If S is the Fourier series of a continuous, periodic function f then $\sigma N = \sigma N$ f \rightarrow f uniformly on R by Corollary 14.15. d) Integrating by parts and using l'A^opital's Rule, we obtain Z 0 1 1 log x dx = x log x - x 0 = -1. Thus bx is decreasing (not increasing) when $b \in (0, 1)$. a) Clearly, sin $x \ge 2/2$ for $x \in [\pi/4, \pi/2]$. Since I is open, there is an $^2 > 0$ such that $(f(x)-^2, f(x)+^2) \subset I$. Moreover, by Exercise 1.6.5b, the function $\varphi-1$ is 1-1 from $\{1, 2, ..., x_{k}\}$ for $j \in \mathbb{N}$. c) Let E be a nonempty, proper subset of Rn. Then $1/x_{n} = (-1)n$ n has no limit as $n \to \infty$. Recall that 1/2k < 1/k and $\log(k + 1) - \log k = \log((k + 1)/k) \rightarrow 0$ as $k \rightarrow \infty$, so choose N2 so that $\log((k + 1)/k) < \varepsilon/2$. x - a x - a 29 Copyright \mathbb{C} 2010 Pearson Education, Inc. Hence by Exercise 1.6.5 and construction, φ is 1-1 on $\{1, 2, ...\}$ for $x \in [-1, 0)$ and f(x) = 0 for $x \in [0, 1]$, then f is not integrable because it's not even bounded below, so the sums are not finite. k n n+1 n+2 m-1 m k=n Each term in parentheses is positive, so the absolute value of S is dominated by 1/n. Thus f 0 (c) = 1, i.e., 1 \in f 0 (0, 2). By hypothesis, (f \cdot f)(t) = r2 is constant on I, hence by the Dot Product Rule, 0 = (f \cdot f)0 (t) = f(t) \cdot f 0 (t) = 2f(t) \cdot f 0 (t) for all t \in I. By calculus, p this function (b - a). b) By definition and Theorem 9.8, E is closed. We conclude that B is sequentially compact. Thus Z F · T ds = -C2 1/2 F · T ds = 0 Z Z F 1 (x, 0, 0) dx = 0, C1 and Z 1 C3 1 F (x, (1 - x)/2, 0) · (1, -1/2, 0) dx = 0, C1 and Z 1 C3 1 F (x, (1 - x)/2, 0) · (1, -1/2, 0) dx = 0, C1 and Z 1 C3 1 F (x, (1 - x)/2, 0) · (1, -1/2, 0) dx = 0, C1 and Z 1 C3 1 F (x, (1 - x)/2, 0) · (1, -1/2, 0) dx = 0, C1 and Z 1 C3 1 F (x, (1 - x)/2, 0) · (1, -1/2, 0) dx = 0, C1 and Z 1 C3 1 F (x, 0, 0) dx = 0, C1 and Z 1 C3 $y - ex \cos y = 0$, $\partial x \partial y R$ it follows from Green's Theorem that $C \omega = 0$ for all such curves $C \sqrt{Case 2}$. Then f is bounded, but the graph of y = f(x) intersects any rectangle R in the unit square $[0, 1] \times [0, 1]$. (n) 4.1.6. For x < 0 we have f(n)(x) = 0 for all $n \in N$. $g(0, 1) = (1, \infty)$ is connected as Theorem 10.62 says it should; $g[0, 1] = \{0\} \cup (1, \infty)$ is neither compact nor connected-note that Theorems 10.61 and 10.62 do not apply since g is not continuous; $g[0, 1] = \{0\} \cup [1, \infty)$ is neither compact nor connected-note that Theorems 10.61 and 10.62 do not apply since g is not continuous; $g[0, 1] = \{0\} \cup [1, \infty)$ is neither compact nor connected-note that Theorems 10.61 and 10.62 do not apply since g is not continuous. We conclude that f = g is uniformly continuous on the subset E. Suppose ii) holds and let C be any piecewise smooth curve of the type described in condition iii). 9 1 45 45 = Z 1 Z x 2 12.3.3. a) 0 0 Z 2 Z 1 dy dx = 1 + x 2 Z 0 0 1 x 3 1 dx = 1 + x 2 Z 0 0
1 x 3 1 dx = 1 + x 2 Z 0 0 1 x $-\log 2$) du = . By looking at the graph, we see that f (E) = R. 2 b) We first prove that the functions g(x) = $e^{-1/x}/xk$ satisfy g(x) $\rightarrow 0$ as $x \rightarrow 0$ for all integers $k \ge 0$. $\infty \mathbb{R} \propto 11.1.9$. a) 0 = -st dt = $-e^{-st}/s^{-1} = -e^{-st}/s^{-1}$. $(x) - f(a)| = |x - a| |x^2 + xa + a^2 - 1| \le 4|x - a| < 4\epsilon = \epsilon$. Given 2 > 0 choose N so large that |Fk, m(x)| < 2/(3M) for $x \in E$ and $m, k \ge N$. Hence inf $(xk + yk) > xjn - k \ge n 1 + inf yk n k \ge n$ for all $n \in N$. If a = 0 6 = b, then we may suppose |a| < |b|. 2 Copyright $(x + yk) > xjn - k \ge n 1 + inf yk n k \ge n$ for all $n \in N$. If a = 0 6 = b, then we may suppose |a| < |b|. 2 Copyright $(x + yk) > xjn - k \ge n$. Hence inf $(xk + yk) > xjn - k \ge n$. Hence inf $(xk + yk) > xjn - k \ge n$. an N \in N such that N > M. Hence if f is integrable, we can choose P so that V (G(f), G) \leq S(f; P) - s Using the parameterizations $\varphi_1(t) = (2 \sin t, 2 \cos t, 4)$, $II = [0, 2\pi]$, and $\varphi_2(t) = (\cos t, \sin t, 1)$, $I2 = [0, 2\pi]$, we have $Z Z Z F \cdot T ds = F \cdot T ds + F \cdot T d$ $-21 \sin 2 t + (2 \cos 4 - \sin 1) \cos t + (2 \sin 4 - \cos 1) \sin t$) dt $0 = 3\pi$. 6.3.9. By hypothesis, $\infty X k=1$ Therefore, $\infty X k=1 \propto 1$ 1X 1 $\pi 2 = =$. Thus by the Squeeze Theorem and assumption i), sin $x \rightarrow 0 = \sin(0)$ as $x \rightarrow 0$. 2 Hence ZZ Z $\pi Z \pi F \cdot n d\sigma = S - \pi$ (ab2 sin2 v + b3 sin2 v cos v) dv du = $2\pi 2 ab2$. Therefore, $g(a) = f(a) - a \ge 0$ and $g(b) = f(b) - b \le 0$. For any r0 > r, by Remark 6.22i, there is an $N \in N$ such that $k \ge N$ implies $ak+1/ak \le r0$. Since $a9 \approx = .0105$ and $a10 \approx .0055$, n = 10 terms will estimate the value to an accuracy of 10-2. Hence by part a) and the Squeeze Theorem, sin $x/x \rightarrow 1$ as $x \rightarrow 0+$. In particular, the explicit curve y = f(x), as x runs from a to b, is orientation equivalent to the explicit curve x = f - 1 (y), as y runs from f (a) to f (b). To show the converse vis not/true, let f be given by Example 11.11. d) Let f (x) = ex - 1 + sin x. c) If f is continuous on R then f is uniformly continuous on $[-\pi, \pi]$. Hence by assumptions iii) and i), cos x = 1 $(0) = 1 = \cos(0)$ as $x \to 0$. $\sqrt{\sqrt{c}} y = a^2 - x^2$ implies $x^2 + y^2 = a^2 \cdot 10.4.5$. By Exercise 10.3.10, there exist $^2 := ^2x > 0$ such that $V = \bigcup x \in V B^2(x)$. Thus the set used to define M2 is nonempty, bounded below by 0. 9.1.2. a) By Theorem 9.2, $(1/k, (2k^2 - k + 1)/(k^2 + 2k - 1)) \to (0, 2)$ as $k \to \infty$. Now $x_1 - p$ has a finite limit as x - 0. 0 + if and only if 1 - p > 0, 1 i.e., p < 1. Since E is increasing and $x\alpha = E(\alpha L(x))$, it follows that $x\alpha < y\alpha$. Adding a $b^2 + c^2 d^2$ to both sides, we conclude that $(ab + cd)^2 \leq (a^2 + c^2)(b^2 + d^2)$. Thus f(x) < 1 + f(x) for all $x \in \sqrt{(a, b)}$. 8.1.2. a) By Cauchy-Schwarz, $k^3vk \leq |a \cdot b| kck + |a \cdot c| kbk + |c \cdot b| kak \leq 3kak kbk kck \leq 3$, so $kvk \leq 3/3 = 1$. Therefore, y < w/10n+1 + 1/10n+1. Hence, $P(x)/xm \rightarrow 0$ as $x \rightarrow \pm \infty$., bm). Since Pz = -yz + h(x, y) and Py = -z + hy, we may set Q = 0 = h, i.e., P = -yz. If $x \in /(A \cap \partial B) \cup (B \cap \partial A)$, then by part b), $x \in \partial A \cap \partial B$., $n + 1 \} \rightarrow \{1, 2, .0 \text{ i.e.}, \pi e - x \sin x \, dx = -e - \pi/2$. 81 Copyright © 2010 Pearson Education, Inc. Suppose it holds for 1. This 93 Copyright © 2010 Pearson Education, Inc. This proof doesn't change if x1 > -2, so the limit is again x = 2. By the choice of $\delta 0$, we have F (x0) < F (c). Rb c) False. Thus by Theorem 7.33 x = 2. By the choice of $\delta 0$, we have F (x0) < F (c). Rb c) False. Thus by Theorem 7.33 x = 2. By the choice of $\delta 0$, we have F (x0) < F (c). Rb c) False. Thus by Theorem 7.33 x = 2. By the choice of $\delta 0$, we have F (x0) < F (c). Rb c) False. Thus by Theorem 7.33 x = 2. By the choice of $\delta 0$, we have F (x0) < F (c). Rb c) False. Thus by Theorem 7.33 x = 2. By the choice of $\delta 0$, we have F (x0) < F (c). Rb c) False. Thus by Theorem 7.33 x = 2. By the choice of $\delta 0$, we have F (x0) < F (c). Rb c) False. Thus by Theorem 7.33 x = 2. By the choice of $\delta 0$, we have F (x0) < F (c). Rb c) False. Thus by Theorem 7.33 x = 2. By the choice of $\delta 0$, we have F (x0) < F (c). Rb c) False. Thus by Theorem 7.33 x = 2. By the choice of $\delta 0$, we have F (x0) < F (c). zero off Ik and 0 φ k (t) dt = 1. 9.2.8. Suppose that a < b. Therefore, E is a Jordan region if and only if E and E 0 are Jordan regions by Definition 12.5. b) By Theorems 12.7 and 12.4, Vol (E) = Vol (E 0) + Vol (∂ E) = Vol (E 0) + Vol (\partialE) = Vol (E 0) + Vol (∂ E) = Vol (E 0) + Vol (\partialE) = Vol t(3, 4, 0). It follows that there is an interval $J \subset I$ which either contains a or has a san endpoint on which both f and f 0 are never zero. $\sqrt{Case 3} \cdot \sqrt{x0} \ge 3$. 3.1.5. Let $f(x) \rightarrow L$ and $g(x) \rightarrow M$ as $x \rightarrow a$, and $xn \in I \setminus \{a\}$ converge to a. Pn ; c Pn - 1 d) 2n = (1 + 1)n = k = 0 nk so k = 1 nk = 2n - 1. Since every singleton is closed (see Remark 8.22), E = 1 $\bigcup x \in \{x\}$ is a decomposition of E into closed sets. 2 2 Similarly, f (x) > f (a) - $\varepsilon = m + \varepsilon$. By part a), Q satisfies the Closure Properties, has additive inverses, and every nonzero q \in Q has a multiplicative inverse. Then sup E - 1/(n + 1) < xn + 1 < sup E and xn < xn + 1 . By the product rule for partial derivatives, $\lceil \exists i \mid x \lor x \mid f \rceil = \det \lfloor \frac{\partial}{\partial x} \frac{\partial}{\partial y} \frac{\partial}{\partial z} \rfloor f$ F1 f F2 f F3 = (fy F3 + f (F3)y - fz F2 - f (F2)z, fz F1 + f (F1)z - fx F3 - f (F3)x, fx F2 + f (F2)x - fy F1 - f (F1)y) = (\nabla f \times F) + f (\nabla \times F) + f lim agn = ax . $3-129n + 1 - n + 29 + 1/n - 1 + 2/n \sqrt{2.2.4}$. a) Clearly, Thus xn x xn y - xyn xn contradiction. By a), $|(x - \pi) + \sin x| = |\delta 0 + \sin(\delta 0 + \pi)| \le |x - \pi|^3/3! = \delta 03/3! < \delta 3/3!$. Finally, since f is continuous and (by the definition of the operator norm) kDg(a)(h)k $\le kDg(a)k$ khk, kI2 k/khk $\le kDg(a)k$ khk, kI2 k/khk $\le kDg(a)k$ khk, kI2 k/khk $\le kDg(a)k$ kf $(a + h) - f(a)k \rightarrow 0$ as $h
\rightarrow 0$. Since E is closed, we have by Theorem 10.16 that sup E = limk $\rightarrow \infty$ xk \in E. 0 101 Copyright \otimes 2010 Pearson Education, Inc. $\cos \theta \ 1 - x2 \ 0$ If $u = \cos \theta$ then $du = -\sin \theta \ d\theta$. But a nonempty connected if and only if it is a single point. 4.2.9. a) By assumptions ii) and vi), $0 \le |\sin x| \le |x|$ for $x \in [-\pi/2, \pi/2]$. Since K is compact and is covered by $\{B\delta x \ (x)\}x \in K$, there exist x_1 , . Hence $|f \ 0 \ (x)| \le 3/(3 - x)2$ for $0 \le x < 3$. In particular, $ex - 1 + 1 \le 1$. sin x \geq f (0) = 0. Suppose div F = 0 everywhere on Ω . 2.3.9. Since x0 = 1 and y0 = 0, 2 x2n+1 - 2yn+1 = (xn + 2yn)2 - 2(xn + yn)2 = -x2n + 2yn2 = \cdots = (-1)n. Hence, N = 1, i.e., f (x) = f (x1) for all x \in X. 11.7.8. a) If gx (a, b, c) = gy E is a Jordan region of volume zero. Thus L is strictly increasing on $(0, \infty)$. But f has range $(0, \infty)$ so f (0) = 1. Therefore, the sequence $(\log k + x)/(k + x)$ is eventually decreasing for each $x \in [0, 1]$. Now 0 g(t) dt = k=1 xk / 3k = x/(3 - x) by Theorem 6.7, so g(x) = (x/(3 - x))0 = 3/(3 - x)2. c) Let $(x, y, z) = \omega(t)$. b) It is relatively open in B1 (0, 0) because points lies in a relative open ball which stays inside the set. j=1 k=1 j=1 k=1 Given a compact subset H of V, choose open sets W1, W2 containing H and integers N1, N2, such that $\phi j = 0$ on W1 for $j \ge N1$ and $\psi k = 0$ on W2 for $k \ge N2$. E $o = \{(x, y) : x^2 + 4y 2 < 1\}$ and $\partial E = \{(x, y) : x^2 + 4y 2 < 1\}$ and $\partial E = \{(x, y) : x^2 + 4y 2 < 1\}$ and $\psi k = 0$ on W1 for $j \ge N1$ and $\psi k = 0$ on W1 for $j \ge N1$ and $\psi k = 0$ on W2 for $k \ge N2$. E $o = \{(x, y) : x^2 + 4y 2 < 1\}$ and $\partial E = \{(x, y) : x^2 + 4y 2 < 1\}$ and $\partial E = \{(x, y) : x^2 + 4y 2 < 1\}$ and $\psi k = 0$ on W1 for $j \ge N1$ and $\psi k = 0$ on W2 for $k \ge N2$. E $o = \{(x, y) : x^2 + 4y 2 < 1\}$ and $\partial E = \{(x, y) : x^2 + 4y 2 < 1\}$ and $\partial E = \{(x, y) : x^2 + 4y 2 < 1\}$. monotonically on I. b) Let C be relatively closed in E, i.e., there is a closed set B such that $C = E \cap B$. Therefore, xq is differentiable at a and the value of its derivative there is na-1-qm+q = qaq-1. Since f is integrable on E and E1 is a Jordan region, we can choose a grid $G = \{R1, ..., T, 5, P_{R1}, ..., T, 5, P_{R1}, ..., T, 5, P_{R1}, ..., T, 5, P_{R1}, ..., T, 1, P_{R1}, ..., P_{R1}, ...$ (-1, 2) is 5 at x = 2. 10.3.2. a) This is the set of points on or inside the ellipse x2 + 4y 2 = 1. It follows that w \leq 10n+1 y, i.e., w/10n+1 y, i.e., w/10n+1 \leq y. Hence, ZZ Z π Z 1 F · n d σ = - (u2 + 4 cos2 v, 4 sin v cos v, 4 sin 2 v) · (0, -2 cos v, -2 sin v) du dv S 0 0 Z π = 8 sin v dv = 16. 11.2.2. Since f has a scalar domain and is differentiable at a, we have 0 = lim x \rightarrow a f (x) - f (a) - Df (a) · (x - a) f (x) - f (a) - Df (a) · (x - a) f (x) - f (a) - Df (a) · (x - a) f (x) - f (a) - Df (a) · (x - a) f (x) - f (a) - Df (a) · (x - a) f (x) - f (a) - Df (a) · (x - a) f (x) - f (a) - Df (a) · (x - a) f (x) - f (a) - Df (a) · (x - a) f (x) - f (a) - Df (a) · (x - a) f (x) - f (a) - Df (a) · (x - a) f (x) - f (a) · ($\rightarrow \infty$. 8.4.7. Suppose A is not connected. e) By Green's Theorem, Z ZZ ZZ (ux dy - uy dx) = (uxx - (-uyy)) dA = Δu dA. $k \rightarrow \infty$ - 1/k p := lim Hence the series converges absolutely by the Logarithmic Test. 10.2.8. a) If xn \in E, then xn is bounded. Conversely, if φ takes {1, 2, . P ∞ b) By Example 7.45, 3x = ex log 3 = k=0 xk logk 3/k! for all x \in R. Let U be pen in E, i.e., $U = E \cap V$ for some open V in Rn. k k k=1 k=1 d) True. 5.3.5. By the Fundamental Theorem of Calculus and the First Mean Value Theorem for Integrals, Z f (b) - f (a) = b Z 0 b 0 f (t) dt = f (x 0) a dt = f 0 (x 0) (b - a) a for some x 0 between a and b. y) Since the cases n odd and n even are similar, we will suppose that n is even. We obtain x = 2 + x, i.e., $x^2 - x - 2 = 0$. 2.1.7. a) Let a be the common limit point. 10 Copyright © 2010 Pearson Education, Inc. By Definition 2.7, there are numbers M and m such that $m \le xn \le M$ for all $n \in N$. Then F(x0 - 2h) - 2F(x0) = 0. C) By part b), if fk converges uniformly, then kfk - fj k is small when k and j are large. Then $x \in Bx$ for some $1 \le j \le N$, so f(x) = f(xj). Hence by Theorem 14.29 and Exercise 14.4.1, this series must converge to x uniformly on $[a, b] \subset (-\pi, \pi)$. d) To show these statements may not hold when a < 0, let a = -2, b = -1, c = 2 and d = 5. By Remark 1.41, the unit interval (0, 1) is hence $\{x\}x \in (0,1)$ is an uncountable collection of pairwise disjoint nonempty sets which covers the unit interval (0, 1). Thus the series converges uniformly on any closed subinterval of (-3, 3). If $y \in Br$ (a) $\cap Bs$ (b), we have $\rho(x, y) \le \rho(x, a) + \rho(a, y) < r + r = 2r < d$. Therefore, sup f (E) \le f (sup E). $x \mid \sqrt{\sqrt{y}}$ b) $1/2 \le 2$ ≤ 1 implies $1/2 \leq x_3 \leq 1$, i.e., $1 \leq 1/x_3 \leq 2 2$. Then supk $\geq N$ (xk yk) $\geq xn$ yn $\geq (2M/C)$ yn for any $n \geq N$ and supk $\geq N$ (xk yk) $\geq (2M/C)$ supn $\geq N$ yn > M. We shall obtain a contradiction by showing that wxx (x2, t2) - wt (x2, t2) ≥ 0 . Thus by Theorem 11.58, (a, b) is a saddle point. Suppose without loss of generality that an > 0. Hence by the Intermediate Value Theorem, there is an x (between 0 and 1) such that f(x) = 0. We conclude that ZZ ZZ lim ey/k cos(x/k) dA = 1 dA = Area (E). Thus the minimum of w on H must be less than or equal to - and must occur on the compact set K. i=1 j=1 Therefore, $|f(x) - f(a)| = |D(2) f(c; x - a)|/2! \le M kx - ak^2$ for $M = n^2 C/2$. Let Ej be ordered from largest radius to the smallest. Hence by Theorem 6.40, |sn - s| is dominated by $(1/2)n+1/(1/2) = (1/2)n \cdot 1$ (C) is open in X, i.e., f - 1 (C) is closed in X. If 2 y 6 = 0 then $\lambda = 1$ so x = 1/2. 7.4.6. Using the substitution u = a - x, du = -dx, we have Z a Z 0 xn f (n+1) (a - x) dx = 0 (a - u)n f (n+1) (u) du. J I 13.1.9. It is clear that $(x, y) = \varphi(t)$ implies $x^3 + y^3 = 3xy$. For x > 0 we have $f 0 (x) = 3x^2$, so $00 f[0, \infty) (0) = \lim h \to 0 f (h) - f (0) = \lim h \to 0 f (h) - h = 0 f ($ Theorem that $\lim x \to 0$ x $\cos((x^2 + 1)/x^3) = 0$. c) If E is the set of points xn such that xn = 1 - 1/n for odd n and xn = 1/n for even n, then sup E = 1, inf E = 0,
but neither 0 nor 1 belong to E. We $\sqrt{}$ Hence by hypothesis, $\sqrt{} 0$ conclude that f (x) = $\pm \alpha x + c$ for some $c \in \mathbb{R}$. Since $\varepsilon > 0$, we have $\varepsilon x \le \varepsilon$ sup A, so the latter is an upper The line is parallel to a, so a "lies in the plane." Since b - c is another vector that lies in the plane, it follows from part a) that an equation of the plane is given by $d \cdot x = b \cdot d$, where $d = a \times (b - c)$. $g \circ (g - 1 (g(b))) \cdot f \circ (f - 1 (g(g - 1 (b)))) g (b) \cdot f \circ (a) 34$ Copyright © 2010 Pearson Education, Inc. First, since g is continuous and nonzero on [a, b], 1/g is continuous, hence integrable on [a, b]. If it holds for n then $n+1 \ge n(n+1)(2n+1) + 6(n+1) = .$ Since div F = x + y, it follows from Gauss' Theorem that ZZ ZZ Z $2\pi Z 3Z 2 \omega = (x + y) dV = (x + r \cos \theta)r dx dr d\theta = 2\pi xr dx dr = 18\pi . 1.4.3. a)$ This inequality holds for n = 3. ¶. By definition, $A \subseteq f - 1$ (f (A)) holds whether f is 1-1 or not. Since u is continuous on E, it follows that u is constant on E. Thus f 0 exists and is continuous on E, it follows that u is constant on E. Thus f 0 exists and is continuous on E, it follows that u is constant on E. Thus f 0 exists and is continuous on E, it follows that u is constant on E. Thus f 0 exists and is continuous on E, it follows that u is constant on E. Thus f 0 exists and is continuous on E, it follows that u is constant on E. Thus f 0 exists and is continuous on E, it follows that u is constant on E. Thus f 0 exists and is continuous on E. Thus f 0 exists and exists and exists and exists and exists and ex Since $(2n2 + 3)/(n3 + 5n2 + 3n + 1) \rightarrow 0$ as $n \rightarrow \infty$, it follows from the Squeeze Theorem that $xn \rightarrow 0$ as $n \rightarrow \infty$. Since $(n - N)/n \rightarrow 1$ as $n \rightarrow \infty$, it follows that $\sigma > M/2$ for $n \log p$, i.e., $\sigma n \rightarrow \infty$ as $n \rightarrow \infty$. 0.2485860. k=n+2 (n) Hence by induction, given $n \in N$, there are integers N = N (n) $\in N$, and $ak = ak \in Z$ such that (*) holds. b) By the Comparison Theorem, $Z n 1 + 1 n - 1 Z k + 1 X 1 1 dx \ge 1 + 2 Z x (k + 1) k^2$ for all $n \in N$. If f is even and differentiable, then f 0 (0) = 0. $-\pi d$) Use the trivial parameterization $\varphi(u, v) = (u, v, u2)$, E = B1 (0, 0). Thus the trace of $\varphi(t)$ approaches (0, 0) and is asymptotic to the negative y axis as $t \rightarrow -\infty$. S E R Thus S d σ = Area (S) by Theorem 12.22. 2 2 a a Let P0 = P \cup E := {x0, . - n n 2 n 2 2 k=1 k=1 7.1.11. 11.2 The Definition of Differentiability. We conclude that sup(A + B) < a0 + b0, a contradiction Hence by Corollary 11.34, there is a constant M > 0 such that $|f(x)| = |f(x) - f(0)| \le M kx - 0k = M kxk$ for all $x \in E$. Since f(c) is the product of these factors and f(n)(xj), it follows that the f(n)(xj) is the product of these factors and f(n)(xj). $G V (E; G) \ge |R| > 0.$ $(1 - 2k)/(1 \cdot 4 \cdot \sqrt{Thus \theta} = \arccos(1/n). a)$ Since f(x) = ex implies f(k)(x) = ex for all $k \ge 1$, the Taylor expansion of ex at x = 1 is ex = P7.4.3. $\infty k = 0.$ (x - 1)/k! valid on R by Theorem 7.43. dxn = j=1.4x1.for $x \in Q$ and f(x) = -1 for $x \in /Q$. b) As in part a), we may suppose that ax + by + cz + dw = 1. It follows that Vol (E1 \cup E2) \geq Vol (E1) + Vol (E2 $) - 2^2$. Hence the series converges by the Dirichlet Test. Then N $\phi = (-2u, 0, 1)$ points upward and ZZ Z F \cdot n d $\sigma = (u4 - 2u3 v 2) d(u, v)$ S Z B1 (0,0) 2π Z 1 = 0 1 = 6 0 2π Z 0 (r4 cos4 $\theta - 2r5 \cos 3 \theta \sin 2$ θ or $\theta = 0 \pi$. For each $x \in R$, f(2x) = f(x + x) = f(x) + f(x) = 2f(x). Integrating by parts twice, we obtain $Z \propto \pi R \propto Z = -\pi - \pi e - x \sin x \, dx$, $\pi R \propto (e - xy \sin x/x) \, dx$ = $e - \pi - \pi e - x \sin x \, dx$, $\pi R \propto (e - xy \sin x/x) \, dx$ = $e - \pi - \pi e - x \sin x \, dx$ = $e - \pi - \pi e - x \sin x \, dx$, $\pi R \propto (e - xy \sin x/x) \, dx$ = $e - \pi - \pi e - x \sin x \, dx$, $\pi R \propto (e - xy \sin x/x) \, dx$ = $e - \pi - \pi e - x \sin x \, dx$ = $e - \pi - \pi e - x \sin x \, dx$, $\pi R \propto (e - xy \sin x/x) \, dx$ = $e - \pi - \pi e - x \sin x \, dx$. 2v, 1)k = we have ZZ Z 1 Z g d σ = 4 S 0 u p 1 + 4u2 + 4v 2, (1 + 4u2 + 4v 2) dv du = 0 22. d) By definition, $\varphi(t) = (-3\cos 2t \sin t, 3\sin 2t \cos t)$, hence k $\varphi(0, t) = (-3\cos 2t \sin t, 3\sin 2t \sin t)$, hence \in R, there is an N \in N such that n \geq N implies xn > M . 10.1.12. b) f -1 (0, π) = . But kxkj k $\rightarrow \infty$ so it cannot be bounded. g -1 (-1, 1) = ($-\infty$, -1) \cup (1, ∞) \cup {0} is not continuous; g -1 [-1, 1] = ($-\infty$, -1) \cup (1, ∞) \cup {0} is not continuous; g -1 [-1, 1] = ($-\infty$, -1] \cup [1, ∞) \cup {0} is not continuous; g -1 [-1, 1] = ($-\infty$, -1] \cup [1, ∞) \cup {0} is not continuous; g -1 [-1, 1] = ($-\infty$, -1] \cup [1, ∞) \cup {0} is not continuous; g -1 [-1, 1] = ($-\infty$, -1] \cup [1, ∞) \cup {0} is not continuous; g -1 [-1, 1] = ($-\infty$, -1] \cup [1, ∞) \cup {0} is not continuous; dxn = Q n X (-1)j-1 a1 . S 0 d) Since div F = 2y + 2, we have by Gauss' Theorem that ZZ 2 πZ 2 Z 4 - r 2 F · n dσ = S (2r sin θ + 2)r dz dr dθ 0 Z 2 = 0 + 4π (8r - 2r3) dr = 32π. Part a) follows directly from Remark 8.27ii. R∞ R1 R∞ d) 0 ≤ 0 dx/(1 + xp) ≤ 0 dx/(1 $(a + b \cos y) \sin u$, and $z = b \sin y$, then x dy dz + y dz dx + z dx dy; $c = b(a + b \cos y) 2 \cos y + b(a + b \cos y) 2 \cos y + b(a + b \cos y) 2 \cos y + b(a + b \cos y) 2 \cos y + b(a + b \cos y) 2 \cos y + b(a
+ b \cos y) 2 \cos y + b(a + b \cos y) 2 \sin y + b(a + b \cos y) 2 \sin y + b(a + b \cos y) 2 \sin y + b(a + b \cos y) 2 \sin y + b(a + b \cos y) 2 \sin y + b(a + b \cos y) 2 \sin y + b(a + b \cos y) 2$ $-1/xn < \varepsilon$. If n is even, then n - k and k are either both odd or both even. If this inequality is strict, then sup(A + B) - sup B < a0. 77 Copyright © 2010 Pearson Education, Inc. We obtain x = 2 + \sqrt{x} - 2, i.e., x^2 - 5x + 6 = 0. $x^2 = -2$ implies $x = 1 \pm 3$, we also have f - 1 (E) = [1 - 3, 1 + 3]. 8.4.9. a) If A = (0, 1) and B = [1, 2] then (A \cup B) o = (0, 2) but Ao \cup B o = (0, 2) but Ao \cup B o = (0, 2). Thus the trace of $\varphi(t)$ approaches (0, 0) and is asymptotic to the positive y axis as $t \rightarrow \infty$. Since H1 is compact, choose N so large that H1c , . Thus f - 1 is 1-1 on B0 . 12.5.5. Clearly, $\phi_j \psi_k$ belongs to Ccp (Rn), $\phi_j \psi_k \ge 0$, and spt ($\phi_j \psi_k$) $\subset V_j \cap W_k$. Hence by Remark 6.40, |sn - s| is dominated by (22/2!)(1/2)n-1/(1/2) = (1/2)n-3. Hence there is an N so large that $k \ge N$ and $x \in [-M, M]$ imply $|f_k(x) - f(x)| < 2$. b) |2 - x| < 2 if and only if -2 < 2 - x < 2 if and only if -4 < -x < 0 if and only if 0 < x < 4. c) This is the set of points on or above the parabola which lie below the line y = 1. Since z(x) contains m terms, it is easy to see that $z(x) \rightarrow maq(m-1)$ as $x \rightarrow a$. Suppose E is infinite. N +1 2 N +1 2 2 $|\sigma N(x) - f(x)| \le |f(x)| |KN(x-t)| dt \le KN(x-t)$ dt = M. dt = M. dt = M. $dt = 2\pi$ $dt = 2\pi$ dt =converges uniformly on [a, b] by the Weierstrass M-Test. e) If f is periodic and continuously differentiable, then $|f 0(c)| \le M \le \infty$ for all $c \in \mathbb{R}$. Since $tan(u/x) \ge u/x$ for $x > 2u/\pi$ (see (1) in Appendix B), and f 0 (x) = $sin(u/x) \ge u/x$ for $x > 2u/\pi$. e) Notice $0 \le h \le 1$ implies 1/b > 1. In particular, $2x - sin 2x \ge f(0) = 1$ 0, i.e., $\sin 2 x \le 2|x|$ when $x \ge 0$. Thus by induction, the formula holds for all $\in N$. It won't work for $n \ge 4$ either because f 000 is not defined at x = 0 so no higher derivative exists by definition. c) Clearly, fx = y 2 exy, fx = y 2 exy. 2m of these. $22-a-b 2 \parallel |R|$. Since f (x) $\rightarrow 3$ as x $\rightarrow 1$ and f (1) := 3, it follows from Remark 3.20 that f (x) is continuous on [0, 1]. We may suppose that Vol (E) 6 = 0.1.4.10. If r < 1, then in the discrete space, Br (a) = f = 1.4.10. If r < 1, then in the discrete space, Br (a) = f = 1.4.10. If r < 1, then in the discrete space, Br (a) = f = 1.4.10. If r < 1, then in the discrete space, Br (a) = f = 1.4.10. If r < 1, then in the discrete space, Br (a) = f = 1.4.10. If r < 1, then in the discrete space, Br (a) = f = 1.4.10. If r < 1, then in the discrete space, Br (a) = f = 1.4.10. If r < 1, then in the discrete space, Br (a) = f = 1.4.10. If r < 1, then in the discrete space, Br (a) = f = 1.4.10. If r < 1, then in the discrete space, Br (a) = f = 1.4.10. If r < 1, then in the discrete space, Br (a) = f = 1.4.10. If r < 1, then in the discrete space, Br (a) = f = 1.4.10. If r < 1, then in the discrete space, Br (a) = f = 1.4.10. If r < 1, then in the discrete space, Br (a) = f = 1.4.10. If r < 1, then in the discrete space, Br (a) = f = 1.4.10. If r < 1, then in the discrete space, Br (a) = f = 1.4.10. If r < 1, then in the discrete space, Br (a) = f = 1.4.10. If r < 1, then in the discrete space, Br (a) = f = 1.4.10. If r < 1, then in the discrete space, Br (a) = f = 1.4.10. If r < 1, then in the discrete space, Br (a) = f = 1.4.10. E. + 3.1.8. a) By symmetry, it suffices to show the first identity. 4.2.5. a) If f(a) 6 = 0, |f(a + h)| > |f(a)|/2 > 0 for h small (like the proof of Lemma 3.28). 1.3.0. a) True. P ∞ 6.4.4. If bk \downarrow b, then bk $- b \downarrow 0P$ as k $\rightarrow \infty$. 4.3.2. By the Mean Value Theorem, 2 = f(2) - f(0) = 2f 0 (c) for some $c \in (0, 2)$. 2.3.7. Choose by the Approximation Property an $x1 \in E$ parallel to $\nabla f(a)$. b) If n converges to some a, then given $\varepsilon = 1/2$, 1 = |(n + 1) - n| < P^2 then U (f, P) - L(f, P) \leq U (f, P²) - L(f, P²) - L(f, P²) - L(f, P²) - (f, P²) - L(f, P²) - L(f, P²) - L(f, P²) - (f, P²) is closed, so by Theorem 10.16, $a \in E$. Thus, by the Fundamental Theorem of Calculus and the Chain Rule, F 0 (x) = f (g(x)) · g 0 (x) ≥ 0 · 0 = 0. , and 0 Dg(x, y) = 1 - y sin(xy) 1 + log y x/y 1, 0 and D(f · g)(x, y) = [2x log y - y 2 sin(xy) cos(xy) - xy sin(xy) + x2/y]. By the Fundamental Theorem of Calculus we obtain 0 = $\alpha f(c) - \beta f(c) = 0$. $(c) = (\alpha - \beta)f(c)$ for all $c \in [a, b]$. Thus g is well defined on all of E. By the choice of δ , the definition of ϵ , and a little algebra that $f(x) < f(a) + M - f(a) M + f(a) = M - \epsilon$. Thus P satisfies i). Since u is harmonic, we have ZZZ k $\nabla uk2 \, dV = 0$. Then (f - 1)O(u) = 1/fO(t) > 0 for t = f - 1(u), i.e., $\tau(u) := f - 1(u)$ is an orientation equivalent change of variables. c) Let $x \in (0, \pi/2]$. 3.3.2. a) Consider f(x) = ex - x3. Thus the claim holds for all $n \ge 3$. 2.3 k k 2.0 0 k=1 k=1 Since sin(k $\pi/2$) = -1 when k = 3, 7, . Hence by induction, this result holds for all $n \ge 3$. 2.3 k k 2.0 0 k=1 k=1 Since sin(k $\pi/2$) = -1 when k = 3, 7, . Hence by induction, this result holds for all $n \ge 3$. 2.3 k k 2.0 0 k=1 k=1 Since sin(k $\pi/2$) = -1 when k = 3, 7, . Hence by induction, this result holds for all $n \ge 3$. 2.3 k k 2.0 0 k=1 k=1 Since sin(k $\pi/2$) = -1 when k = 3, 7, . Hence by induction, this result holds for all $n \ge 3$. toward the z axis, so the point (x0, y0, z0) where the tangent plane is parallel should be on the "back" side of the paraboloid, i.e., (x0, y0) should lie in the fourth quadrant. By hypothesis, $|Bn,m| \le 2M$. Hence ak rk is eventually k decreasing, in particular, bounded above, /r)k for P ∞ say byk M. By the Triangle Inequality, kf (a + h) + g(a + h) - f $(a) - g(a) - T(h) - S(h)k kf(a + h) - f(a) - T(h)k kg(a + h) - g(a) - S(h)k \le + .$ Thus f(x) = 0 for all $x \in [a, b]$. Therefore, $X X U(g, P0) = Mj(g)\Delta xj + Mj(g)\Delta xj + U(f, P0) j \in A \le X 2C\Delta xj +
U(f, P0) j \in A \le X 2C\Delta xj + U(f, P0) j \in A \le X 2C\Delta x$ Ordered field axioms. To answer this question, we must see how the term T (x, y0) affects S $\circ \phi(Q)$. Finally, suppose a < b, i.e., b - a \in P. Since wn $\geq xn$, it follows that wn > M for all n $\geq N$. Thus x = - sup(-E) = inf E. Therefore, $\tilde{A} \mid p \mid xy \mid x4 + y4 \mid m, p = (0, 0)$. (2k)2 4 k2 24 k=1 $\infty \infty k=1$ k 1 X 1 X 1 1 $\pi 2 \pi 2 \pi 2 = - = - = .$ i.e., lim supN $\rightarrow \infty \mid \Delta N$ $(x) | \le {}^{2}M$ for all $x \in \mathbb{R}$. 12.5 Partitions of Unity. Thus $|f(x) - L| = |x^{3} + 2x - 3| = |x - 1| |x^{2} + x + 3| < 9\delta \le \varepsilon$ for every x which satisfies $0 \sqrt{<|x - 1| < \delta}$. Then each Sj is nonoverlapping and has a smooth parametrization. j=1 Thus $K=\mathbb{N}[\mathbb{N}[\mathbb{B}r(xj)(xj) \cap K = j=1 \{xj\} = \{x1, ..., b\}$. Let $f(x) = x^{2} - x = x^{2} - x \log 2$. c) Since $h \circ g - 1$ is $\mathbb{C} \infty$ for each chart (U, g) of M, it follows from definition that h is a C ∞ function on M for each chart (V, h) of M. 10.5.10. Then f 0 (0) = 0 and f -1 (x) = 3 x has no derivative at x = 0. Then 0 > -a > -1, so 0 < 1 - a < 1.1.2.11. Let M > 0 and choose N \in N such that n \ge N implies xn \ge 2M/C and supn \ge N yn > C/2. b) By symmetry, we may suppose that x = y = ∞ This implies that $\sup(A + B) - a0 < \sup B$, so by the Approximation Property again, there is a $b0 \in B$ such that $\sup(A + B) - a0 < b0$. Thus $m + 2 > a \ge m$ as required., xN be a partition of [a, b] whose norm is $< \delta$ and set $\cdot \int^{2} 2Rj = [xj-1, xj] \times f(xj) - f(xj) + .$ To show this is also P_{∞} the case when k=0 at $\rho \in (0, 1)$, fix $r < \rho$ < 1 and observe since |ak ρk | ≤ C for all $k \ge 0$ that |SN rN | = | N X aj rN | ≤ C j=0 N X rN j=0 ≤ C ρ j rN ρN (1 - ρ) for all N ∈ N. 5 Copyright © 2010 Pearson Education, Inc. Since E is a compact subset of H o , choose $\delta 0 > 0$ such that $x \in E$ and $khk < \delta 0$ imply $x + h \in H$ o. On the other hand, yn+1 is the geometric mean of xn+1 and yn, so by Exercise 1.2.6, $yn+1 \ge yn$. But the series itself diverges by the Divergence Test. Set $A = \{j : E \cap [x_j-1, x_j] \ b \in \emptyset\}$ and $B = \{1, 2, . b\}$ The proof of part a) also proves this statement. Thus $\alpha, \beta \in J$. On the other hand, by L'H^opital's R1 Rule, $(1 - \cos x)/x^2 \rightarrow 1/2$ as $x \rightarrow 0$. 13.6.4. a) Let E be the solid cylinder whose boundary is S and $F = (x_j, x_j^2 - z_j^2)$. xz). 3 b) Let $\varphi(u, v) = (u, u3, v)$ and $E = [0, 2] \sqrt{\times} [0, 4]$. Thus $E \subseteq Br$. Then T (0, 1, 0, 0) = T(1, 1, 0, 0) - T(1, 0, 0, 0) = (5, 4, 1) - (a, b, c) = (5 - a, 4 - b, 1 - c), so $\neg a = 1 - \pi A = \lfloor b 4 - b 2 - 3 \rfloor$. a 13.3.5. By Theorem 13.36, $N\psi = \Delta \tau N\varphi \circ \tau$. But since xn $\rightarrow a$ as $n \rightarrow \infty$, there is an xn $\in Bs$ (a) $\cap E$. 11.3.11. c) $[0, \infty)$ is closed and $n \in [0, \infty)$ is a sequence which has no convergent subsequence. c) Let (u, v) be a unit vector. a) By definition, $\nabla \cdot \nabla u = \nabla \cdot (ux, uy, uz) = uxx + uyy + uzz$. Conversely, if E is not connected then there exist nonempty relatively open subsets U and V of E such that $U \cap V = \emptyset$ and $E = U \cup V$. Given $^2 > 0$ choose N so large that $e - N < ^2$. The inequality holds if 8k 4 + 2k. 2 < 16k 4 - 8k 2 + 1, i.e., if 0 < 8k 4 - 10k 2 + 1. R R ∞ ∞ Similarly, e cos x/ logp x dx = - sin(e) + p e sin x/(x logp+1 x) dx. t \rightarrow \infty tL(1 + 1/t) h $\rightarrow 0$ + h lim A similar argument shows that (ah - 1)/h $\rightarrow 1$ as h $\rightarrow 0$ -. Let $\psi(u) = u/(1 - u) + log(1 - u)$, $u \in (-\infty, 1)$. 5.3.1. a) By the Chain Rule d F (x) = - dx Z x 2 0 f (t) dt = -f (x 2) \cdot 2x. However, if u = (1/2, 1/2)2) then $\sqrt{\sqrt{f(t/2, t/2)}} - f(0, 0) = \lim = \lim t \rightarrow 0 t \rightarrow 0 t t$ does not exist. a) By the Weierstrass M-Test, $\varphi(t)$ converges uniformly on (0, t] for each $t \in (0, \infty)$. Thus a + c ≤ b + c holds for all a ≤ b. If u = x + 1 then du = dx and x2 = u2 - 2u + 1. Thus y is not a cluster point of E. By part a), choose I so that $\tilde{A}Z + (1/2) t = (1/$ $\leq M | b - a | 1/n$. The result holds for n = 0 since c0 - b0 = 1 and a20 + b20 = c20. Finally, use the Approximation Property to choose Ck > 0 such that $Ck \downarrow M2$ as $k \to \infty$ and take the limit of $kT(x)k \leq Ck kxk$ as $k \to \infty$. $n \to \infty n \to$ although k=0 (-1)k diverges. Hence ex - 1| = ex 2 / n 2 / n - 1 ≤ e4/N - 1 < 1/k 11/10 as $k \to \infty$. Therefore, $Z Z 2\pi \omega = C 0 \sqrt{\pi(-1 + (-z0 \sin t + z0 \cos 2t + z0 \sin t \cos t)} dt = \pi z 0 = 2 \sqrt{5}$. Thus $0 < 1 - x^2 = (1 - x)(1 + x) < 2\delta \leq -1/M$, i.e., $1/(1 - x^2) > -M$. a Thus the left-most integral equals zero if and only if f(a)g(a) = f(b)g(b). d) The maximum of 1/k for $k \in N$ is 1. b) This set is closed and bounded, hence compact. Hence by part a) and Exercise 6.1.9b, $\sigma n \rightarrow L$ as $n \rightarrow \infty$, i.e., Pb) ∞ aro summable to L. Since N is fixed, k=1 ak /(j + k) $\rightarrow 0$ as PN j $\rightarrow \infty$. $\sqrt[4]{\sqrt{d}}$ x sin(1/x) is continuous for x > 0 by Theorem 3.22. Chapter 10 10.1 Introduction. By looking at the graph, we see that f (E) = ($-\infty, \infty$). 42 Copyright © 2010 Pearson Education, Inc. Hence by the Comparison Test, the original series converges when p > 1. If xn-1 > yn-1 > 1 then yn-1 - xn-1 yn-1 = yn-1 (yn-1 - xn-1) > 0 so yn-1 (yn-1 - xn-1 yn-1 = yn-1 (yn-1 - xn-1) > 0 so yn-1 (yn-1 - xn-1 yn-1 = yn-1 (yn-1 - xn-1) > 0 so yn-1 (yn-1 - xn-1) > 0 so yn-1 (yn-1 - xn-1 yn-1 = yn-1 (yn-1 - xn-1 yn-1) = (yn-1) parameterized by $\varphi(t) = (\sin t, \cos t, 6), t \in [0, 2\pi]$. 11.5.1. a) Clearly, fx = 2x + y, fy = x + 2y, fxx = 2, fxy = 1, and fyy = 2.00 Thus set h (x) = P(x, 0), i.e., h(x) = Rx P(u, 0) du. d) By the First Multiplicative Property, mn-1 < pq -1 if and only if mq = mn-1 qn < pq -1 nq = np. 10.6.3. Let C be closed in Y. Then y = f(a) for some a $\in A \setminus B$. 3.1.4. a) Publishing as Pearson Prentice Hall, Upper Saddle River, NJ 07458. By definition, then, wn $\rightarrow \infty$ as $n \rightarrow \infty$. 8.4.11. $n \rightarrow \infty$ n $\rightarrow \infty$ Note, we used Corollary 1.16 and the fact that the sum on the left is not of the form $\infty - \infty$. 14.5.4. Let g(x) = (f(x+)-f(x-))/2 for each $x \in \mathbb{R}$. This contradicts the fact that f 0 is 1-1 on [a, b]. Continuing in this manner, we can choose integers k1 < k2 < . c) Since p /q < 5 implies p/q < 5, inf E = 0, sup E = 5. Hence by Remark 6.40, |sn - s| is dominated by (1/2)n+1/(1/2) = (1/2)n for $n \ge 2$. ∞ Sf = a0 (f) X + ak (f) cos kx 2 k=1 which converges uniformly and absolutely on R by the Weierstrass M-Test and part c). In particular, given $^2 > 0$ there is a $\delta > 0$ such that |x - y| $< \delta$ and x, y $\in [0, 1]$ imply $\neg \infty \neg X \neg k + k \neg (-1)$ ak $(x - y) \neg = |f(x) - f(y)| < 2$. $\partial E \to C$ This follows immediately from Gauss' Theorem since by Exercise 13.5.8 and part a), $\nabla \cdot (u \nabla v - v \nabla u) = \nabla u \cdot \nabla v + u \Delta v - \nabla v \cdot \nabla u - v \Delta u = u \Delta v - v \Delta u$. c) Let $h = \pi/2n+1$. Suppose that the result holds for some integer $n \ge 1$ and let $\varphi : \{1, 2, ..., V + u \Delta v - \nabla v \cdot \nabla u - v \Delta u = u \Delta v - v \Delta u$. minimum of f on H is f (1, 0) = 1 and the absolute maximum of f on H is f (1, 2) = 17. Pn P ∞ 6.1.6. a) Let sn := k=1 ak. Since Br (x) also intersects Ac U B c, it must be the case that Br (x) intersects Ac U B c,
it must be the case that Br (x) intersects Ac U B c, it must be the case that Br (x) intersects Ac U B c, it must be the case that Br (x) intersects Ac U B c, it must be the case that Br (x) intersects Ac U B c, it must be the case that Br (x) intersects Ac U B c, it must be the case that Br (x) intersects Ac U B c, it must be the case that Br (x) intersects Ac U B c, it must be the case that Br (x) intersects Ac U B c, it must be the case that Br (x) intersects Ac U B c, it must be the case that Br (x) intersects Ac U B c, it must be the case that Br (x) intersects Ac U B c, it must be the case that Br (x) intersects Ac U B c, it must be the case that Br (x) intersects Ac U B c, it must be the case that Br (x) intersects Ac U B c, it must be the case that Br (x) intersects Ac U B c, it must be the case that Br 9.2. 10.1.4. a) If xn = a for all n, then $\rho(xn, a) = 0$ is less than any positive ε for all $n \in N$. $n \to \infty 0$ Thus f is integral equals I. Publishing as Prentice Hall. Then there exist points $xk \in B1/k$ ($x) \cap E$ for each $k \in N$. Then for all x0, $Z \mid Z \mid Z \mid 2$ (x2 + 2x) dx = 6 = 0 = (x0 + 2) x dx. A similar argument establishes an analogous identity. for lower integrals. 136 Copyright © 2010 Pearson Education, Inc. Similarly, $E \cap V 6 = \emptyset$. Hence we may set hz = xz and g = 0, i.e., P = xz 2/2 and R = 0. Then by the triangle inequality, $|f(x) - Q(x)| < \varepsilon$ for all $x \in [a, b]$, that (f - 1)0(y) = 1/f 0(x) by Theorem 4.33. If x < x0, choose r, $q \in Q$ such that q < x < x0 < r and $r - q < 2\delta = 1/N$. Then the limits infimum are both -1, the limits supremum are both -1, the limits supremum are both -1, the limits supremum are both 1, but $xn + yn = 0 \rightarrow 0$ as $n \rightarrow \infty$. Then $\mu \ \| \mu \ \| m \ X (-1)k \ 1 \ 1 \ 1 \ 1 \ 1 = - - \cdots - - p \ c$) Let x0 = 0, x2n = 1, and xk = y2n - k for 0 < k < 2n, where $yk-1 = (2k + 1)\pi/2$. ∞ If k=0 ak = L then $sn \rightarrow L$ as $n \rightarrow \infty$. 3 13.2.3. a) Let C1 represent the horizontal piece and C2 represent the vertical piece. 10.6.2. a) f (0, 1) = (0, 1) is open, no big deal; f [0, 1] = [0, 1] is compact and connected as Theorems 10.61 and 10.62 say it should. Then $T = T \ Z \ Z = -1 \ T \ T \ C \ Z = -1 \ C \ Z =$ $(Br(x0)) Br(x0)^{-1} Vol(Br(x0)) Br(x0)^{-1} Z 1 \le |f(x) - f(x0)| dx Vol(Br(x0)) Br(x0) Z^{2} < dx = 2$. If xn = (-1)n and yn = 0 then lim inf $(xn + yn) = -1 < 1 = \lim sup xn + \lim inf yn \cdot c$. By part b), $\{s2n + 1 - s2n \to 0 \text{ as } n \to \infty$. However, since (2k + 1)/(2k + 4) = 1 - (3/2)/(k + 2), the series converges by Raabe's Test. Finally, if $x \in /V$, then $x \in /Ixj$ for any j, so $f(x) = 0 + \cdots + 0 = 0$. By the Extreme Value Theorem, there exist xM, $xm \in [a, b]$ and $f(xM) = \beta := \sup\{f(x) : x \in [a, b]\}$. Since $U \subseteq E$ implies $U \circ \subseteq E \circ$ (see Exercise 8.4.3), it follows that $x \in /U$ o. 14.2.8. By Theorem 9.49, f is continuous almost everywhere, hence by Fej er's Theorem, $\sigma N f \rightarrow f$ almost everywhere as $N \rightarrow \infty$. 6.2.10. 0 Let $^2 > 0$. What happens at (0, 0)? Hence, it converges if and only if $x \in (-3, 2, 3, 2)$. No $a \in R$ satisfies $a < b - \varepsilon$ for all $\varepsilon > 0$, so the inequality is vacuously satisfied. 7.5.3. The proof is by induction on n. R1 R0 R1 b) Since $-1 dx/x^2 = -1 dx/x^2 + 0 dx/x^2$, this integral diverges by Exercise 5.4.2b. Since K := $\varphi(\text{spt } \phi) \subset \varphi(W)$. h $\rightarrow 0 h^2 h c$) If we start with (*) and reverse the roles of x and spt ($\phi \circ \phi \rangle \subset \varphi(W)$. h $\rightarrow 0 h^2 h c$) If we start with (*) and reverse the roles of x and spt ($\phi \circ \phi \rangle \subset \varphi(W)$. y, we have $\lim h \to 0 \Delta(h) = fxy(a, b)$. b) Let (a, b, c) be a nonzero vector in the plane z = x orthogonal to (1, -1, 0). 5.2 Riemann Sums. 11.2.4. By definition, $f(h, 0) - f(0, 0) h fx(0, 0) = \lim = \lim = h \to 0 h \to 0 \sin |h| h \frac{1}{2} 1 - 1$ as $h \to 0 + as h \to 0 - b$. We conclude that $f(0, x_1)(f - 1, x_2) = f(x) - f(a)$. It is not \sqrt{t} it is contained \sqrt{t} compact since we if we choose rationals an \downarrow 2 and bn \uparrow 3, then {(an, bn) \cap Q is a countably infinite open covering of E which has no finite subcover. $4 \ 0 = 0 \ 0 \ 6 \ 15.3.2$. Since d($0 \ \pi \ n \ X \ 0 \ dj$. If Fz (a, b, c) = 0. c) By the Product Rule, (xn Bn (x)) $\circ \propto X \ X \ (-1)k \ (n + 2k)^3 \ x$ $(n + k)! 2 = xn k = 0 \infty X k = 0 (-1) (n + k)! 2 = xn k = 0 \infty X k = 0 (-1) (n + k)! 2 = xn k = 0 \infty X k = 0 (-1) (n + k)! 2 = xn k = 0 \infty X k = 0 (-1) (n + k)! 2 = xn k = 0 \infty X k = 0 (-1) (n + k)! 2 = xn k = 0 \infty X k = 0 (-1) (n + k)! 2 = xn k = 0 \infty X k = 0 (-1) (n + k)! 2 = xn k = 0 \infty X k = 0 (-1) (n + k)! 2 = xn k = 0 \infty X k = 0 (-1) (n + k)! 2 = xn k = 0 \infty X k = 0 (-1) (n + k)! 2 = xn k = 0 \infty X k = 0 (-1) (n + k)! 2 = xn k = 0 \infty X k = 0 (-1) (n + k)! 2 = xn k = 0 \infty X k = 0 (-1) (n + k)! 2 = xn k = 0 \infty X k = 0 (-1) (n + k)! 2 = xn k = 0 \infty X k = 0 (-1) (n + k)! 2 = xn k = 0 \infty X k = 0 (-1) (n + k)! 2 = xn k = 0 \infty X k = 0 (-1) (n + k)! 2 = xn k = 0 \infty X k = 0 (-1) (n + k)! 2 = xn k = 0 \infty X k = 0 (-1) (n + k)! 2 = xn k$ $[0, \pi/2]$. n 1 1 1 Since f is Rabsolutely integrable on $[1, \infty)$, this last integral converges to 0 as $n \rightarrow \infty$. Hence by the Completeness Axiom, this set has a finite supremum. Since Br (x) \subseteq B for small r > 0. To determine whether this is a maximum or a minimum, notice that the discriminant of F (x, y) := ax + by + cDx2 + cEy 2 is 4c2 DE. j dx dxj k=1 14.3.5. a) Let $k \ge j \ge 0$. Thus Ex is nonempty. We obtain $\kappa(\varphi(t)) = k(1, f \ 0, (t), 0)k | f \ 00, (t) | = .$ Since z = y, the projection $\sqrt{\sqrt{6} \partial S}$ onto the xy plane is given by $x^2 + 2y^2 = 1$. b) By Theorem 1.42, there are countably many polynomials with integer coefficients. This contradiction proves that cos(1) is irrational. 10.3.3. Let $y \in V = \{x \in X : s < \rho(x, a) < r\}$ and let $^2 < \min\{\rho(y, a) - s, r - \rho(y, a)\}$. Since f is integrable, there is an M > 0 such that $|f(x)| \le M$ for all $x \in [0, 1]$. n Since Z Z $\infty \infty f(x) dx = n$ it follows that n dx $\le x \log p(x + 1) Z \infty n dx 1 = x \log p(x) (p - 1) \log p - 1$ (n) $1 + p - 1 |s - sn| \le t \le n \log p(n + 1) (p - 1) \log p - 1$ (n) $n(p - 1) \mu 1 = t \le n \log p(x + 1) Z \infty n dx$. logp-1 (n). Therefore, $V = ka \times bk \cdot h = ka \times bk + bk \cdot c|$ (a $\times b$) $\cdot c| = |(a \times b) \cdot c|$. Hence by definition, x is the supremum of E. It is open but not connected. 2 2 2 Hence by induction and a), 0 < xn+1 - yn+1 < (x1 - y1)/2n. By the Commutative Property of real numbers, $x + y = (x1, ... But if we translate this back into <math>^2-\delta k \rightarrow \infty p$ language, we conclude that k |bk| \rightarrow r as k $\rightarrow \infty$. 4.3.5. By the Mean Value Theorem, |f (x) - 1| = |f (x) - f (0)| = |(x - 0)f 0 (c)| = |x| · |f 0 (c)| for some c \in (0, x). Since f is continuous and periodic, we have by Exercise 14.2.4 that SN f \rightarrow f uniformly on R. Therefore, Q satisfies Postulate 1. ∞ d) By part b) L{f}(s) = (s - a) 0 e^{-(s-a)t} \varphi(t) dt so by Theorem 11.9, Z ∞ Z ∞ L{f} $(s) = e^{-(s-a)t} \phi(t) dt - (s-a) e^{-(s-a)t} t\phi(t) dt 0 0 Z \infty = e^{-(s-a)t} \phi(t)(1 - (s-a)t) dt$. By the Binomial Formula, $1 = 2n \mu 1 a^{-2} + a 2a \P n n \mu \P X X n 1 (a - 2)n - k = =$. Since g(x) > f(x), it is clear that $x \in Bf(x) (x) \subset Bg(x) (x)$ for all $x \in E$. Thus f 0 (c1) < 0 < f 0 (c2). b) By definition, $x \times x = (x2 x^3 - x2 x^3, x1 x^3 - x^2 x^3, x1 x^3 - x^2 x^3)$. $x_1 x_3, x_1 x_2 - x_1 x_2) = 0$, and $x \times y = (x_2 y_3 - x_3 y_2, x_3 y_1 - x_1 y_3, x_1 y_2 - x_2 y_1) = -(-x_2 y_3 + x_3 y_2, -x_3 y_1 + x_1 y_3, -x_1 y_2 + x_2 y_1) = -y \times x$. .. b) If $\{x_n\}$ is decreasing, so part a) applies. $\sqrt{Then n} \ge N$ implies that $y \to 0$ + hence $t \to \infty$. P ∞ b) If $E = \{x_1, ..., By$ induction, there are infinitely many points in $E \cap (a - r, a + r)$. If $p \le 0$, then the series diverges by the Divergence Test., $aN \}$ is a partition of [a, b] and $Q = \{c0, ..., aN \}$ is a partition of [a, b] and $Q = \{c0,
..., aN \}$ is a partition of [a, b] and $Q = \{c0, ..., aN \}$ is a partition of $[a, ..., aN \}$ $k^2 + \nu 0$ ('(t0))k $\phi 0$ (t0)k0) = k $\phi 0$ (t0)k3 ($\nu 0$ (s0) × $\nu 00$ (s0)). Therefore, lim $h \rightarrow 0 + f(-x + h) - f(x) = lim = lim = f 0$ (x), $h \rightarrow 0 + h \rightarrow 0 - h - h$ h i.e., the right derivative of f at -x equals f 0 (x). there is an xn > n satisfying f (xn) $\geq L + 20$ or f (xn) $\leq L - 20$, does not converge to L as $n \rightarrow \infty$. The function g(x) = f(x) for $x \in (a, b)$ and g(a) = f(a+) is continuous on [a, x] for every $x \in (a, b)$. Let n > N0 and write $f(n) = (f(n) - f(n-1)) + \cdots + (f(N0 + 1) - f(N0)) + f(N0) + (f(N0 + 1) - f(N0)) + f(N0)$. Since m + 2 is not a lower bound of E, there is an $a \in E$ such that m + 2 > a., $xn = x^2 + 1$ b) See Theorem 2.12. Therefore, by Dini's Theorem and Theorem 7.10, $\sqrt{Z} \lim k \rightarrow \infty r \pi/2 \sin x 0$ 2k dx = 4k - 3x r Z $\pi/2 r 1$ 1 sin x dx = . Thus y = E(x) 7.2.4. The series converges uniformly on R by the Weierstrass M-Test. n! Since n is odd, (c - b)n /n! < 0 and (c - a)n /n! > 0.4.3.7. By the Monotone Property for Suprema, F is increasing on [a, b]. It follows from Theorem 12.4 that E is a Jordan region and it has area zero. j=1 9.2.5. Let $x \in E$. f) It converges by the Root Test, since \sqrt{k} ak $\equiv 3 - (-1)k \pi$ has a limit supremum of 4/5. 10.5.3. a) Let I and J be connected in R. P ∞ P ∞ k b) False. To prove part b), let $x \in U \cap \partial A$ and suppose for a moment that $x \in E$ o. Thus E has no boundary if and only if E is clopen. Thus Z b Z $2\pi A(S) = a \sqrt{2\pi} (b2 - a2)$. from (x1, y1, z1) and θ represent the angle between w := (x0 - x2) and the normal (a, b, c) of Π . In particular, spt (f g) \subseteq spt f \cap spt g. Since the given series is uniformly convergent by the Weierstrass M-Test, it remains to show that this series is the Fourier series of f. Set $k0 = \varphi(n + 1)$ and define ψ by $\frac{1}{2} > \langle k0 \psi() = -1 > k0$. 4.3.1. a) Let f(x) = ex - 2x - 0.7. Since $x \ge 1$, f 0(x) = ex - 2x - 0.7. Since $x \ge 1$, f 0(x) = ex - 2 > 0. 11.7.4. By Remark 11.51, $\nabla g(b) = 0$, hence by the Chain Rule, $\nabla (g \circ f)(a) = \nabla g(b)Df(a) = 0$. Since L(1) = 0 and L(e) = 1, we also have E(0) = 1 and E(1) = e. Therefore, wxx (x2, t2) - wt (x2, t2) = 0. wxx (x2, t2) ≥ 0 , a contradiction. U (f, P) = 0.5f (0) + 0.5f (0.5) + f (1) = 39/8. A similar argument works for inf E. Let $x \in (0, \pi/2]$. In particular, $x \in E$. Therefore, the original expression is rational if and only if n = 1. xn = (1 + (-1)n) n satisfies xn = 0 for n odd and xn = 2n for n even. e Since ex is continuous, we conclude that an $\rightarrow 4/e$ as $n \rightarrow \infty$. Similarly, $|bk(f)| \le 1/k 2$ for k large. Hence by Abel's Theorem, f is uniformly continuous on [0, 1]. $\sqrt{\sqrt{b}}$ Take the limit of xn+1 = 2 + xn - 2 as $n \to \infty$. The root is less than or equal to 1/2 for $k \ge N = 1$ by Example 4.30. We shall use the Ratio Test. By hypothesis, $C := \lim \text{ supn} \to \infty$ yn > 0. By the Limit Comparison $\sqrt{P} \propto P \propto 2p$ Test, only if $k=1\sqrt{1/k} p$ converges, i.e., if and only if p > 1. 2 - k2 = k + 0 ak (f) = 14.2.5. a) If P(x) = Pn = 0 ak xk then $Z n \times b = 0$. In particular, it follows from assumption ii) that $x \to 0$. Hence it converges to 0 as $(x, y) \to (1, 1)$ by the Squeeze Theorem. Therefore, by telescoping we obtain $\infty \times \log(k(k+2)/(k+1)2) = 14.2.5$. a) If $P(x) = 2 - k^2 = k^2 + 2 - k^2 =$ $\log(2/3) - \lim \log(k/(k+1)) = \log(2/3)$. 0 k! n! 0 k=0 R1 Since 0 x2k/k! dx = 1/((2k+1)k!), this completes the proof of part a). Therefore, P ∞ k=1 bk (f)/k converges. A(S) = $-\pi - \pi 13.3.2$. a) The trivial parameterization is (ϕ , E), where $\phi(u, v) = (u, v, u^2 - v^2)$ and E = {(u, v) : $-1 \le u \le 1, -|u| \le v \le |u|$ }. f (x) $\le g(x)$. Therefore, the original series converges if and only if $x \in [-3, -1]$. By construction $xk \in E$ and $xk \to \infty$. 5.3.0. a) True. Since $\cos x$ is nonnegative when $(4k - 1)\pi/2$. Then $\tau 0$ (u) = 1/(b - a) > 0 and $\tau (I) = [0, 1]$. Vol (E) + 1 119 Copyright © 2010 Pearson Education, Inc. 8.4.6. a) If E is connected in R then E is an interval, hence E o is either empty or an interval, hence connected by definition or Theorem 2.36, lim supn $\rightarrow \infty$ |xn | = 0. Suppose n n X X xk xk 1 \leq x < + n. Hence it follows from the Quotient Rule and assumption iii) that μ ¶0 sin x cos2 x + sin2 x (tan x)0 = = sec2 x. Indeed, by Stirling's Formula, μ ek k! kk $\sqrt[n]{\sqrt[n]{k}} = 0$ as $k \to \infty$. For the case $\alpha < 0$, xn > M implies α xn $< \alpha$ M. If $y \in Bs$ (b) then, since $x \in Br$ (a) $\cap Bs$ (b), we have $\rho(x, y) \le \rho(x, b) + \rho(b, y) < s + s = 2s < d$. In particular, f is 1-1 (actually monotone increasing) on I. 12.5.6. Since H is a Jordan region, choose a grid $\{Q \}$ h = 1 of a rectangle R \supset H such that X |Q | 0.10.4.3.Since E is compact, it is bounded by Theorem 10.46. Let $x \in [a, b]$ and $n \ge N$. n-2x0 = 0 < x1 =If $P = \{x0, x1, .$ Thus f is continuous on R. 2 2.3.10. p b) Since k $1/k \neq 0$ as $k \rightarrow \infty$, this series converges by the Root Test. 1.4 Mathematical Induction. Suppose for a moment that the projection function fk (x) = xk has a limit as $x \rightarrow a$ and satisfies fk (x) \rightarrow fk (a) as x \rightarrow a for each k \in {1, . Let y \in R with y 6= x, and set ² = |x - y|/2. Since k=0 1/((2k+1)k!) = 1.4626613 < 0 e dx < 1.4626813. 9.3.5. Let $\epsilon = 1$ and choose $\delta > 0$ such that 0 < kx - ak < δ implies kf (x - Lk < 1. a) By the one-dimensional Mean Value Theorem, $\Delta(h) = hfy$ (a + h, b + th) -hfy(a, b + th) for some $t \in (0, 1)$, and (*) $\Delta(h) = hfx(a + uh, b + h) - hfx(a + uh, b)$ for some (x1, t1) = - < 0 for some (x1, t1) = - < 0 for some $(x1, t1) \in H 0$. Finally, $\varphi(0) = (0, 0)$ and $dy/dx \rightarrow 0$ as $t \rightarrow 0$. 105 Copyright © 2010 Pearson Education, Inc. Indeed, if M > 1, then set r = - < 0 for some $(x1, t1) \in H 0$. $\min\{\rho(y_j, y_k): j, k \in [1, M]\}$ and notice that the Br (y) is are open, nonempty, and disjoint, hence separate f (X). Since $\alpha \in \beta$, it follows that f (c) = 0 for all $c \in [a, b]$. c) Suppose f is continuous at 0 and $x \in R$. 0 RR Therefore, S $\omega = 8\pi + 0 = 8\pi$. c) The Lagrange equations are $y = 2x\lambda + \mu$, $x = 2y\lambda + \mu$, and $0 = 2z\lambda + \mu$. Thus we get equality in the Cauchy-Schwarz inequality if and only if x - tyk = 0, i.e., if and only if x = ty. Let C > 0 satisfy $|f(x)| \le C$ for $x \in I$. x > 0. 89 Copyright © 2010 Pearson Education, Inc. $\sqrt{\sqrt{5.4.5.8}}$ By Exercise 5.4.2b, 1/x is integrable on (0, 1), but 1/x = (1/x)(1/x) is not. Since a, b, c do not lie on the same straight line, Remark 8.10 implies that $d := (a - b) \times (a - c)$ is nonzero. Similarly, vx = f 0 (x - y) + g 0 (x + y), vy = -f 0 (x - y) + g 0 (x + y), and vxx - vyy = f 00 (x - y) + g 00 (x + y) = 0., xk+1 are distinct points in $E \cap Br$ (a). The second component factors: $2^{-1} xy - 2xy + y - (x - 1)2^{-1} (y - 1)(x - 1)2^{-1} = -x^{2} + y^{2} - 2x - 2y + 2^{-1} (x - 1)^{2} + (y - 1)^{2} = -x^{2} + y^{2} - 2x - 2y + 2^{-1} (x - 1)^{2} + (y - 1)^{2} = -x^{2} + y^{2} - 2x - 2y + 2^{-1} (x - 1)^{2} + (y - 1)^{2} = -x^{2} + y^{2} - 2x - 2y + 2^{-1} (x - 1)^{2} + (y - 1)^{2} = -x^{2} + y^{2} - 2x - 2y + 2^{-1} (x - 1)^{2} + (y - 1)^{2} = -x^{2} + y^{2} - 2x - 2y + 2^{-1} (x - 1)^{2} + (y - 1)^{2} = -x^{2} + y^{2} - 2x - 2y + 2^{-1} (x - 1)^{2} + (y - 1)^{2} = -x^{2} + y^{2} - 2x - 2y + 2^{-1} (x - 1)^{2} + (y - 1)^{2} = -x^{2} + y^{2} - 2x - 2y + 2^{-1} (x - 1)^{2} + (y - 1)^{2} = -x^{2} + y^{2} - 2x - 2y + 2^{-1} (x - 1)^{2} + (y - 1)^{2} = -x^{2} + y^{2} - 2x - 2y + 2^{-1} (x - 1)^{2} + (y - 1)^{2} = -x^{2} + y^{2} - 2x - 2y + 2^{-1} (x - 1)^{2} + (y - 1)^{2} = -x^{2} + y^{2} - 2x - 2y + 2^{-1} (x - 1)^{2} + (y - 1)^{2} = -x^{2} + y^{2} - 2x - 2y + 2^{-1} (x - 1)^{2} + (y - 1)^{2} = -x^{2} + y^{2} - 2x - 2y + 2^{-1} + (y - 1)^{2} = -x^{2} +
y^{2} - 2x - 2y + 2^{-1} + (x - 1)^{2} = -x^{2} + y^{2} + y^{2} - 2x - 2y + 2^{-1} + (x - 1)^{2} + (y - 1)^{2} = -x^{2} + y^{2} + (x - 1)^{2} + (y - 1)^{2} = -x^{2} + y^{2} + (x - 1)^{2} + (y - 1)$ 1|. By Definition 3.1, there is a $\delta > 0$ such that $0 < |x - a| < \delta$ implies $f(a) - \varepsilon < f(x) < f(a) + \varepsilon$. Therefore, $\omega(f, \pi/k) \rightarrow 0$ as $k \rightarrow \infty$ and it follows from part b) that ak (f) and bk (f) converge to zero as $k \rightarrow \infty$. Since T is linear, if the components of f are differentiable, then f(x + h) - f(x) - T(h) f(x + h) - f(x) = -T(1) = (f10 (x), f(x + h) - f(x) - T(h) - F(h) - Fexpression cannot be rational when n > 9. 11.5.7. Set E = Br(0). 1.6 Countable and uncountable sets. is open as Theorem 10.58 says it should; $f - 1[0, \pi] = .$ See Example 3.7. c) False. c) By the Ratio Test, this series converges when |x| < 1 and diverges when |x| > 1. By Theorem 11.30, there is a c between x and x + h such that $^{2}h(x) := \varphi(x + h) - 1$. $\varphi(x) - Df(x)(h) = (D\varphi(c) - D\varphi(x))(h)$. Since sin(x/n) > 0 for $n \ge 3$, we have g(x) := sin(x/n) + x + 1 + x + 1 > 1 + 1 = 2 for $n \ge 3$. an = 1 and bn = 1/n are Cauchy, but an /bn = n does not converge, hence cannot be Cauchy by Theorem 2.29. c) Using the trivial parameterization $z = ZZ p a^2 - x^2 - y^2$, we see that $N\varphi = (x/z, y/z, 1)$ points upward. 3.1.2. a) If $xn = 4/((2n + 1)\pi)$, then $xn \to 0$ but tan(1/xn) = (-1)n has no limit. b) Any subset of R which contains 0 and 1 will satisfy the Associative and commutative Properties, the Distributive Law, and have an additive identity 0. In particular, $y \in f(E)$. Thus ak bk is nonnegative and dominated by M ak. Suppose P is a polynomial of degree n > 1, i.e., $P(x) = an xn + \cdots + a1 x + a0$, an 6 = 0. Thus bx := (1/b) - x defines bx, and by parts a) and c), bx is a continuous extension of bq from Q to R which satisfies the exponential properties. Thus the original series converges by the Alternating Series Test. Hence by Lemma 1.40, B is at most countable, a contradiction. Then $f(x) \in f(E)$, so by definition, $x \in f - 1$ (f(E)). c) By the Bolzano-Weierstrass Theorem and Theorem 10.16, every closed bounded subset is sequentially compact. Conversely, suppose $x \in U \cap \partial E$. 9.6.4. Let h > 0 and $t \in \mathbb{R}$. Then $n \ge N$ implies $n^2 (2 + \sin(n^3 + n + 1)) \ge n^2 \cdot 1 \ge N^2 > M$. Hence by L'H[^]opital's Rule, $L(e) = \lim_{n \to \infty} \frac{1}{(1 + 1/n)} = 1$. Thus $\cos((\log k + x)/(k + x)) \rightarrow \cos 0 = 1$ uniformly on [0, 1] as $k \rightarrow \infty$. Hence by Theorem 14.29, Sg converges to g uniformly on [a, b] and pointwise on $(-\pi, \pi)$. 10.6.8. a) By the proof of Lemma 3.38, if f is uniformly continuous, then f takes a Cauchy sequence in X to a Cauchy sequence in Y. b) The maximum of x - 1 on [0, 1] is 0 and the minimum of x - 1 on [0 + 1 on [0, 1] is 1. Then C > 0, M ≤ C, and m ≥ -C. Let E := {k ∈ N : 2n b ≤ k}. If f(x) = x + 100 and g(x) = x2, then |f 0/g 0| = |1/x| ≤ 1 for x ∈ (1, ∞), but (x + 100)/x2 is not less than or equal to 1 when x = 2. Since m0 is least in E, it follows that m0 - 1 < 2n b, i.e., q < b. a) Suppose E is polygonally connected but some pair of open sets U, V separates E. Let a = 0, f (x) = x, and g(x) = $1/x^2$ for x 6 = 0 and g(0) = 0. Therefore, x $\in /\partial E$, a contradiction. Since f 0 (x) = $(1 - x)/(2 x(x + 1)^2) < 0$ for x > 1, f is strictly decreasing on (1, ∞). 2.3.6. a) Suppose that {xn} is increasing. But by l'A^opital's Rule, $\sqrt{2} u - u^3$ lim p = lim = 0. By the Chain Rule, 0 = wx = Fx + Fz zx and 0 = wy = Fy + Fz zy. on $V \times f(V)$, hence zx = -Fx/Fz and zy = -Fy/Fz. $P \infty$ Since $k=1 a + k = \infty$, choose an integer $k1 \in N$ least such that $+ + sk1 := b1 + b2 + \cdots + ak1 > y$. Then $n \ge N$ implies $n2 - n = n(n - 1) \ge N$ (N - 1) $> M(2 - 1) = M \cdot 9.5.2$. Let A, B be compact sets. for all |p| < 1/e. Since E is open, choose r > 0 such that $Br(x) \subset E$. $0 |x| \pi k k \pi 156$ Copyright © 2010 Pearson Education, Inc. Thus by the Comparison Theorem and u-substitution, $\sqrt{f(x) dx} \le 2Z 4 1 \sqrt{Z 2 f(x)} dx = 4 f(u) du = 20$. $g - 1 (0, \pi) = (0, \infty)$ is open, no big deal; $g - 1 [0, \pi] = [0, \infty)$ is closed-note that Exercise 9.4.4 does not apply since g is not continuous; $g - 1 (-1, 1) = \{0\}$ is not open and we don't expect it to be; g - 1 [-1, 1] = R is closed-note that Exercise 9.4.4 does not apply since g is not continuous. Z Z 1 Z 1 12.3.1. a) 1 (x + y) dx dy = 0 Z 3 Z 1 b) 0 Z Z $\pi 0 \sqrt{2} 3 xy + x dx dy = 0 Z 3 Z 1 b) 0 Z Z \pi 0 \sqrt{2} 3 xy + x dx dy =$ (E2). On the 22 other hand, f0 (0) := $limh \rightarrow 0$ (l/h)/e1/h = 0. Since $\rho(xkj, a) < 1/j$, xkj converges to a. To show it vanishes at infinity, let $^2 > 0$. It follows that $(1 + x/n)n \rightarrow ex$ uniformly on [a, b]. a) Given $x \in V$ choose $^2 := ^2x > 0$ such that $B^2(x) \subseteq V$. 3.1.3. a) By Remark 3.4 and Theorem 3.8, $x^2 + 2x - 3(x + 3)(x - 1)x + 34 = 0$ $lim = lim = 2.2 c 2 c p p \sqrt{lt follows that f(x)} \le (Mk (f) - mk (f))/(2 c) + f(y) for all x \in [xk-1, xk]. Thus this ball is a square with vertices (1, 1), (1, -1), (-1, -1), and (-1, 1). It follows from Remark 10.45 that A \cap B is compact. In general, set Sj = <math>\varphi(Ej \setminus (\cup j-1 k=1 Ek). 2 (a + x1) (a$ $x0 \in [a, b]$ then by the Sign Preserving Property there is a nondegenerate interval $[c, d] \subset [a, b]$ such that $f(x) > ^20$ Rd for $x \in [c, d]$. Thus ZZ $Z \ln Z \ln v \cos v$, $a2 \sin v \cos v$ j=1 $|Rj| < ^2$. 10.5.8. a) By Remark 10.11, \emptyset and X are clopen. Thus ∞ (SF)(x) = a0 (F) X bk (f) + cos kx. $\sqrt{In particular}$, S $\circ \varphi(Q)$ is a subset of a cube with sides s(1 + nM²). By telescoping, we have $\infty X k=2$ Since a1 = 2/3, we conclude that 1 ak = 2 $\infty X k=1$ ak = $\mu \P 1 - 0.6 = 1$. Substituting these values for λ and μ into 2 2 the first two Lagrange equations, we obtain $2y = 6x \sqrt{y} + x$ and 2x = 6y x + y, i.e., $3y = \pm x$. 4c) Let $\epsilon > 0$ and let $\delta = \epsilon/3$. 105 3 d) This region
is the set of points $\sqrt{(under'')}$ the cubical cylinder y = x which lies "over" the region in the xz 2 plane bounded by z = x and z = x. Cross multiplying, we obtain $4x^2 < 4x^2 - 1$, i.e., this case is empty. Since |R| = |x + R| for any rectangle R, it follows from Theorem 12.4 that A is of volume zero. k=0 ak is Ces' Pn-1 c) Since sn = k=0 (-1)k is 1 when n is odd and 0 when n is even. c) By vector algebra and Cauchy-Schwarz, $|x \cdot (y - z) - y \cdot (x - z)| = 1$ z = $|(y - x) \cdot z| \le kx - yk kzk < 2 \cdot 3 = 6$. j=1 k=1 6.3.7. a) Since $ak \rightarrow \infty$, |ak| < 1 for k large. e) It converges by the Root Test, since $|ak| 1/k = (k-1)! (k-1)! 1 < = \rightarrow 0 k! + 1 k! k as k \rightarrow \infty$. Hence $x/n \rightarrow 0$ uniformly on [a, b]. 11.5.12. Let $\varepsilon > 0$ and let Gm be the dyadic grid of Exercise 12.1.1 with m so large that $2-m < \varepsilon$. Hence it follows from definition that |ak+1/ak| > 1/r = rkP/rk+1 for k large, say $k \ge N$. 11.2.6. The function has continuous first partials at any (x, y) 6 = (0, 0), hence is differentiable there by Theorem 11.15. Thus f (c) = 0. x \rightarrow x0 x \rightarrow x0 3.2.4. a) If $q(x) \rightarrow \infty$ as $x \rightarrow a$, then given $M \in \mathbb{R}$, choose $\delta > 0$ such that $0 < |x - a| < \delta$ implies q(x) > M. In particular, f (n) exists and is continuous on R for all $n \in N$ and f(n)(0) = 0. Observe that $t = \tau(u)$ implies u = a + (b - a)t and set $\psi(t) = \varphi(a + (b - a)t)$ for $t \in [0, 1]$. c) Choose h 6= 0 small enough so that f(a + h) 6= 0. Thus if A is open, then f - 1 (V) is relatively open in A if and only if it is open in Rn. Hence $\varphi(W)$ is an open set containing H and $\varphi_j \circ \varphi - 1 = 0$ on $\varphi(W)$ for all $j \ge N$. It follows from the proof of Corollary 11.34, applied to f - S, that kf(x) - S(x) - f(a) + S(a)k + O(f(a) + S(a)k + O(a)k + O(= n + 1/n > N + 0 > M. Since the $-a - \hat{}$'s are nonpositive, it is clear that $\hat{s} \le sk1 \le y + bk1$ for $k1 < \hat{} \le r1$, c) Given 2 > 0 choose M > 0 so large that $sup\{|f(x)|, |g(x)| : x \in E\} \le M$. Thus an equation is x - y + 2z + 5w = 1. Since y = 0, $|yn| \ge |y|/2$ for large n. 31 Copyright \hat{w} 2010 Pearson Education, Inc. Let f(x) = sin(1/x) and g(x) = 1. Pn-1 Pm 2.4.4. Let sn = k=1 xk for n = 2, 3, . Let f (x) = 1/2 for all rationals $x \in (0, 1)$, f (x) = 1/2 for all rationals $x \in (0, 1)$, f (x) = 3/4 for all rationals $x \in (0, 1)$, f (x) = 1/2 for all 0 = a - b, i.e., such a vector must have the form (a, a, a), a 6 = 0. d) Apply part b) to u = v. Then it follows from part a) that $\infty X k = 0$ ak $rk \ge (1 - r) \infty X$ Sk $rk \ge (1 - r)M k = N \infty X rk = M rN$. b) Similarly, by Theorem 2.9ii f (xn)g(xn) $\rightarrow 0$ for all $xn \in I \setminus \{a\}$ which converge to a. Thus ∇f (a, b, c) and $\nabla g(a, b, c)$ are parallel, i.e., () i j k (0, 0, 0) = $\nabla f(a, b, c)$ are parallel, i.e., () i j k (0, 0, 0) = $\nabla f(a, b, c)$ are parallel, i.e., () i j k (0, 0, 0) = $\nabla f(a, b, c)$ are parallel, i.e., () i j k (0, 0, 0) = $\nabla f(a, b, c)$ are parallel, i.e., () i j k (0, 0, 0) = $\nabla f(a, b, c)$ are parallel, i.e., () i j k (0, 0, 0) = $\nabla f(a, b, c)$ are parallel, i.e., () i j k (0, 0, 0) = $\nabla f(a, b, c)$ are parallel, i.e., () i j k (0, 0, 0) = $\nabla f(a, b, c)$ are parallel, i.e., () i j k (0, 0, 0) = $\nabla f(a, b, c)$ are parallel, i.e., () i j k (0, 0, 0) = $\nabla f(a, b, c)$ are parallel, i.e., () i j k (0, 0, 0) = $\nabla f(a, b, c)$ are parallel, i.e., () i j k (0, 0, 0) = $\nabla f(a, b, c)$ are parallel, i.e., () i j k (0, 0, 0) = $\nabla f(a, b, c)$ are parallel, i.e., () i j k (0, 0, 0) = $\nabla f(a, b, c)$ are parallel, i.e., () i j k (0, 0, 0) = \nabla f(a, b, c) are parallel, i.e., () i j k (0, 0, 0) = $\nabla f(a, b, c)$ are parallel, i.e., () i j k (0, 0, 0) = \nabla f(a, b, c) are parallel, i.e., () i j k (0, 0, 0) = \nabla f(a, b, c) are parallel, i.e., () i j k (0, 0, 0) = \nabla f(a, b, c) are parallel, i.e., () i j k (0, 0, 0) = \nabla f(a, b, c) are parallel, i.e., () i j k (0, 0, 0) = \nabla f(a, b, c) are parallel, i.e., () i j k (0, 0, 0) = \nabla f(a, b, c) are parallel, i.e., () i j k (0, 0, 0) = \nabla f(a, b, c) are parallel, i.e., () i j k (0, 0, 0) = \nabla f(a, b, c) are parallel, i.e., () i j k (0, 0, 0) = \nabla f(a, b, c) are parallel, i.e., () i j k (0, 0, 0) = \nabla f(a, b, c) are parallel, i.e., () i j k (0, 0, 0) = \nabla f(a, b, c) are parallel, i.e., () i j k (0, 0, 0) = \nabla f(a, b, c) are parallel, i.e., () i j k (0, 0, 0) = \nabla f(a, b, c) are parallel, () i j k (0, 0, 0) = \nabla f(a, b, c) are parallel, () i j k (0, 0, 0) = \nabla f(a, b, c b, c) × $\nabla q(a, b, c) = det | fx(a, b, c) |$ (-1/2, 1/2). Then n \geq N and x \in [a, b] imply $|x/n| \leq \max\{|a|, |b|\}/N < 2$. The trace looks like a sine wave traced vertically on the plane y = x. Therefore, Bn -b converges to 0 as n $\rightarrow \infty$. To show that q is continuous on X, let $^2 > 0$ and choose $\delta > 0$ such that x, $y \in D$ and $\rho(x, y) < \delta$ imply $\tau(f(x), f(y)) 0$, $|x \cos y/3| - x + y| \leq 1/3| - x + y| < 1/3| - x$ and $Z 1 \sqrt{3} 0^{-1} 3 dx 3 = -(1 - x)2/3^{-0} = < \infty$, $xn = x^{-1} x^{-1} + x^{-1}$ $\sup |b| = \lim \inf |b| k k \rightarrow \infty k| = r$. Finally, if H is a compact subset of $\varphi(V)$ then $\varphi(T)$ is a compact subset of V. \sqrt{b} If these surfaces intersect, then $z = 1 - z^2$, i.e., $z = (-1 \pm 5)/2$. Since E is sequentially compact, choose xnk $\rightarrow b$ as $k \rightarrow \infty$. b) If x = 0, then the norm of x/kxk is 1, so $^{\circ}\mu \P^{\circ}kT$ (x) $k \circ x \circ ^{\circ} \leq M1$. We obtain k=1 as bk = k=1 sk (bk bk-1). In particular, $y \in f(E)$ as required. e) Since a > 0 implies $\log a x \ge 1$ for $x \ge 0$, $f(x) \ge xp$ for all $x \in [1, \infty)$. Hence by Theorem 8.32, $Ao \subseteq Bo \cdot \sqrt{d}$ By the Archimedean Principle, given $\varepsilon > 0$ there is an $N \in N$ such that $N > 1/3\varepsilon$. 5.3.12. Let U := Ux0 and $V := \bigcup \{Uy : Uy \cap Ux0 = \emptyset\}$. Let $\delta > 0$ be so small that $\delta/^2 0 < r2$ and suppose $|f(x0)| \le \delta$. Thus ZZ Z 2 π Z 1 π F · n d σ = - (u, v, u2 + v 2) · (-2u, -2v, 1) d(u, v) = r3 dr d\theta = . Suppose that neither x nor y is zero. Since $\varphi(0) = x0$ and $\varphi(1) = x0 + a$, C contains x0 and x0 + a. Notice that Df (a)(y) \in R3 so this cross- product makes sense under the identification of 3 × 1 matrices with vectors in R3. Thus Py = fy and we may set f = 0, Qx = 1. Then $(f \lor g)(x) = g(x)$ and |f(x) - g(x)| = g(x) - f(x) so $f(x) + g(x) + |f(x) + g(x)| 2g(x) = g(x) = (f \lor g)(x)$. 36 Copyright © 2010 Pearson Education, Inc. By Taylor's Formula, x2 y2 + fxy (a, b)x + fy (a, b)x + fy (a, b)x + fy (a, b)x + fx (a, b)x + fy (a, b)x + fx (a, some $(c, d) \in L((a, b); (a + x, b + y))$. Chapter 14 14.1 Introduction. $\sqrt{Taking the \sqrt{supremum over x}} \in [xk-1, \sqrt{xk}]$ and then the infimum over $y \in [xk-1, \sqrt{xk}]$ and then the infimum over $y \in [xk-1, \sqrt{xk}]$ and then the infimum over $y \in [xk-1, \sqrt{xk}]$ and then the infimum over $y \in [xk-1, \sqrt{xk}]$ and then the infimum over $y \in [xk-1, \sqrt{xk}]$ and then the infimum over $y \in [xk-1, \sqrt{xk}]$ and then the infimum over $y \in [xk-1, \sqrt{xk}]$ and then the infimum over $y \in [xk-1, \sqrt{xk}]$ and then the infimum over $y \in [xk-1, \sqrt{xk}]$ and then the infimum over $y \in [xk-1, \sqrt{xk}]$ and then the infimum over $y \in [xk-1, \sqrt{xk}]$ and then the infimum over $y \in [xk-1, \sqrt{xk}]$ and then the infimum over $y \in [xk-1, \sqrt{xk}]$
and then the infimum over $y \in [xk-1, \sqrt{xk}]$ and then the infimum over $y \in [xk$ any compact set. If $x \in /[-M, M]$ then $|fk(x) - f(x)| \le |fk(x)| + |f(x)| \le 2|f(x)| < 2$. Consequently, $p|hk| f(h, k) - f(0, 0) - Df(0, 0) \cdot (h, k) = \sqrt{2}$. Formula, k=1 ak bk = bn sn + k=1 sk (bk - bk - 1). Taking the supremum of this last inequality over all kxk = 1, we conclude that $M1 \le M2$. Pn ; ¢ Pn ; ¢ 1.4.2. a) 0 = (1 - 1)n = k = 0 nk (n-1)k = k = 0 nk (-1)k = 0bk1 = a + k1, bk1 + 1 = -a1, 10.5.11. Since $E = E \cup \partial E$, it follows from the sequential characterization of continuity and our construction that g is a continuous extension of f to E. By Theorem 3.6, f (qn) \rightarrow f (x0). A similar argument shows there is an x2 > x0 such that f 0 (x2) < 0. ∞ Now $|xn+1 - xn| \leq |xn+1 - xn| \leq |xn+1 - xn| \leq |xn+1 - xn| = -a1$. k=m+1 rk+1 = rm+2/(1 - r). e) By parts c) and d), Vol (E1 \cup E2) = Vol ((E1 \cap E2)) \cup (E1 \cap E2)) (E1 (\cap E2)) (E1 \cap E2)) (E1 (\cap E2)) (E1 \cap E2)) (E1 (\cap E the triangle inequality imply $\| e^{-1} e^{$ ak (f) cos kh - bk (f) sin kh. i.e., f -1 (x) = (x + |x - 2| - |x - 4|)/3. Since $\delta < 1$, $|x - 2| < \delta$ implies |x + 1| < 7. 10.5.9. Let E be a nonempty, proper subset of X. $3 - \pi/2$ 3 C1 $\sqrt{70}$ parameterize C2, set $\psi(t) = (\cos t, -\cos t, \sin t/3)$ and $J = [\pi/2, 3\pi/2]$. Thus by part c), Z 1 x dy dz + y dz dx + z dx dy 3 $\partial E Z \pi$; $\psi 2\pi = b(a + b \cos v) 2 \cos v + b2 ($ $\cos v$ sin 2 v dv 3 $-\pi \notin 2\pi i = 0 + 2ab2 \pi + 0 = 2\pi 2 ab2$. But E is closed, so the limit of the x i's, namely a, must belong to E, a contradiction. Hence I \cap J is empty or an interval, hence connected by definition or Theorem 10.56. 5.4 Improper Riemann Integration. We conclude by Theorem 7.12 that ∞X dj f dj (x) = (ak (f) cos kx + bk (f) co sin kx). By Exercise 2.5.6a and Theorem 2.36, lim sup (xn yn) \leq (lim sup xn) (lim sup yn) = x lim sup (xn yn) \leq (lim sup (xn yn) \leq (lim sup (xn yn)) \leq (lim sup (xn yn) $(a + b)n = n \mu$ $(a + b)n = n \mu$ $(a + b)n = n \mu$ $(a + b)n = xn0 x \rightarrow x0$ for all $n \in N$. Hence by Theorem 3.8, lim $xn = (lim x)n = xn0 x \rightarrow x0$ for all $n \in N$. Hence by Theorem 3.8, lim $xn = (lim x)n = xn0 x \rightarrow x0$ for all $n \in N$. Hence by Theorem 3.8, lim $xn = (lim x)n = xn0 x \rightarrow x0$ for all $n \in N$. Hence by Theorem 3.8, lim $xn = (lim x)n = xn0 x \rightarrow x0$ for all $n \in N$. Hence by Theorem 3.8, lim $xn = (lim x)n = xn0 x \rightarrow x0$ for all $n \in N$. Hence by Theorem 3.8, lim $xn = (lim x)n = xn0 x \rightarrow x0$ for all $n \in N$. Hence by Theorem 3.8, lim $xn = (lim x)n = xn0 x \rightarrow x0$ for all $n \in N$. Hence by Theorem 3.8, lim $xn = (lim x)n = xn0 x \rightarrow x0$ for all $n \in N$. Hence by Theorem 3.8, lim $xn = (lim x)n = xn0 x \rightarrow x0$ for all $n \in N$. Hence by Theorem 3.8, lim $xn = (lim x)n = xn0 x \rightarrow x0$ for all $n \in N$. Hence by Theorem 3.8, lim $xn = (lim x)n = xn0 x \rightarrow x0$ for all $n \in N$. Hence by Theorem 3.8, lim $xn = (lim x)n = xn0 x \rightarrow x0$ for all $n \in N$. 4, z = 4, oriented in the clockwise direction when viewed from high up the positive z axis, and the circle C2 described by $x^2 + y^2 = 1$, z = 1, oriented in the counterclockwise direction when viewed from high up the positive z axis, and the circle C2 described by $x^2 + y^2 = 1$, z = 1, oriented in the counterclockwise direction when viewed from high up the positive z axis, and the circle C2 described by $x^2 + y^2 = 1$, z = 1, oriented in the counterclockwise direction when viewed from high up the positive z axis. dx + Rb(U) = q(x) dx. c) $|x^3 - 3x + 1| < x^3$ if and only if $-x^3 < x^3 - 3x + 1 < x^3$ if and only if 3x - 1 > 0 and $2x^3 - 3x + 1 > 0$. Apply part a) to 2 := 2/2 - rt1 to choose a compact set $K \subset H = 0$ and $b \in A$. We conclude by the choice of P that p p U (f, P) - L(f, P) $\sqrt{U(f, P) - L(f, P)} \sqrt{U(f, P) - L(f, P)} \sqrt{(f, P) - L(f, P)} = (g 0 (f(a)) \cdot f 0 (a)) = (g 0 (f(a)) \cdot f 0 (a)) + (g 0 (f(a)) \cdot f 0 (a)) = (g 0 (f(a)) \cdot f 0 (a)) + (g 0 (f(a)) \cdot f 0 (a)) = (g$ then 1/g is continuous on E and bounded by 1/20. Hence $\sqrt{\frac{1}{2}}(1-1-4x2)/2x x 6=0 \text{ f}-1(x) = 0 x = 0$, $\sqrt{2} 2 2 1/2 13.6.5$. Let F = (P, Q, R) and $\varphi(u, v) = (u, v, 0)$. b) The trivial parameterization of z = y 3, x2 + y 2 ≤ 3, has normal (0, -3y 2, 1), whose induced orientation on C is counterclockwise (the wrong way). Moreover, g is continuous by the Sequential Characterization of Limits. Thus ac \leq bc holds for all a \leq b and c \geq 0. Since f is 1-1, a = f -1 ({y}). Hence by definition, f (x) = P ∞ by(k)Theorem k (0)x /k! for x \in R. If k > N := max{N1 , N2 }, then k log(k + 1) - log k, 1/2k) - (0, 0)k2 = log2 ((k + 1)/k) + 1/22k < c2 /4 + 1/k 2 < c2 . Hence by Stokes's Theorem and Exercise 5.1.4, Z ZZ F T ds = curl F · n d\sigma = 0 ∂S S implies F · T = 0 everywhere on ∂S . Therefore, 1 $\partial 2$ u 1 ∂u $\partial 2$ u + + 2 = fxx + fyy = 0. a c) By part b) and Exercise 5.1.4, f 2 (x) = 0 for all $x \in [a, b]$, thus f (x) = 0 for all $x \in [a, b]$, thus f (x) = 0 for all $x \in [a, b]$. Thus either a > b, b > a, or a = b. Hence these quantities are equal. Since $-n \in N$, we have by algebra and part a that xn - xn0 x - n - x - n n $= 0 \cdot x x 0 \rightarrow nx - n - 1 \cdot x 2n = nxn - 1 \cdot x 2n = nxn - 1$. Since 1/k lim sup |a\alpha = lim sup |a\alpha | \alpha / k = a0 k | k \righta \infty + a0 k | k \rin \n \righta \infty + a0 k | k \righta then there is nothing to prove. Let x, $y \in [0, \infty)$ and suppose $|x - y| < \delta$. 3.3.6. Let f(x) = 1 if $x \in Q$, f(x) = 0 if $x \in P$. Using the there is nothing to prove. Let x, $y \in [0, \infty)$ and suppose $C \subseteq E$ contains all its limit points which stay in E. a) Let $x \in R$. Using the that the sequence x2 f (k/(k 2 + x)) is decreasing for each $x \in [0, 1]$ for $k \ge k0$. Since $\mu p^{-}ak+1 = 1 = 1 + 1 \rightarrow ak^{-}|p| k|p| P^{\infty}$ as $k \rightarrow \infty$, k=1 k p /
pk converges absolutely when |p| < 1. Chapter 11: Differentiability on Rn 11.1 11.2 11.3 11.4 11.5 11.6 11.7 Partial Derivatives and Partial Integrals... 99 The Definition of Differentiability. ..102 Derivatives, Differentials, and Tangent Planes.....104 The Chain Rule.....107 The Mean Value Theorem and Taylor's Formula...... ...108 The Inverse Function Theorem..... ...111 Optimization.117 Riemann Integration on Jordan Regions......119 Iterated Integrals.... .122 Change of Variables ...130 The Gamma Function and Volume..... ..131 Chapter 13: Fundamental Theorems of Vector Calculus 13.1 13.2 13.3 13.4 13.5 13.6 Curves..... ..135 Oriented ..140 Oriented Surfaces.. ..143 Theorems of Green and Gauss..... .150 Chapter 14: Fourier Series 14.1 14.2 14.3 14.4 Curves. ..137 Surfaces. ..147 Stokes's Theorem. ...157 Growth of Fourier Coefficients..... 14.5 Introduction. .156 Summability of Fourier Series.... .159 Convergence of Fourier Series...... .163 Copyright © 2010 Pearson Education, Inc. Let f(x) = kxk. Without loss of generality, we suppose the former. E $o = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$, $e = \{(x, y) : y > x^2, 0 < y < 1\}$ $-1 + 2x2 \sqrt{p} \sqrt{y} - 22 = -2 \sqrt{-11} + 2/x2 + 1x2 + 2 + x2$ as $x \to \infty$. By the quadratic formula, y = f(x) implies $x = (1 \pm 1 - 4y 2)/2y$. 1 (3 - x - y) dy dx = 2 - 1 p p b) The curves x = y/2 and y = x2/4 at (2, 1), x = y/2 and y = 3 - x at (1, 2). Solving for x, we obtain x = xn - 1 - f(xn - 1)/f 0(xn - 1) as promised. $P \propto 2 d$) True. $N \rightarrow \infty 2N \equiv 0 N = 0 N \Rightarrow \infty 2N \equiv 0 N \equiv 0 N$ + f g 0 h + f gh0. Since f is increasing, f (x) \leq f (sup E) for every x \in E. Since K is compact and is covered by {Br(x) (x)}x \in K, there exist x1, . Hence by Dini's Theorem the series converges uniformly on [a, b] and can be integrated term by term. By Exercise 11.2.8, Dg(t) = u for all t and by definition, h0 (0) = lim t \rightarrow 0 f (a + tu) - f (a) = Du f (a). Hence $|ak| p \le |ak|$ for k large and it follows from the P0∞as k → p Comparison P∞ Test that k=1 |ak| converges. A similar proof, using part a) in place of Corollary 10.59, shows that f -1 (A) \cap E is relatively closed in E for all relatively extreme point of H. On the other hand, f is not continuous because along the path x = 0 the limit is zero, but along the path $y = x^2$ the limit is 1/2. Then f is uniformly continuous on $(0, \infty)$ and g is positive and bounded, but f (x)g(x) is not continuous at x = 2 so cannot be uniformly continuous. b) Set Bn = $(b1 + \cdots + bn)/n$. Thus f $(x) = 0 + 0 + \cdots + 0 = 0$ 0. Therefore, lim inf $(xn + yn) \le \lim \sup xn + \lim \inf yn$. e) By parts c) and d), gn (x) := f(x) + x/n is increasing on (a, b). By the choice of r0, xk belongs to $E \cap Br(a)$ and is distinct from the xj 's, $1 \le j < k$. By algebra and telescoping $\infty X(ak - ak + 2) = k = 1 \infty X(ak - ak + 2) = k = 1 \infty X(ak - ak + 2) = (a1 - a) + (a2 - a)$. In particular, $\sqrt{x} - \log x$ $-0.6 \ge f(4) = 2 - \log 4 > 0$. Thus $x \in /\partial U$, a contradiction. Thus y = f(b) for any $b \in B$. Hence it follows from the P ∞ Monotone Convergence Theorem and hypothesis that xn convergence Theor $n \rightarrow \infty$. 11.1.11. Since $f(x) := \sin x - 2x/\pi$ has no local minima in $[0, \pi/4]$ and $f(0) = 0 < f(\pi/4)$, we also have $\sin x \ge 2x/\pi$ for $x \in [0, \pi/4]$. Hence dist $(A, B) = \rho(x_0, y_0) > 0$. Thus there is a $\delta > 0$ such that $|x - y| < \delta$ and $x, y \in [0, \pi/4]$. Hence dist $(A, B) = \rho(x_0, y_0) > 0$. Thus there is a $\delta > 0$ such that $|x - y| < \delta$ and $x, y \in [0, \pi/4]$.

such that $p \propto \infty X X q(-1)k(-1)k = cos(1) = +$. Since Mj $(f + g) \le Mj (f) + Mj (g)$, we have Z b $(U) (f(x) + q(x)) dx \le U (f, P) + U (g, P) \le U (g, P) = U (g, P) + U (g, P) = U (g, P)$ $< \min\{ky - ak - s, r - ky - ak\}$. If $xn \to -\infty$ as $n \to \infty$, then choose $N \in N$ such that $n \ge N$ implies $xn < -1/\epsilon$. 1.3.1. a) Since $x^2 + 2x - 3 = 0$ implies x = 1, -3, inf E = -3, sup $E = \sqrt{1.10.1.10}$. $\sqrt{\sqrt{2.2.8}}$. a) Take the limit of xn+1 = 1 - 1 - xn, as $n \to \infty$. In particular, we conclude that $f(x)/g(x) \to \infty = B$ as $x \to a$ through I. Since $0 \le x | \sin(1/x)| \le x$, it follows from the Squeeze Theorem that $f(x) \rightarrow 0 =: f(0)$ as $x \rightarrow 0+$. a) If $\psi(t) = ta + b$, then $\psi(0, t) = a$ and (t) = kak(t - t0) for all t. $\partial u \partial v \partial u \partial v \partial$ (0) + sL(f)(s). Now f (xkj \in BECO (a) so xkj \in f -1 (BECO (a)) \cap B. Since each V α contains at most one x β , it follows that E is countable, a contradiction. $\beta n - \alpha n \beta n - \alpha n \beta n - \alpha n \beta - \alpha$ Moreover, since φ is nonzero, its reciprocal is continuous on [a, b], hence integrable there. Thus the trace of $\varphi(t)$ lies in the fourth quadrant and is asymptotic to the line y = -x as $t \rightarrow -1-$. Since 9! = 362, 880 and 11! = 39, 916, 800, it follows that $n \ge 5$. b) False. S ∂ S 13.6.3. a) Since div $F = x^2 + y^2 + z^2$, we have by Gauss' Theorem that ZZ ZZZ Z $2\pi (x^2 + y^2 + z^2) dV = F \cdot n d\sigma = S B1 (0,0,0) 0 Z 0 \pi Z 1 \rho 4 \sin \phi d\rho d\phi = 4\pi/5$. Suppose E is not connected. 6.5.2. a) p > 1 (see Exercise 6.2.4). Thus set a := xN . $3\sqrt{R \pi/2 R1 c}$ Using the substitution $u = \sin x$, $du = \cos x dx$, we have $0 \cos x/3 \sin x dx = 0 u - 1/3 du = 3/2$. By looking at the graph, we see that $f(E) = (1, \infty)$. 60 Copyright © 2010 Pearson Education, Inc. Thus $xP \in E$ if and only if x has a ternary expansion whose digits never 1. 3.3.0. a) True. Then xn-1 = 2xn-1 1 + xn-1 2 < = xn < = 1. Since 1/2n \approx 0.00049 for n = 10 and \approx 0.00049 for n = 11. Therefore, 0 (1 - cos x)/x2 dx is a Riemann integral. Hence 1 - a is real and by (6), 1 - a < 1 - a. Hence by Theorem 14.29, Sf converges to f almost everywhere on $[-\pi, \pi]$. Hence, Vol ($\varphi(Qj) \ge C^2 |\Delta \varphi(x)| < |\Delta \varphi(x)| + \eta |Qj|$ for j large. 3 2 b) Since cos x = (1 + cos 2x)/2, we have by orthogonality that a0 (cos 2 x) = 1/2, and all other Fourier coefficients of cos 2 x are zero. \sqrt{b}) Let $\varphi(t) = (-1, \cos t, \sin t/2)$ and $I = (-1, \cos t, \sin t/2)$. $[0, 2\pi]$. e) Let L = 0 and suppose that $\varepsilon > 0$. d) Since the minimum of $x^2 + x - 1$ on (-1, 0) is -1.25, -1 < x < 0 implies $|x^3 - 2x + 1| = |x^2 + x - 1| |x - 1| < 5|x - 1|/4$. Similarly, $x^0 \in V$ also leads to a contradiction. Since N X N $|R_j| = j=1^2 X |x_j - x_j - 1| = 2$, $b - a_j = 1$ we conclude by Theorem 12.4 that Vol (G(f)) = 0. If {Va} } is an open covering of A \cup B, then it is a covering of A and B. If S is oriented, we can repeat the entire process making sure that not only are the (φx , Ex) smooth, but also "orientable." 13.5 Theorems of Green and Gauss. If A is at most countable, then by Lemma 1.40 there is a function f which takes N onto A., n + 1}, then φ takes {1, 2, . 11.1.10. Thus f (xn) \rightarrow 0 as n $\rightarrow \infty$, and f (a) := 0 continuously extends f from (a, b] to [a, b]. 2 2 2 (2k - 1) k (2k) 6 24 8 - 6.3.10., VM covers A \cup B. f (a + h) f (a) f (a) f (a + h) f (b) f (a) f (a + h) f (a + h) f (a) f (a + h) f (a + $f(x_0)^- \leq \delta < r2$. $\sqrt{x^2 3.3.1.a}$ By Theorem 3.24, e and sin x are continuous on R. On the other hand, if $x \in A$, then $\varepsilon \in B$, so $\varepsilon x \leq \sup B$, i.e., sup B/ ε is an upper bound for A. 121 Copyright © 2010 Pearson Education, Inc. 8.3.8.a) Given $x \in V$ choose 2 := 2x > 0 such that $B^2(x) \subseteq V$. P ∞ Since k1 is least, sk1 $-1 \leq y$, hence sk1 $\leq y + bk1$ In particular, E is a Jordan region of volume zero. 3 13.4.4. By Theorem 13.36, $N\psi = \Delta \tau N\varphi \circ \tau$. By the Trichotomy Property, either x > 0, -x > 0, or x = 0. If $xn-1 \ge 1$ then $1 = 2 1 + xn-1 2xn-1 \le xn \le x = xn \le 1$. It follows from Definition 9.1i), given any r > 0, there is an N such that $k \ge N$ implies kxk - ak < r. If y = 3x-7 then x = (y + 7)/3. P Take the limit this identity as $n \rightarrow \infty$, Pof $\infty \infty$ bearing in mind that sn is bounded and bn $\rightarrow 0$ as $n \rightarrow \infty$. If k=1 ak converges then sn $\rightarrow s$ for some $s \in \mathbb{R}$. By definition, $\frac{1}{2} 2 u/v v 6 = 0$ f (tu, tv) $- f(0, 0) u 2 v D(u,v) f(0, 0) = \lim = \lim 4 2 = t \rightarrow 0 t \rightarrow 0 u t + v 2 t 0 v = 0$. Thus z 1 Z 2 x 2 f (x + 1) dx = 0 (u 2 f (u) - 2u f (u) + f (u)) du = 9 - 2 \cdot 6 + 5 = 2. Thus set V = B6 (a) and M = max{kf (a)k, kLk + 1}. 14.4.4. a) Fix N \in N and r \in (0, 1). (4m - 1)! (4m + 1)! 32 Copyright © 2010 Pearson Education, Inc. Then Å (n+1) (fg) ! 0 n μ ¶ X n (k) (n-k+1) = fg + fg k k=0 n μ ¶ n μ ¶ X n (k) (n-k) = ((fg)) = fg k k=0 n μ ¶ n μ ¶ X n (k+1) (n-k) X n (k) (n-k+1) = fg + fg k k=0 n μ ¶ n μ ¶ X n (k) (n-k) = ((fg)) = fg k k=0 n μ ¶ n μ ¶ X n (k) (n-k) = ((fg)) = fg k k=0 n μ ¶ n μ ¶ X n (k) (n-k) = ((fg)) = fg k k=0 n μ ¶ n μ ¶ X n (k) (n-k) = ((fg)) = fg k k=0 n μ ¶ n μ ¶ X n (k) (n-k) = ((fg)) = fg k k=0 n μ ¶ n μ ¶ X n (k) (n-k) = ((fg)) = fg k k=0 n μ ¶ n μ ¶ X n (k) (n-k) = ((fg)) = fg k k=0 n μ ¶ n μ ¶ X n (k) (n-k) = ((fg)) = fg k k=0 n μ ¶ n μ ¶ X n (k) (n-k) = ((fg)) = fg k k=0 n μ ¶ n μ ¶ X n (k) (n-k) = ((fg)) = fg k k=0 n μ ¶ n μ ¶ X n (k) (n-k) = ((fg)) = fg k k=0 n μ ¶ n μ ¶ X n (k) (n-k) = ((fg)) = fg k k=0 n μ ¶ n μ ¶ X n (k) (n-k) = ((fg)) = fg k k=0 n μ ¶ n μ ¶ X n (k) (n-k) = ((fg)) = fg k k=0 n μ ¶ n μ ¶ X n (k) (n-k) = ((fg)) = (fg) k k=0 n \mu ¶ n μ ¶ X n (k) (n-k) = ((fg)) = (fg) k k=0 n \mu ¶ n μ ¶ N n (k) (n-k) = ((fg) h (n-k) + (fg) k k=0 n \mu ¶ n μ ¶ n μ ¶ N n (k) (n-k) = ((fg) h (n-k) + ((fg) h ($n+1 X \mu n + 1\P = f(k) g(n+1-k) k(n) 0 k=0$ by Lemma 1.25. 9.6 Applications. Hence by part b), there is a point (x2, t2) $\in K$ where the absolute minimum of w occurs. Then L(x) < L(y) and $\alpha L(x) < \alpha L(y)$. 9.6.6. Since g is continuous, any point of discontinuity of f is a point of discontinuity of f is a point (x2, t2) $\in K$ where the absolute minimum of w occurs. Then L(x) < L(y) and $\alpha L(x) < \alpha L(y)$. then $x = q - r \in Q$, a contradiction. b) Let $(U, g) \in A$ be a chart at x. Therefore, $\{xn\}$ is Cauchy, so converges to some $c \in R$. b) The closure is [0, 1], the interior is E, the boundary is $\{1/n : n \in N\} \cup \{0\}$. Therefore, (φ, I) is a smooth curve. These efforts include the development, research, and testing of the theories and programs to determine their effectiveness. Hence, the second case cannot hold. Since P 2 n-1 |ec xn /n!| $\leq 3/n!$, it follows that |ex - k=0 x2k /k!| $\leq 3/n!$ for all x $\in [-1, 1]$. Note f (-1, 0) = -1. Set f (x, y) = $\varphi 3$ (g(x, y)). $\sqrt{6.6.5}$. If p > 1 is infinite, let q = 2. Conversely, if E 0 6= \emptyset then since E 0 is open it must contain a ball, hence a rectangle R. By part a), $\partial E \subseteq E$. 5.3.7. a) Since 1/t is continuous on $(0, \infty)$, it follows from the Fundamental Theorem of Calculus that L(x) is differentiable at each point $x \in (0, \infty)$ with L(x) is differentiable at each point $x \in (0, \infty)$ with L(x) is differentiable at each point $x \in (0, \infty)$ with L(x) is differentiable at each point $x \in (0, \infty)$ with L(x) is differentiable at each point $x \in (0, \infty)$ with L(x) is differentiable at each point $x \in (0, \infty)$ with L(x) is differentiable at each point $x \in (0, \infty)$ with L(x) is differentiable at each point $x \in (0, \infty)$ with L(x) is differentiable at each point $x \in (0, \infty)$ with L(x) is differentiable at each point $x \in (0, \infty)$ with L(x) is differentiable at each point $x \in (0, \infty)$ with L(x) is differentiable at each point $x \in (0, \infty)$ with L(x) is differentiable at each point $x \in (0, \infty)$ with L(x) is differentiable at each point $x \in (0, \infty)$ with L(x) is differentiable at each point $x \in (0, \infty)$. Let $x \in f - 1$ (I). b) $\Omega f(t - h, t + h) = 0$ for t = 0 when h is small, and = 1 when t = 0. This verifies the claim. Hence, kxk $\leq M$ for all $x \in K := E$. 6.3.8. a) The middle inequality is obvious since the infimum of a set is always less than or equal to its supremum. Since $|x||x^2 - y^2||f(x, y)| = \leq |x|, x^2 + y^2$ it follows from the Squeeze Theorem that f is continuous at (0, 0) with f (0, 0) = 0. k log (k + 1) k log k p k=2 k=2 But by the Integral Test, this last series converges when p > 1. Since any cluster point of A is a cluster point. However, bk does not converge as $k \rightarrow \infty$. Then f (n+1)(x) = PN k=n+2 (-kak/x f k+1 (n+1) 2) e-1/x + PN 1 (0) := lim x $\rightarrow 0$ x k+3 -1/x2) e 3.4.4. a) Given $^2 > 0$ choose N so large that $x \ge N$ implies $|f(x) - L| < ^2/3$. b) If C is relatively closed in E then $C = E \cap A$ for some closed set A. Vol (E) + 1, it follows that I1 is small, hence f is integrable. 13.1.8. By hypothesis, there exist closed,
nonoverlapping intervals J1, . Hence sin x/xp is improperly integrable on $[1, \infty)$ for all p > 0. b) Differentiating term by term, Bn0 $(x) = \infty X (-1)k (n + 2k)^3 x (n+2k-1)k = 0$ and Bn00 $(x) = 2k!(n + k)! 2 \infty X (-1)k (n + 2k)(n + 2k - 1)^3 x (n+2k-2)k = 0$ there is a $\delta > 0$ such that $\rho(x, y) < \delta$ and $x, y \in E$ imply τ (f (x), f (y)) < ². G G Rj $\cap E6 = \emptyset$ Rj volume zero, then by definition there is a finite collection of rectangles $\{R_j : j = 1, r_1 \leq s_1 < r_2 < r_2 < t \rightarrow \infty$ b) Fix h > 0 and let y = ah - 1. Conversely, if X is not connected then there exist nonempty open sets U and V such that $U \cap V = \emptyset$ and $X = U \cup V$. Hence 0 = -xy dx converges uniformly on [a, b]. b) If E is closed in Y then Y \ E is open in Y, so by part a), Y \ E = U \cap Y for some U open in X. c) By part b) and Theorem 11.8, L{f} exists and is continuous on (a, ∞). Applying this inequality to x = 1/(N \ (1/(2N)) \le 1, a contradiction. Similarly, a - = 0 if a < 0. a) Let C1 be the "outside curve. 0 b) Let H be the solid hyperboloid whose boundary is S, let A be the upper semidisk {(x, z) : $z \ge 0$, $x_1 + z_2 \le 1$ }, and B be the upper semiannulus {(x, z) : $z \ge 0$, $x_1 + z_2 \le 2$ }. Therefore, x = 0, 1 $\le x_1 + z_2 \le 2$. Therefore, x = 0, 1 $\le x_1 + z_2 \le 2$. substitution $d\eta = \omega$ we have $Z Z Z Z \eta \omega = d(\eta \omega) = d\eta \cdot \omega + (-1)r \eta d\omega = d\eta \cdot \omega = \omega^2$. Fix h with norm so small that $f(a + h) \times g(a) - T(h) = f(a + h) \times g(a) - T(h)$ $(h) + (f(a + h) - f(a)) \times Dg(a)(h)$ (f $(a + h) - f(a) - Df(a)(h)) \times g(a) =: I1 + I2 + I3$. In particular, y k2 is least such that sr2 < x, then x + inf b' $\leq \geq r1$ sup s' < x. By the Extreme Value Theorem, f has an absolute minimum on [-N, N], say f (xm) = m. Since f is a finite sum of C ∞ functions, it is C ∞ on R. 13.2.1. a) Let (x, y, z) = $\phi(t)$. If $\lambda = 0$, then $y\sqrt{=-2x}$ and constraint implies $x = \pm 1/5$. Hence, by our opening observation, f(2k-1)(0) = 0 for all $k \in N$. P9 b) Notice that 3/n! < 10-5 holds when $n \ge 9$. $xjnn \rightarrow f1j1(a)$. Hence by the Sequential Characterization of Limits, $h(x) \rightarrow L$ as $x \rightarrow a$. Since f is bounded and $\{fn\}$ is uniformly bounded, there is an M > 0 such that $max\{|fn(x) - f(x)| : x \in [a, b], n \in N\} \le M$. Therefore, $\lim supn \rightarrow \infty xn = 0$ by Theorem 2.36. There is an M > 0 such that $max\{|fn(x) - f(x)| : x \in [a, b], n \in N\} \le M$. set of points on the circle $(x - 1)^2 + y^2 = 1$ or on the x axis between x = 2 and x = 3. Since $sin(1/yk^2) = (-1)k$, we have $Var(\phi) \ge n 2X - 1 |\phi(xk) - \phi(xk-1)| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x^2k - 1 |x^2k + x^2k - 1 |x^2k + x^2k - 1| = k = 1 n 2X - 1 n |x^2k + x$ it should; f - 1 [-1, 1] = [0, 1] is relatively closed in [0, ∞) as Exercise 10.6.4 says it should. 0 b) If $\varphi(t) = (a \cos t, b \sin t)$ and I = [0, $\pi/2$], then $k\varphi 0$ (t)k = k(-a sin t, b cos t)k = p a2 + (b2 - a2) cos2 t, 135 Copyright © 2010 Pearson Education, Inc. It follows that f (x) $\in \{f(x1), \ldots, f(x)\}$ (b) If $\varphi(t) = (a \cos t, b \sin t)$ and I = [0, $\pi/2$], then $k\varphi 0$ (t)k = k(-a sin t, b cos t)k = p a2 + (b2 - a2) cos2 t, 135 Copyright © 2010 Pearson Education, Inc. It follows that f (x) $\in \{f(x1), \ldots, f(x)\}$ (b) If $\varphi(t) = (a \cos t, b \sin t)$ and I = [0, $\pi/2$], then $k\varphi 0$ (t)k = k(-a sin t, b cos t)k = p a2 + (b2 - a2) cos2 t, 135 Copyright © 2010 Pearson Education, Inc. It follows that f (x) $\in \{f(x1), \ldots, f(x)\}$ (b) If $\varphi(t) = (a \cos t, b \sin t)$ and I = [0, $\pi/2$], then $k\varphi 0$ (t)k = k(-a sin t, b cos t)k = p a2 + (b2 - a2) cos2 t, 135 Copyright © 2010 Pearson Education, Inc. It follows that f (x) $\in \{f(x1), \ldots, f(x)\}$ (b) If $\varphi(t) = (a \cos t, b \sin t)$ and I = [0, $\pi/2$], then $k\varphi 0$ (t)k = k(-a sin t, b cos t)k = p a2 + (b2 - a2) cos2 t, 135 Copyright © 2010 Pearson Education, Inc. It follows that f (x) $\in \{f(x), \ldots, f(x)\}$ (b) If $\varphi(t) = (a \cos t, b \sin t)$ (c) If bc by the Multiplicative Property. 9.1.3. a) By the Cauchy-Schwarz inequality and the Squeeze Theorem, kxk · yk k \leq kxk k kyk k \leq 0 as k $\rightarrow \infty$. Thus $\omega f(t) = 0$ if t 6 = 0 and $\omega f(0) = 1$. Choose k \in N such that $k \leq \beta < k + 1$ and let $x \in (0, 1)$. A similar argument proves that the left derivative of f at x is even. Since $f/(-2) = \sqrt{-8}$, it follows that $\sqrt{-8}$ f(E) = [-8, 1]. Then there are numbers x1 < x2 < x3 in [a, b] such that $f(x_0) < f(x_1) = 2$ is a local minimum if k and j are even, $f((2k + 1)\pi/2, j\pi) = -2$ is a local minimum if k and j are odd, and $((2k + 1)\pi/2, j\pi) = 2$ is a local minimum if k and j are odd, and $((2k + 1)\pi/2, j\pi) = -2$ is a local minimum if k and j are even, $f((2k + 1)\pi/2, j\pi) = -2$ is a local minimum if k and j are even, $f((2k + 1)\pi/2, j\pi) = -2$ is a local minimum if k and j are odd, and $((2k + 1)\pi/2, j\pi) = -2$ is a local minimum if k and j are even, $f((2k + 1)\pi/2, j\pi) = -2$ is a local minimum if k and j are even, $f((2k + 1)\pi/2, j\pi) = -2$ is a local minimum if k and j are even, $f((2k + 1)\pi/2, j\pi) = -2$ is a local minimum if k and j are even, $f((2k + 1)\pi/2, j\pi) = -2$ is a local minimum if k and j are even, $f((2k + 1)\pi/2, j\pi) = -2$ is a local minimum if k and j are even, $f((2k + 1)\pi/2, j\pi) = -2$ is a local minimum if k and j are even, $f((2k + 1)\pi/2, j\pi) = -2$ is a local minimum if k and j are even, $f((2k + 1)\pi/2, j\pi) = -2$ is a local minimum if k and j are even, $f((2k + 1)\pi/2, j\pi) = -2$ is a local minimum if k and j are even, $f((2k + 1)\pi/2, j\pi) = -2$ is a local minimum if k and j are even, $f((2k + 1)\pi/2, j\pi) = -2$ is a local minimum if k and j are even, $f((2k + 1)\pi/2, j\pi) = -2$ is a local minimum if k and j are even, $f((2k + 1)\pi/2, j\pi) = -2$ is a local minimum if k and j are even, $f((2k + 1)\pi/2, j\pi) = -2$ is a local minimum if k and j are even, $f((2k + 1)\pi/2, j\pi) = -2$ is a local minimum if k and j are even, $f((2k + 1)\pi/2, j\pi) = -2$ is a local minimum if k and j are even, $f((2k + 1)\pi/2, j\pi) = -2$ is a local minimum if k and j are even, $f((2k + 1)\pi/2, j\pi) = -2$ is a local minimum if k and j are even, $f((2k + 1)\pi/2, j\pi) = -2$ is a local minimum if k and j are even, $f((2k + 1)\pi/2, j\pi) = -2$ is a local minimum if k and j are even, $f((2k + 1)\pi/2, j\pi) = -2$ is a local minimum if k and j are even, $f((2k + 1)\pi/2, j\pi) = -2$ is a local minimum if k and j are even, $f((2k +
1)\pi/2, j\pi) = -2$ is a local minimum if k an is a finite real number. Since $E \cap (a - r, a + r)$ contains infinitely many points, so does $E \cap (a - r, a + r) \setminus \{a\}$. , $|xk - a|\}$, there is a continuous function g on [a, b] such that f(x) = g(x) for all $x \in I$. Then a < b and c < d but ac = -4 is not less than bd = -5, a2 = 4 is not less than b2 = 1, and 1/a = -1/2 is not less than 1/b = -1. c) Suppose $x \in \partial(A \cap B)$. Indeed, the set E := B1 (0, 0) \cup B1 (1, 0) is connected but not convex. 2 k k=1 By Theorem 14.29, SF converges to F uniformly on R, in particular, at $x = 0.2 - a a \sqrt{2}$ implies (x - 1)2 + y = 2 = 1. Clearly, it takes {1, 2, . Since sin 3 $\theta = -1/2$ is not less than 1/b = -1. c) Suppose $x \in \partial(A \cap B)$. Indeed, the set E := B1 (0, 0) \cup B1 (1, 0) is connected but not convex. 2 k k=1 By Theorem 14.29, SF converges to F uniformly on R, in particular, at $x = 0.2 - a a \sqrt{2}$ implies (x - 1)2 + y = 1. Clearly, it takes {1, 2, . Since sin 3 $\theta = -1/2$ is not less than 1/b = -1/2 is $\sin \theta(1 - \cos 2 \theta)$, it follows that $Z \pi/3 Z 1/2 \sin 3 \theta f(\cos \theta) d\theta = -0 (1 - u^2) f(u) du = 3 - 7 = -4$. dy d(x, z) = 3 E D 5 - 4x - 2z D Using the change of variables $x = 2 + r \cos \theta$, $z = 1 + r \sin \theta$, dx dy = r dr d θ , we conclude ZZ $2\pi Z \omega = 3 S 0 2 (4 - r^2) r dr d\theta = 24\pi$. In particular, if n > N, then $\sqrt{k} \sup k > n$ ak $\leq (aN r 0 - N) 1/k \cdot r 0$. 3.3.11. Since f 0 (x) $= -(x \log x) - 2$ (p logp $-1 x + \log x) \le 0.55$ Copyright @ 2010 Pearson Education, Inc. However, 0 < y < x is satisfied by the other pair because s > 2 t > 0. Therefore, lim supn $\rightarrow \infty xn = \infty$ by Theorem 2.36. 5.4.8. Let $b \in [1, \infty)$ and n > 1. Thus f (2, 2, 4) = 48 is the minimum. Taking the limit of these inequalities as $p \rightarrow 1 + we$ see that this inequality holds for p = 1 too. By induction, there exist infinitely many points in $E \cap Br$ (a). Hence by assumptions iii) and vi), $0 \le 1 - \cos x \le 1 - \cos$ $h_{0}(x) = 1/r$ for all t. b) If f: $M \to Rk$ is C p and G: $Rk \to R^{2}$ is C p, then so is (G \circ f) $\circ h-1 = G \circ (f \circ h-1)$ for every chart (h, V) of M. c) By the Outient and Chain Rules, g 0 (x) = f(x_{3}) \cdot 1 - x \cdot 3x_{2} f 0 (x_{3}) \cdot f 2 (x_{3}) \sqrt{so g 0 (32)} = (f(2) - 6f 0 (2))/f 2 (2) = (1 - 3\pi)/2. Suppose t > t0 is near to and $x0 = \psi(t0)$. Since $Q(x)/xm \rightarrow bm$, we conclude that $P(x)/Q(x) \rightarrow 0$ as $x \rightarrow \pm \infty$. x2 + y2 = r2 is such a curve for r sufficiently small. Fix $n \ge 2$. If x = 1 then f(y) = 1 + 3y - y3 has critical points of H. S E But on E, n = (0, 0, 1), and the third component of $\nabla \times F$ is $\partial \partial (y \cos z 3) - (x \sin z 3) = 0$. Thus by Green's Theorem and Theorem 11.2, Z ZZ F · T ds = (fx - fxy) dA = 0. Since g(U) is an open set containing h(x), we can choose a $\delta > 0$ such that $B\delta(h(x)) \subset g(U)$. By l'H^oopital's Rule, $x \log(1/x) \rightarrow 0$ as $x \rightarrow 0+$, so use Theorem 3.40. 14.3.1. Since $\sin(k + \alpha)x = \sin kx \cos \alpha x + \cos kx \sin \alpha x$, we have $Z \pi f(x) \sin(k + \alpha)x dx = \pi bk$ (f (x) $\cos \alpha x$) + πak (f $(x) \sin \alpha x$ $-\pi$ for all $k \in N$. $|bn| \le k=0$ n X k=0 |1/2 A 2 |ak| n X |1/2 2 |bk| k=0 for all $n \in N$. Therefore, n+1 n $\sqrt{x} = 2$, -1. By the Mean Value Theorem, there is a c between x1 and x2 such that f (x2) - f (x1) = f 0 (c)(x2 - x1). $-\pi/2$ 13.5.7. By Exercise 12.2.3 and Gauss' Theorem, ZZ ZZ 1 1 lim $F \cdot n \, d\sigma = \lim div F \, dV r \rightarrow 0$ Vol (Br (x0)) $r \rightarrow 0$ Vol $(Br(x0)) \partial Br(x0) = div F(x0) = div F(x0)$ and induction, $|xn+1 - xn| = |F(xn) - F(xn-1)| \le r|xn - xn-1| \le r$ for $k \ge N$., f(xN) for all $x \in X$. By (7), $0 \le a2 < b2$. b) Since $\lim y \to 0$ f(x, py) = 1/2 and $\lim x \to 0$ f(x, y) = 1, this function has no limit as (x, y) $\to (0, 0)$. Let $M := \sup\{|x|k : x \in [a, b], k = 0, 1, . d\}$ Since f(0, 1) = 26 = 0, and it follows from the Inverse Function Theorem that D(f - 1)(-1, 0) = (Df(0, 1)) - 1 = -1/2 - 1/2. 1. By the Extreme Value Theorem, $g(x) \ge 20 > 0$ for $x \in [a, b]$. (These xk 's have been chosen so that φ achieves its maximum variation.) Since sin(1/xk) = (-1)k, parts a) and b) imply that n n n X X 4 X 8k 2 + 2 8 $|\varphi(xk) - \varphi(xk-1)| = |x^2k + x^2k - 1| = 2 \le 2 0$ for all $x \in R$, ex is convex on R. x 5.3.9. Using the substitution y = f(x) and integrating by parts, we have Z Z f(b) f(a) - 1 (y) dy = a b b f(x) dx + R f(b) f(a) - 1 (y) dx + R f(b) f(a) - 1 (y αx and f (x) sin αx are integrable on $[-\pi, \pi]$, it follows from the Riemann-Lebesque Lemma that the integral converges absolutely and uniformly by the Weierstrass M-Test. It must also be bounded. P ∞ Finally, the derived series k=1 cos(x/(k + 1)) to the vertice of the converges absolutely and uniformly by the Weierstrass M-Test. It must also be bounded. P ∞ Finally, the derived series k=1 cos(x/(k + 1)) to the vertice of the vertice of the converges absolutely and uniformly by the Weierstrass M-Test. It must also be bounded. P ∞ Finally, the derived series k=1 cos(x/(k + 1)) to the vertice of the vertic 1)/(k(k + 1)) converges uniformly on R by the Weierstrass M-Test. Hence, by the point-normal form, $d \cdot (x - a) = 0$ is an equation of the plane through a, b, c. Then there is a kj+1 > r. By hypothesis, this means $L - \varepsilon < f(x) \le q(x) \le h(x) \le L + \varepsilon$. R1 $\sqrt{11.1.8}$. a) Since $|\cos(x^2 + y^2)| \le 1$ for any $y \in (-\infty, \infty)$ and $0 dx/x < \infty$, it follows from the Weierstrass-M R1 $\sqrt{\text{Test}}$ that 0 cos(x2 + y 2)/x dx converges uniformly on ($-\infty$, ∞). d) This is the set of points between the two branches of the hyperbola x2 - y 2 = 1, $-1 \le y \le 1$ and $\partial E = \{x_2 - y_2 = 1, -1 \le y \le 1\}$ and $\partial E = \{x_2 - y_2 = 1, -1 \le 1\}$ and $A = \{x_2 - y_2 = 1, -1 \le 1\}$ and $A = \{x_2 - y_2 = 1, -1 \le 1\}$ and $A = \{x_2 - y_2 = 1, -1 \le 1\}$ and $A = \{x_2 - y_2 = 1, -1 \le 1\}$ and $A = \{x_2 - y_2 = 1, -1 \le 1\}$ and $A = \{x_2 - y_2 = 1\}$ and A = $-yk2 - x \cdot x| = |-2x \cdot y + y \cdot y| = |(-2x + y) \cdot y| \le 2 \cdot 1 = 2$. Since the 1/bk 's are decreasing, we have by telescoping that $\infty - \mu \P \mu \P^T X^- 1 1^- 1 1^2 - ak^T \le 2 \sup |c| < . d$) Let $L = \infty$ and suppose
without loss of generality that M > 0. This contradicts the fact that $e - N y \rightarrow 1$ as $y \rightarrow 0$. Suppose f is integrable on [a, b], g is integrable on [c, d], h(x, y) := f(x)g(y), and $R := [a, b] \times [c, d]$. Let Q be the product $\sqrt{}$ of intervals [aj, bj], where bj - aj = s for all j. e) Use the trivial parameterization $\varphi(t) = (t, f(t), 0)$ and apply part d). The rest of these identities follow in a similar way from corresponding properties of real numbers. Therefore, Z Z Z bn Z 1 1 bn $f(x) dx - fn(x) dx \le |f(x) - fn(x)| dx + |f(x)| dx = 0 \le x \le bn + M (1 - bn) \le t = 2$. e) Given any partition P of [a, b], S(f; P) - s(f; P) is the area of a collection of rectangles which covers G(f). Finally, $2 \pi a0 (x^2) = Z \pi x^2 dx = 0 2\pi 2$. In 2 2 particular, $y = (s \pm s - 4t)/2$ and $x = (s \mp s - 4t)/2$. Hence, for 2 sufficiently small and j large, Vol $(\varphi(Q_j)) e^2 |\Delta \varphi(x)| > |\Delta \varphi(x)| > |\Delta \varphi(x)| > |\Delta \varphi(x)| > |\Delta \varphi(x)| = 0$ is an integer. A similar argument shows that sup $B \le \varepsilon$ sup A. Thus choose N0 such that $\varepsilon N0 < \varepsilon/2$. Thus $(\sigma \circ h) \circ g - 1 = \sigma \circ (h \circ g - 1)$ and $g \circ (\sigma \circ h) - 1 = (g \circ h - 1) \circ \sigma - 1$ are C p when $h \circ g - 1$ and $g \circ h - 1$. are, and the Jacobians $\Delta(\sigma \circ h) \circ g - 1 = \Delta \sigma \Delta h \circ g - 1$ and $\Delta g \circ (\sigma \circ h) - 1 = \Delta g \circ h - 1 \Delta \sigma - 1$ are positive when A is oriented. Using Lagrange's integral form of the remainder term for the one-dimensional Taylor's Formula, we obtain f (x) - f (a) = F (1) - F (0) = Z 1 p - 1 X 1 (j) 1 F (0) + (1 - t)p - 1 F (p) (t) dt. Case 3. 88 Copyright © 2010 Pearson Education, Inc. 11.5.6. a) By the Chain Rule, g 0 (t) = fx (tx + (1 - t)a, y)(x - a) + fy (a, ty + (1 - t)b)(y - b). Then f and g are uniformly continuous on (0, 1), but (f/g)(x) = 1/x is not uniformly continuous on (0, 1), but (f/g)(x) = 1/x is not uniformly continuous on (0, 1). Thus here, f - 1 (x) = arcsin(\pi - x). By Theorem 9.8, xk \rightarrow a and xk $\in C \subseteq B$ implies a $\in B$. In particular, f (x0) = 0, a contradiction. Thus E is at y = 1. Therefore, the original series diverges. n! (n + 1)! Since x > 0 implies xn+1 > 0 and ec > 1, it follows that ex > 1 + x + ··· + xn . But E $\subset \cup \alpha \in A V \alpha$, hence by Lindel of's Theorem, E $\subset \cup \alpha \in A V \alpha$, hence by Lindel of's Theorem, E $\subset \cup \alpha \in A V \alpha$, hence by Lindel of A. 14.2.4. a) If (Sf)(x0) converges to M, then by Remark 14.6, ($\sigma N f$)(x0) $\rightarrow M$ as N $\rightarrow \infty$, i.e., M = L. d) Since and ec > 1, it follows that ex > 1 + x + ··· + xn . But E $\subset \cup \alpha \in A V \alpha$, hence by Lindel of A. 14.2.4. a) If (Sf)(x0) converges to M, then by Remark 14.6, ($\sigma N f$)(x0) $\rightarrow M$ as N $\rightarrow \infty$, i.e., M = L. d) Since and ec > 1, it follows that ex > 1 + x + ··· + xn . But E $\subset \cup \alpha \in A V \alpha$, hence by Lindel of A. 14.2.4. a) If (Sf)(x0) converges to M, then by Remark 14.6, ($\sigma N f$)(x0) $\rightarrow M$ as N $\rightarrow \infty$, i.e., M = L. d) Since and ec > 1, it follows that ex > 1 + x + ··· + xn . But E $\subset \cup \alpha \in A V \alpha$, hence by Lindel of A. 14.2.4. a) If (Sf)(x0) converges to M, then by Remark 14.6, ($\sigma N f$)(x0) $\rightarrow M$ as N $\rightarrow \infty$, i.e., M = L. d) Since and ec > 1, it follows that ex > 1 + x + ··· + xn . But E $\subset \cup \alpha \in A V \alpha$, hence by Lindel of A. 14.2.4. a) If (Sf)(x0) converges to M = L. d) Since and ec > 1, it follows that ex > 1 + x + ··· + xn . But E $\subset \cup \alpha \in A V \alpha$ is the follows that ex > 1 + x + ··· + xn . But E $\subset \cup \alpha \in A V \alpha$ is the follows that ex > 1 + x + ··· + xn . But E $\subset \cup \alpha \in A V \alpha$ is the follows that ex > 1 + x + ··· + xn . But E $\subset \cup \alpha \in A V \alpha$ is the follows that ex > 1 + x + ··· + xn . But E $\subset \cup \alpha \in A V \alpha$ is the follows that ex > 1 + x + ··· + xn . But E $\subset \cup \alpha \in A V \alpha$ is the follows that ex > 1 + x + ··· + xn . But E $\subset \cup \alpha \in A V \alpha$ is the follows that ex > 1 + x + ··· + xn . But E $\subset \cup \alpha \in A V \alpha$ is the follows that ex > 1 + x + ··· + xn . But E $\subset \cup \alpha \in A V \alpha$ is the follows that ex = 1 + x + ··· + xn . But E $\subset \cup \alpha \in A V \alpha$ is the follows that ex = 1 + x + ··· + xn . But E $\subset \cup \alpha \in A V \alpha$ is the follows that ex = 1 + x + ··· + xn . But E $\subset \cup \alpha \in A V \alpha$ is the follows that ex = 1 + x + ··· + xn . But E $\subset \cup \alpha \in A V$ easy induction proves that 9n > n for all $n \in N$, we have 9-n < 1/n. 11.4.5. By the Chain Rule, $ur = fx \cos \theta + fy \sin \theta$, $vr = gx \cos^2 \theta + fy \sin^2 \theta$, $vr = gx \cos^2 \theta + fy \sin^2 \theta + fy \sin^2 \theta$, $vr = gx \cos^2 \theta + fy \sin^2 \theta + fy \sin$ the Heine-Borel Theorem. Since E is convex, $x 0 \in L(x; y) \subseteq E$. k=1 j=1 Taking the limit of this inequality as $N \rightarrow \infty$ we obtain the desired inequality as $N \rightarrow \infty$ we obtain the desired inequality as $N \rightarrow \infty$ we obtain the desired inequality. If f(c) > 0, then c - a > 0 and c - b < 0 imply that $f 0(x_1) > 0 > f 0(x_2)$. 2.2 Limit Theorems. $P \infty 6.1.0.$ a) False. Since 10n+1 y < 10, $E \subseteq \{0, 1, ..., E = 1, ..., 2 = f - 1$ (c), and apply the Inverse Function Theorem. We want to solve for u, v, w, so we must compute (4) 5u 2xv 1 $\partial(F1, F2, F3) = det (2uy 5v 4 1) = 4w3 (25u4 v 4 - 4uvxy)$. By parts, Z 1 0⁻¹ ex f (x) dx = -0 1 ex f (x) dx = ex f (x) dx = -0 1 ex f (x) dx = ex f (x) dx = -0 1 ex f (x) dx is always 1. Since B1/2 (a) contains xk for infinitely many k's, choose k2 > k1 such that xk2 \in B1/2 (a). 7 Copyright © 2010 Pearson Education, Inc. Let a $\in \partial E$ and let xn $\in E$ with xn $\rightarrow a$ as n $\rightarrow \infty$. 10.2.7. Modify the proofs of Theorem 3.24, replacing the absolute value signs with the metric ρ . n $\rightarrow\infty$ n $\rightarrow\infty$ 4.3.10. Hence by Theorem 11.22, a normal to the tangent plane at (a, b, c) is e = (Fx (a, b, c)/Fz (a, b, c), Fy (a, b, c)/Fz (a, b, c), 1). In particular, $f(q) = f(q \cdot 1) = fq(1)$ for all $q \in Q$. Hence, Mj (f - fN) and mj (f - fN) are both less than ε , and it follows that I1 $\leq 2 \varepsilon 2Vol(E) + 2X |Rj| \leq Rj \cap E6 = \emptyset \varepsilon V (E, G)$. By Corollary 5.23 and the Fundamental Theorem of Calculus, it suffices to prove that f - 1and f 1/m are integrable for all $m \in N$. Hence by Theorem 9.15 (the limit of the sum is the sum of the limits), P (x) \rightarrow P (a)
as $x \rightarrow a$. 14.2.7. a) Suppose P (x) \equiv an $xn + \cdots + a1 x + a0$ is a polynomial on R and $\varepsilon > 0$. It might be better to change the surface. Finally, if E is closed and bounded then (by the Heine-Borel Theorem) E is compact. Hence by the Archimedean Principle, there is an $n \in N$ such that 2n > 1/(b - a). If x > M, then $|f(x) - L| \le 1/x^2 < 1/M 2 = \epsilon$. 2k + 3 2 d It converges by the Ratio Test, since |ak+1| 2k + 1 = -0 |ak| (2k + 1)(2k + 2) ask $\rightarrow \infty$. Clearly, a0 (f) = $\sqrt{2} \sqrt{\sqrt{n} 2} \pi \cos 2x \, dx = 2 \sin 2\pi$. It's a little easier to resort to the $2-\delta$ definition of continuity.) Define g(x) := f(x) when x \in D and g(x) as above when x \in X \ D. Thus by the Heine-Borel Theorem, E is compact. Therefore, sn is Cauchy by hypothesis. Let $^2 > 0$ and use the Approximation Property to choose N \in N such that supk \geq N xk < s + 2 . 134 Copyright © 2010 Pearson Education, Inc. d) The absolute value of the ratio of successive terms of this series is given by $\sqrt{\sqrt{k}}$ k + 1|x + 2|/((k + 1)k + 2). By induction, then, there exist infinitely many points xnk in Br (a) \cap E. Therefore, |f(x) - f(x)| < 2/(2(b - a)), i.e., $\kappa(x0) = 0$ for all t, i.e., $\kappa(x0)$ the open upper half plane and the open lower half plane. Then (f v g)(x) = f (x) - g(x) = g (x) - g(x) = g (x) - g (x) - g (x) = g (x) follows that $a \in C$. 4.2.3 (x α) $0 = (e\alpha \log x) = \alpha \alpha \log x / x = \alpha \alpha - 1$ for all x > 0. Then by the Second Multiplicative Property, $4x^2 > 4x^2 - 1$, i.e., 0 > -1. 27 \sqrt{c} Because these surfaces intersect at z = 3/2, a parameterization of this "spanish olive half" is given by (ψ , B) where $\psi(u, v) = (3 \cos u \cos v, 3 \sin u \cos v, 3 \sin v) \sqrt{and B} = [0, 2\pi] \times [\pi/4, 3\pi)$ $\pi/2$]. a b f -1 (x) dx = xf (x) a = bf (b) - af (a). $\partial E E 13.4.6$. The tetrahedron T has three faces, T1 in x = 0, T2 in y = 0, and T3 in z = 0. Therefore, μ (n 3 x n $\rightarrow \infty$. The function f (x) = x for x $\in \mathbb{N}$ and f (x) = 1 for x $\in \mathbb{N}$ and f (x) = 1 for x $\in \mathbb{N}$ and f (x) = 1 for x $\in \mathbb{N}$ and f (x) = 1 for x $\in \mathbb{N}$ and f (x) = x for x $\in \mathbb{N}$ and f (x) = 1 for x \in \mathbb{N} and f (x) = 1 for x $\in \mathbb{N}$ and f (x) = 1 for x $\in \mathbb{N}$ and f (x) = 1 for x \in \mathbb{N} and f (x) = 1 for x $\in \mathbb{N}$ and f (x) = 1 for x \in \mathbb{N} and f (x) = 1 for x \in \mathbb{N} and f (x) = 1 for x $\in \mathbb{N}$ and f (x) = 1 for x \in \mathbb{N} and f (x) = 1 for x \in \mathbb{N} and f (x) = 1 for x \in \mathbb{N} and f (x) = 1 for x \in \mathbb{N} and f (x) = 1 for x \in \mathbb{N} and f (x) = 1 for x \in \mathbb{N} and f (x) = 1 for x \in \mathbb{N} and f (x) = 1 for x \in \mathbb{N} and f (x) = 1 for x \in \mathbb{N} and f (x) = 1 for x \in \mathbb{N} and f (x) = 1 for x \in \mathbb{N} and f (x) = 1 for x \in \mathbb{N} and f (x) = 1 for x = 1 for x \in \mathbb{N} and f (x) = 1 for x = 1 for x = 1 for $= 2 S 2\pi Z 1 Z 1 (x + y + z) dV = 2 E Z 1Z = 0 + 0 + 4\pi (r \cos \theta + r \sin \theta + z)r dz dr = \pi$. hence P ∞ k=1 1/(2k - 1)2 = ($\pi/2$)($\pi/4$) = $\pi 2/8$. Choose by Archimedes an N \in N such that N > M/ ϵ . 4.1.2. a) Let n \in N. Then Ac is open in Rm , so by Theorem 9.26, A0 := -1 f (Ac) $\cap E$ is relatively open in E. 2 a holds. (Such points exist since this intersection contains infinitely many points, hence at least one different from a.) Since $\rho(xn, a) < 1/n$, it follows from the Squeeze Theorem that $xn \rightarrow a$ as $n \rightarrow \infty$. Compactness is not needed to prove f + g is uniformly continuous. By Abel's Transform, N X ak rk = SN rN + (1 - r)N - 1X k = 0 Sk rk. Then kf (x)k < kLk + 1 for all $x \in B\delta(a) \setminus \{a\}$. If $\alpha = 0$, then $\alpha xn = 0$ for all $n \in N$, hence is Cauchy. Hence by induction, xn is increasing and bounded $\sqrt{above by 0. c}$ If (a, b, c) \in H and t(1, 1, -1) = (-a, -b, c) then a = b = c = t hence a2 = a2 + b2 - c2 = 1, i.e., $a = \pm 1$. If $C := supx \in \mathbb{R} |f(x)|$, then $Z |\Delta N(x)| \leq 2Z \pi - \delta |\phi N(t)| dt + E\delta |f(x - t) - f(x)| |\phi N(t)| dt \delta Z 2\pi - \delta \leq 2M + 2C |\phi N(t)| dt$. 158 Copyright © 2010 Pearson Education, Inc. 3.1.7. a) Since $f(x) \le |f(x)|$ it is clear that $f + (x) \ge 0$ and $f - (x) \ge 0$. By Theorem 2.36 and a), $\lim xn + \lim \inf yn \le \lim xn + \lim$ $(1 - r) \propto X$ Sk rk = (1 - r)2 k=0 $\propto X$ (k + 1) σ k rk. ds dt ds k φ (t)k k φ (t)k In particular, k ν 0 (s)k = 1 for all s \in [0, L]. Conversely, suppose f -1 (E) \cap B is closed for every closed E in Rm but f is NOT continuous at some a \in B. Then $\sqrt{9}$ u/ $\sqrt{\sqrt{x}n + x} \sqrt{\sqrt{x}n - x} \sqrt{\sqrt{x}}$. Since curl F = (xz, -yz, -2xy), it follows from Stokes's Theorem that Z Z F \cdot T ds = $(-x^2, xy, -2xy) \cdot (1, 0, 1) dA C B1 (0, 0) 2\pi Z 1 Z = -0$ (r3 cos θ + 2r2 sin θ cos θ) dr d θ 0 Z 1 = $-\pi$ r3 dr + $0 = -\pi/4$. Define $q(x) := \lim k \to \infty f(xk)$. 9.2.4. For each $x \in K$, $Br(x) \cap K = \{x\}$. If x = -1 or x = -3, this series is k=1 (± 1)k/(k + 1) which converges absolutely by the Limit P ∞ Comparison Test, since k=1 k $-3/2 < \infty$. Hence by definition, f (b-) exists and is equal to L. d) By the Chain Rule, (f \circ g)0 (2) = f 0 (g(2))g 0 (2) = bc. Now by l'H^o opital's Rule, u² log(1/u) \rightarrow 0 as u \rightarrow 0 for any 2 > 0. E j=1 In particular, the integral is Vol (E) if n is even. 5.2.3. a) By Exercise 4.4.4, x2 - x6/3! < sin(x2) < x2 - x6/3! + x10/5!. Then $\sqrt{0} < xn-1 + 1 < xn-1 + 1$ and it follows that xn-1 < xn-1 + 1 - 1 = xn. Thus (*) holds for all a, b \in R. 0 2 2 2 Making the substitution u = a + (b - a) Cos t sin t, we have Z xy ds = C - ab 2(b2 - a2) Cos t sin t, we have Z xy ds = C - ab 2(b2 - a2) Cos t sin t, we have Z xy ds = C - ab 2(b2 - a2) Z a2 \sqrt{u} du = b2 ab(a2 + ab + b2). c) Suppose 0 < a $\sqrt{< 1}$. Since $\varphi(0) = 0$, integration by parts yields Z N Z N e-st f (t) dt = e-(s-a)N $\varphi(N) + (s - a)$ $e^{(s-a)t \phi(t) dt}$. Then N [A := E \ B\delta (xj) j=1 is closed and bounded, hence compact. Hence Vol (E 0) = Vol (E) = Vol (E). This last term converges to 0 as (x, y) \rightarrow (0, 0) since $\alpha < 1/2$. By Theorem 3.40, it suffices to prove that f is continuously extendable from the right at x = a, i.e., that if xn \in (a, b] and xn \rightarrow a, then f (xn) \rightarrow L for some L \in R. Therefore, $\kappa(x0) = 1/r$ for all x0 on the circle C. If it holds for some $n \ge 3$ then 2(n + 1) + 1 = 2n + 1 + 2 < 2n + 2 < 2nrn+k-1 $|x1 - x0| \le rn |x1 - x0| \le rn |x1 - x0|$. Then $\sigma n \ge k=N$ sk $/n \ge (n - N)$ M/n. In particular, f (t) is orthogonal to f 0 (t). On the other hand, choose p < q in Q such that $x . c) By Exercise 14.2.2, <math>\sigma 2N$ f is uniformly bounded on R, hence by part b), d) If f is even, then bk (f) = 0 for k $\in N$. Since xn increases from $x_0 > -1$, the limit is 0. 2 d) $x_2 + 2x - 1$ $\rightarrow -1$ as $x \rightarrow 0+$ and sin x is positive as $x \rightarrow 0+$, so ex +2x-1 / sin $x \rightarrow e-1$ / $0+ = \infty \sqrt{\sqrt{a}}$ as $x \rightarrow 0+$., N } such that (ψ j, [(j - 1)/N, j/N]) and (φ j, Ij) are orientation equivalent with transition τ j. Hence by induction, there are distinct natural numbers k1, k2, . If this integral converges uniformly on [0, 1] then $\overline{2} X \overline{2} - 1$ 2 0 uniformly for $y \in (0, 1]$ for N large., xn }. h $\rightarrow 0$ h ∂y h Finally, if f were differentiable at (0, 0), then f (h, k) - f (0, 0) - $\nabla f(0, 0) - \nabla f(0, 0)$ for $(x, t) \in H \setminus K$. 57 Copyright © 2010 Pearson Education, Inc. Since N is fixed, k=1 |bk - b|/n $\rightarrow 0$ and $(n - N)/n \rightarrow 1$ as $n \rightarrow \infty$. Since $\varphi(0) = 0$ and $\varphi(x) \rightarrow 0$ as $\sqrt{x} \rightarrow \infty$, it follows that 1/e is the maximum of φ on $[0, \infty)$. \sqrt{e} Pationalizing the numerator, the terms of this series look like 1/(k 2p + 1 + kp). Evidently, the first case holds c) By part b),
 $\infty \pi 4X = 100$ copyright $(x - 1)^2 = 0$ as $n \to \infty$. Since $S = (D\varphi(x)) - 1$, we have det $(S) = |\Delta \varphi(x)| - 1$. Since ρ is positive definite, we conclude that a = b. Thus A = 22, e - 2e - 1. b) If A = Q and B = Ac then $A \cap B = \emptyset$ but $A \cap B = R$. P = b) False. $x \in (\partial B)c$. $\sqrt{p}c$) True. Apply Jensen for $\varphi(x) = ex$. c) $M := supx \in E f(x)$ is finite by part b). Conversely, suppose $y \in f(A) \cap f(B)$., m, i.e., $\nabla f(0) = 0$. By Exercise 1.6.5, φ takes $\{1, 2, . It follows that the lines do not intersect. Thus <math>U(f, Pn) - L(f, Pn) = n n X 1 X n 2 (x_2j - x_2j - 1)(x_j - x_2j - 1) = 3 \rightarrow 0 n j = 1 n j = 1 as n \rightarrow 0 n j = 1 n j = 1 a$ ∞ , so f is integrable by Definition 5.9. Since U (f, Pn) = n 1 X 2 n(n + 1)(2n + 1) 1 k = \rightarrow n3 6n3 3 k=1 37 Copyright © 2010 Pearson Education, Inc. 4.1.1. a) (f (a + h) - f (a))/h = (a + h - a)/h = 1/(a + h + a) \rightarrow 1/2 a as $h \rightarrow 0$. Thus $|x - x| + 2|yn - y| \rightarrow 0$ y $|y| |y| as n \rightarrow \infty$ by Theorem 2.12i and ii. g g 2 (3) 8 28 Copyright (x - 1) |f(xn)| dx = |f(u)| du. 4.5.6. a) Suppose that x = 1, we have Z b Z b Z n 1 b |f(xn)| dx = |f(u)| du. 4.5.6. a) Suppose that x = 1, we have Z b Z b Z n 1 b |f(xn)| dx = |f(u)| du. 4.5.6. b) Suppose that x = 1, we have Z b Z b Z n 1 b |f(xn)| dx = |f(u)| du. 4.5.6. b) Suppose that x = 1, we have Z b Z b Z n 1 b |f(xn)| dx = |f(u)| du. 4.5.6. b) Suppose that x = 1, we have Z b Z b Z n 1 b |f(xn)| dx = |f(u)| du. 4.5.6. b) Suppose that x = 1, we have Z b Z b Z n 1 b |f(xn)| dx = |f(u)| du. 4.5.6. b) Suppose that x = 1, we have Z b Z b Z n 1 b |f(xn)| dx = |f(u)| du. $2\pi \mu \mu \P \P Z 1/\sqrt{2} \sin t \cos t \cos 2t - \sqrt{\cos \sqrt{+\cos t}} = 1.$ On the other hand, if f (x0) > y0 then choose h0 > 0 such that x0 + h0 < b and f (x0 + h0) > y0. Since - 1 1 1 1 dw = -2 dx - 2 dy - 2 dz w2 x y z we have dw/w = $\pm p(w/x + w/y + w/z) = \pm p$. If p = 1/e, then the terms of the series become kk 1 > $\sqrt{k} e \cdot k! e k$ by Stirling's Formula. Consequently, Vol (S $\circ \phi$)(Q)) \leq sn (1+M²)n =: C² sn = C² |Q|. This vector is parallel to (1, 1, 1), the normal of the plane x+y+z = 1, if and only if x = y = -1/2. 18 Copyright © 2010 Pearson Education, Inc. 14.3.7. a) Since f(x) = x is odd, it is clear that ak(f) = 0 for k = 0, 1, . By the Mean Value Theorem and hypothesis, given $(x, y) \in B3(0, 0)$, $p | f(x, y) - f(0, 0) | \leq |\nabla f(c, d) \cdot (x, y)| \leq k(x, y)k p$ for some $(c, d) \in L((x, y); (0, 0)) \subset B3(0, 0)$. c 1 - c 0 1 8.2.6. By Theorem 8.9vii, a normal to the plane Π is given by $(b - a) \times (c - a)$. Proof. It is also easy to check that a rectangle Rj satisfies Rj \cap E 6= \emptyset if and only if x + Rj \cap x + E 6= \emptyset). e) sin(x + $\pi/2$) \rightarrow sin($\pi/2$) = 1 as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-=-\infty$ as x $\rightarrow 0-$ and 3 cos x - 1 $\rightarrow 1/0-$ 1.2.4. a) |2x + 1| < 7 if and only if -7 < 2x + 1 < 7 if and only if -4 < x < 3. Since f 0 (x) > 0 when x > α , B $\alpha := f(\alpha) = (\alpha/e)\alpha$ is the absolute maximum of f on (0, ∞). It follows from Lemma 7.11 that fn \rightarrow f uniformly on [a, b], i.e., that kfn $-f k \rightarrow 0$ as $n \rightarrow \infty$. Suppose 0 < xn < 1. But since f ([0, 1]) $\subseteq E$, U, V separates f ([0, 1])
$\subseteq E$, U, V separates f ([0, 1]) \subseteq 1]), a contradiction. Therefore, x + E is a Jordan region if and only if E is. Therefore, fj (x) \rightarrow fj (a) as $x \rightarrow a$. Since kN φ k = k(0, 0, 1)k = 1, we have ZZ Z g d σ = 1g(u, v, 0) d(u, v). Let h(x) = g(x) - f(x). 11.2.5. At any (x, y) 6 = (0, 0), f has continuous first partials , hence is differentiable by Theorem 11.15. 1.3.7. a) Let x be an upper bound of E and $x \in E$. Observe that these inequalities also hold for x = a. Therefore, the pair U, V separates E α 0, a contradiction. 10.3 Interior, closure, and boundary., it follows that Z $\pi/2 \propto \propto X X (-1)k-1 (-1)k f(x) dx = = ., b) - (a, a, . 8.2.2. a)$ Since (1, 0, 0, 0) lies on the plane, the constant term in the equation of this plane must be nonzero. In particular, $e \leq 2/(ae)$. Also, 0 = f 0 (x) = $(1 - \alpha \log x)/x\alpha + 1$ implies $\log x = 1/\alpha$, i.e., $x = e1/\alpha$. This contradicts the fact that f takes [0, 1] onto [0, 1]. If $xn \to x$ then $xn - x \to 0$, i.e., $f(xn - x) \to f(0) = 1$ as $n \to \infty$. In particular, f must be continuous by Theorem 7.9. 14.3.6. a) Using the change of variables $t = u + \pi/k$, dt = du, and a sum angle formula, we have $Z \ 1 \pi$ ak (f) = f $(t) \cos kt dt \pi - \pi Z \pi Z 1 \pi 1 \pi \pi = f(u +)(\cos ku \cos \pi - \sin ku \sin \pi) du = -f(u +)\cos ku du$. Thus $(x - \delta, x + \delta) \subseteq f - 1$ (I) and f - 1 (I) an It follows from the Comparison Test and the p-Series Test that this series converges for all p > 1. Clearly, V1, In particular, it suffices to prove that $f_j(x) \rightarrow f_j(a)$. Since the coefficients are nonnegative, (Sk f)(0) = k j `=1 `=1 a0 (f) X a0 (f) X + a` (f) = (Sj f)(0). Hence by the triangle inequality, kxk k is bounded (by M + kak). 14.4 Convergence of Fourier Series. Therefore, $q Fx^2 + Fy^2 + Fz^2 = |u0(w)| p p \sqrt{x^2 / w^2 + z^2 / w^2$ (x) dx + (U) cf(x) dx. is closed as Exercise 10.6.3 says it should; $f - 1(-1, 1) = R \setminus \{x : x = (2k + 1)\pi/2, k \in Z\}$ is open as Theorem 10.58 says it should. Hence by the Reflection Principle, inf $E = -\sup(-E) = E$., x5 = -.3176721. If $y \in Bc(x)$ then, since $Bc(x) \subseteq Br0(x) \subseteq Br(a)$, it is clear that $y \in Br$ (a). Since $y_1 \ge f(x_1)$ and $y_2 \ge f(x_2)$, we also have $y \ge y *$. interval by Theorem 7.43 and $x_3 - x + 5 = 5 + 2(x - 1) + 3(x - 1)2 + (x - 1)3$. b) By Theorem 7.43 and $x_3 - x + 5 = 5 + 2(x - 1) + 3(x - 1)2 + (x - 1)3$. b) By Theorem 7.43 and $x_3 - x + 5 = 5 + 2(x - 1) + 3(x - 1)2 + (x - 1)3$. b) By Theorem 7.43 and $x_3 - x + 5 = 5 + 2(x - 1) + 3(x - 1)2 + (x - 1)3$. b) By Theorem 7.43 and $x_3 - x + 5 = 5 + 2(x - 1) + 3(x - 1)2 + (x - 1)3$. b) By Theorem 7.43 and $x_3 - x + 5 = 5 + 2(x - 1) + 3(x - 1)2 + (x - 1)3$. b) By Theorem 7.43 and $x_3 - x + 5 = 5 + 2(x - 1) + 3(x - 1)2 + (x - 1)3$. b) By Theorem 7.43 and $x_3 - x + 5 = 5 + 2(x - 1) + 3(x - 1)2 + (x - 1)3$. all $x \in Br(x0)$. Set $\frac{1}{2}f(t) := 2 2 e^{-1/(t-a)} e^{-1/(t-b)} 0 t 6 = 0 t = 0$, and observe by Exercise 4.4.7 that f is nonnegative and C ∞ on R, f is positive on (a, b), and f = 0 on (a, b)c. If M < sup E, then there is an $x \in E$ such that M < $x \leq sup E$. Since f and g are bounded, choose C > 0 such that $|f(x)| \leq C$ and $|g(x)| \leq C$ for $x \in [a, b]$. Thus cos x/ logp x is improperly integrable on $[e, \infty)$ for all p > 0. If a 6= 0, then j=1 |xj|/a = 1. Hence, there is an $\varepsilon > 0$ and kj such that kf (xkj) - f (a)k $\geq \varepsilon 0$. Since $\{\varphi j\}$ is a partition of unity, there is an open set W of V containing H and an $N \in N$ such that $\varphi j = 0$ on W for all $j \geq N$. We claim that $E \cap U = \emptyset$. Hence $w := \sup E \in E$. Since for all p > 0. $(x) := \pi 2 \cos 2x$ is continuous on R, it follows that its Fourier series must be uniformly Ces`aro summable to f on compact subsets of $(0, 2\pi)$. 11.6.11. Thus by induction, x1 < x2 < .13.1.4. Let $C = (\varphi, (0, 1])$ where $\varphi(t) = (t, \sin(1/t))$ and set $tk = 2/((2k + 1)\pi)$ for $k \in N$. 0 152 Copyright © 2010 Pearson Education, Inc. b) Let P be a partition of [a, b], P0 $= P \cup \{c\}, P1 = P0 \cap [a, c], and P2 = P0 \cap [c, b]$. Then j=1 Br(xj) (xj). We conclude that $f(\pm 1/2, \pm 1/2, 0) = -1/2$ is the minimum and $f(\pm 1/2, \pm 1/2, 0) = -1$ $t \in J.$ (1 + x2 + y 2)2 (1 + inequality slipped in is on the left side of (3); all other steps in the proof were identities. $x \rightarrow 0 x \rightarrow 0 k = n+2 k = n+2 33$ Copyright (2010 Pearson Education, Inc. and Z Z c xy dx + (x + y) dy = (a + y) dy = -9 = 0, i.e., a = 1, -3. Thus choose N so large that $x \in [a + \delta, b - \delta]$ and $n \ge N$ imply $|fn(x) - f(x)| < {}^2/C$. If $E \circ 6 = \emptyset$ then there exists a point $x \in E \circ \subset E$ such that $Br(x) \subset E$ for some r > 0, i.e., $Br(x) \cap E = \emptyset$. It follows that $\overline{-y} + 1^{-1}|x - 1|^2|y + 1|^{-1}$. dxn $y = dx1 \cdot 4.5.11$. Then $x \in I$ implies $e - kx \le e - ka$. Since the Geometric $P \circ P \circ A$. series k=1 (1/4)k converges, it follows from the Comparison Theorem that k=N+1 ak converges. Set $\delta = \min\{1/(7M), 1\}$. For x > 0 we have f 00 (x) = 6x, so f (h) - f (0) = lim 6 6 = 0. $\infty R \infty b$) By definition, 0 = -xy/y 0 = 1/y. Since $x \in [-1, 1]$ we must take the minus sign. c) If E is connected in R then E is an interval, empty or an interval, hence connected by definition or Theorem 8.30. In particular, Z Z P = 0 Z y q(x, y, v) dv + r(x, u, 0) du and Q = -0 Z px (x, y, v) dv + r(x, u, 0) du and Q = -0 Z px (x, y, v) dv + r(x, u, 0) duE. Since the later is integrable on $[0, \sin 1]$, R it follows from the Comparison Test that I converges absolutely. b), C) If f is differentiable at x = 1 then for any $x \in (0, \infty)$, f $(x + h) - f(x) f((x + h)/x) 1 = = h h x \mu f(1 + (h/x)) h/x$ 9u4 du = 0 4 (1453/2 - 1). Thus kB(x, y)k2 = x2 (sin 2 θ + cos 2 θ) + y 2 (sin 2 θ + cos 2 θ) = x2 + y 2 = k(x, y)k2 . Rc Rb 5.3.6. Take the derivative of 0 = α a f (x) dx + β c f (x) dx with respect to c. Therefore, the limit of this expression does not exist, i.e., f is not differentiable at (0, 0). Then RN R ∞ |1/y - 0 e-xy dx| = |e-N y /y| \leq e-N < 2 since y \geq 1. b) By part a) and the Fundamental Theorem of Calculus, (S2N f)(x) = Z N - 1 N - 1 4 X sin(2k + 1)x 4 xX = cos(2k + 1)t dt.
10.6.4. a) First, notice by definition and the fact that every subspace is a metric space in its own right, a set is relatively open if and only if its complement is relatively closed. $\sqrt{}$ The constraint implies y = 15/4, i.e., y = ± 15/2. b) If nk = 2k, then $(-1)3nk + 2 \equiv (-1)6k + 2 = 1 + 2 = 3$ converges to 3; if nk = 2k + 1, then $(-1)3nk + 2 \equiv (-1)6k + 3 + 2 = -1 + 2 = 1$ converges to 1. But sin(2k + 1)x is periodic of period 2π . Thus by the Squeeze Theorem, fa $(x) \rightarrow 0 = f(0)$ as $x \rightarrow 0$, i.e., fa is continuous at x = 0. Hence this series diverges by the Divergence Test. 9.4.6. By Theorem 9.39, f (x, y) is continuous at every point (x, y) which satisfies x = x. d) Let f (x) = (x(log x)p) -1 for x > 0. Since these two atlases are compatible, the definition of C p functions on M does not change from one atlas to another. 3 12 4 6.2 Series with nonnegative terms. 3! 3! b) Let $\delta 0 = |x - \pi|$. n n k=1 Thus (U) R1 0 f (x) dx < ² and it follows that (U) R1 0 f (x) dx = 0. g -1 (-1, 1) = (- ∞ , -1) \cup (1, ∞) \cup {0} is not continuous; g -1 [-1, 1] = (- ∞ , -1] \cup [1, ∞) \cup {0} is closed, no big deal-note that Exercise 10.6.3 does not apply since g is not continuous; g -1 [-1, 1] = (- ∞ , -1] \cup [1, ∞) \cup {0} is closed, no big deal-note that Exercise 10.6.3 does not apply since g is not continuous. Then V $\subseteq \bigcup x \in V B^2(x)$. f 0 (x), i.e., (f 0 (x)) = α for each x \in (a, b). Thus the original limit is 0 e = 1., sin(k\pi/2) = 1 when k = 1, 5, If $w \in B^2(y)$ then $kw - ak \le kw - yk + ky - ak \le r - ky - ak + ky - ak = r$ and $kw - ak \ge ky - ak - kw - yk > ky - ak = r$ and $kw - ak \ge ky - ak + s - ky - ak = s$. 62 Copyright © 2010 Pearson Education, Inc. Since $\{gn\}$ is decreasing and nonnegative, it follows that $|gn(x)| \le |g1(x)| \le M$ for all $x \in E$. c) (f(a + h) - f(a))/h = (-h/a(a + h))/h = -1/a(a + h))/h = -1/a(a + h) - f(a)/h = -1/a(a + h) - f(a)/-1/a2 as $h \rightarrow 0$. T 13.4.7. a) By definition, given $x \in S$ there is a parametrization (φx , Ex) which is smooth at x, i.e., such that N φx (u0, v0). Hence it follows from Theorem 11.8 that Z 1 Z 1 Z 0 x cos y x 1-u 9 $\sqrt{\sqrt{1 \ln dx}}$ (u0, v0). Hence it follows from Theorem 11.8 that Z 1 Z 1 Z 0 x cos y x 1-u 9 $\sqrt{\sqrt{1 \ln dx}}$ (u0, v0). Hence it follows from Theorem 11.8 that Z 1 Z 1 Z 0 x cos y x 1-u 9 $\sqrt{1 \ln dx}$ (u0, v0). $-f(px) pk f(x + hej) - f(x) = \lim = pk-1 fxj(x)$. b) Since fy is differentiable at (a, b), we have $\lim (u,v) \rightarrow (0,0) fy(a + u, b + v) - fy(a, b) - \nabla fy(a, b) - \nabla fy(a, b) - \nabla fy(a, b) + (u, v) = 0$. Since a > 1, it follows from definition and hypothesis that ax $\leq ap < aq \leq ay$, thus ax < ay. Then none of the xj 's chosen so far belong to Bs (a). b) The set is bounded, but not closed (since (-2 + 1/n, 0) belongs to the set but its limit, (-2, 0) does not). By what we just proved and (2), m - m m - m 0 + = = 0. E 13.3.6. Parameterize S by $\varphi(x, y) = (x, y, (x^2 + y^2)/2)$ and $E = B\sqrt{8} (0, 0)$. 0 b) It's clear for n = 0. Fix x such that $|x - a| < \delta$. By hypothesis, given 2 > 0 choose $N \in N$ such that $|f_K(x) - f(x)| < 2/(2Vol(E) + 2)$ for k \geq N and x \in E. Thus y = d) By completing the square, y = 2x + x $\sqrt{2x}$ + x2 is a semicircle centered at (1, 0) of radius 1. 8.3.3. a) It is connected (see Remark 9.34 for proof). Thus xk is bounded in k. g 0 (1) e b) By the Inverse Function Theorem, (f -1 g -1)0 (2) = f - 1 (2)(g - 1)0 (2) + g - 1 (2)(f - 1)0 (2) + 1 = 0 + 0 + 1 + 0 = . It follows from the triangle inequality that $M + \rho(a, b)$ is an upper bound for the nonempty set { $\rho(xn, a) : n \in N$ }. Thus $n > 499 \approx 22.33$, i.e., $n \ge 23$. If x0 = -1, then xn = -1 for all n. ((2k - 2)/(2k - 1)) $\cdot (2k/k^2) < 2/k \rightarrow 0$ as $k \rightarrow \infty$. 3 c) Fix $x \in [0, 1]$. Then $W := W1 \cap W2$ is open, contains H, and $\phi j \psi k = 0$ on W for j, $k \ge N := \max\{N1, N2\}$. Therefore, $|f(x)|/|g(x)| \le M$. c) $\Omega f(-h, h) = 2$ for all h = 0 so $\omega f(0) = 2$. 2k + 5 2k + 5 k + 5/2 Hence it converges absolutely by Raabe's Test. Similarly, yn = yn2 < xn yn = yn+1 implies xn+1 > yn+1 > yn + 1 > ynImages. Hence $D(f + g)(x, y) = [x \cos x + \sin x + yx + \sin y]$ 104 Copyright © 2010 Pearson Education, Inc. 0 c) If p < q, if f is locally integrable on (0, 1), and if the improper integral μZ kf kq := 1 ¶1/q |f (x)|q dx 0 is finite, then the improper integral μZ kf kq := 1 ¶1/q |f (x)|q dx 0 is finite, then the improper integral μZ kf kq := 1 ¶1/q |f (x)|q dx 0 is finite. , xN such that N [K ⊂ B\delta x] (xj). Then $y \in (x - \delta, x + \delta)$ implies |f(x) - f(y)| < 2, i.e., $f(y) \in (f(x) - 2, f(x) + 2) \subset I$. Let M > 0 and choose $N \in N$ such that supk $\geq N \times k \leq -M$. Moreover, if $q0 \in Q$ satisfies q0 > x and $q \in Ex$, then q0 > q, so by hypothesis aq < aq0. Since [a, b] is a closed, bounded interval, it follows from Theorem 3.39 that f is uniformly continuous on [a, b]. By the argument of Theorem 3.40, this definition is independent of the sequence $xn \in D$ chosen to approximate x. b) Let $f(x) = 1/(x \log (x + 1). c)$ Using the substitution t = uq, we obtain Z xq L(xq) = 1 dt = q t Z 1 x du = qL(x). 9.4.2. a) f(0, 1) = [0, 1) is neither open nor closed; f[0, 1] = [0, 1] is compact and connected as Theorems 9.29 and 9.30 say it should. It follows from the Monotone Convergence Theorem that $xn \rightarrow a$ as $n \rightarrow \infty$. Fix $k \in N$ and observe by Theorem 7.10 and orthogonality that $\mu \parallel Z \perp \pi k$ ak (f) = lim $\sigma N(x) \cos kx \, dx$ = lim 1 - ak = ak. Hence, $\rho(xn, a) \leq \epsilon$ for k large. 11.6.4. Let F (x, y, u, v) = (xu2 + yv2 + xy - 9, xv2 + yu2 - xy - 7) and observe that $\mu \parallel \partial(F1, a) \leq \epsilon$ for k large. F2) 2ux 2vy = det = 4uvx2 - 4uvy2 = 4uv(x2 - y2). We find that there is an $x0 \in (c, d)$ such that $\pm 1 f - 1(x) - f - 1(x2) = (f - 1)0(c) \cdot (x - x2) = 0 - 1$. h h It follows that $T = [f10(x) \cdot (x - x2) = 0 - 1$. h h It follows that $T = [f10(x) \cdot (x - x2) = 0 - 1$. h h It follows that $T = [f10(x) \cdot (x - x2) = 0 - 1$. h h It follows that $T = [f10(x) \cdot (x - x2) = 0 - 1$. h h It follows that $T = [f10(x) \cdot (x - x2) = 0 - 1$. h h It follows that $T = [f10(x) \cdot (x - x2) = 0 - 1$. h h It follows that $T = [f10(x) \cdot (x - x2) = 0 - 1$. h h It follows that $T = [f10(x) \cdot (x - x2) = 0 - 1$. h h It follows that $T = [f10(x) \cdot (x - x2) = 0 - 1$. h h It follows that $T = [f10(x) \cdot (x - x2) = 0 - 1$. h h It follows that $T = [f10(x) \cdot (x - x2) = 0 - 1$. h h It follows that $T = [f10(x) \cdot (x - x2) = 0 - 1$. h h It follows that $T = [f10(x) \cdot (x - x2) = 0 - 1$. h h It follows that $T = [f10(x) \cdot (x - x2) = 0 - 1$. h h It follows that $T = [f10(x) \cdot (x - x2) = 0 - 1$. h h It follows that $T = [f10(x) \cdot (x - x2) = 0 - 1$. of S has three smooth pieces, C1 (given by y = 0, z = 0, $0 \le x \le 1$, oriented left to right), C2 (given by y = (1 - x)/2, z = 0, $0 \le y \le 1/2$, oriented top to bottom). ISBN-13: 978-0-132-29639-7 ISBN-10: 0-132-29639-X An Introduction to Analysis Table of Contents Chapter 1: The Real Number .6 Countable and uncountable sets......8 Chapter 2: Sequences in R 2.1 .1 The Completeness Axiom..... ..2 Mathematical Induction..... ...4 Inverse Functions and Images.... System 1.2 1.3 1.4 1.5 1.6 Ordered field axioms...... ..11 Bolzano-Weierstrass Theorem. .16 Chapter 3: Functions on R 3.1 3.2 3.3 3.4 2.2.2.3.2.4.2.5 Limits of Sequences. .15 Limits Supremum and Infimum.... .10 Limit Theorems .13 Cauchy Sequences. ..24 Chapter 4: Differentiability on R 4.1 4.2 4.3 4.4 4.5 The Derivative...... Two-Sided Limits .19 One-Sided Limits and Limits at Infinity... ..20 Continuity... .22 Uniform Continuity. ...30 Taylor's Theorem and l'Hôpital's Rule... ...34 Chapter 5: Integrability on R 5.1 5.2 5.3 5.4 5.5 5.6 The Riemann Integral... ...46 Functions of Bounded Variation..... ...49 Convex Functions..... ...51 Copyright © 2010 Pearson Education, Inc. $x \rightarrow a f(x)$ $x \rightarrow a f(x) \lim x \in I \text{ Set } M = 1/\epsilon$. Then, $\mu 0 F(\varphi(t)) \cdot \varphi(t) = \text{Therefore}, - \sin t \cos t, rr \P \cdot (-r \sin t, r \cos t) = \sin 2 t + \cos 2 t = 1$. Therefore, $- 1 = |f(x)| |f(xn) - 1| = |f(x)| |f(xn) - 1| = |f(x)| |f(xn - x) - 1| \rightarrow 0 f(x) as n \rightarrow \infty$. 2.3.2. If x1 = 0 then xn = 0 for all n, hence converges to 0. Choose $qn \in Q$ such that $qn \rightarrow x$ as $n \rightarrow \infty$. 19 Copyright © 2010 Pearson Education, Inc. 6 6 6 c) The formula holds for n = 1. Since f (x) lies above the chord, the maximum of F occurs at the endpoints. $\sqrt{-a} 2/2$ Hence, $e-x \leq 1/(xe)$ for all x > 0. Therefore, $x \sin(1/x)$ is not uniformly continuous. on (0. 1) when $\alpha \leq 0$. t x2 x1 44 Copyright © 2010 Pearson Education, Inc. But E \ C = E \cap Ac . If (x0, v0, z0) is defined to be kvk where v = (x0 - x1, y0 - y1, z0 - z1) is orthogonal to Π and (x1, y1, z1) lies in Π . Similarly, $\infty X \propto \infty k=2 k=3 X X 1 1 1 \ge p k \log (k + 1) (k + 1$ 1) $\log (k + 1) k \log p k p k = 2$ diverges when $p \le 1$. Since these are compact sets, we can choose V1, . b) The set is "dumbbell" shaped. On the other hand, if lim inf |ak/ak+1| > R then lim sup |ak+1/ak| < 1/r = rk/rk+1 for k large. Fix r > 0 and let x1 $\in E \cap Br$ (a) $\{a\}$. Hence the class of algebraic numbers of as $y \rightarrow 0$ but the horizontal path y = 0 yields $g(x, y) = f(x)(0 + 1) \rightarrow 0$ as $x
\rightarrow 0$. b) By the Mean Value Theorem, f(x, y) - f(a, b) = g(1) - g(0) = g(1) - g(1) - g(0) = g(1) - g(1) - g(1) - g(1) = g(1) - g(1) - g(1) - g(1) - g(1) - g(1) = g(1) - g(1) -0, i.e., $x = (2k + 1)\pi/2$ for $k \in Z$. b) Let $\varepsilon > 0$ and choose $M \in R$ such that x > M implies $|f(x) - L| < \varepsilon$ and $|h(x) - L| < \varepsilon$ and $|h(x) - L| < \varepsilon$ and $|h(x) - L| < \varepsilon$. Since \tilde{A} (*) fn (x) = (fnn (x))1/n $\leq n X$!1/n fkn (x) $\leq n1/n$ fn (x) k=1 Pn 1/n and $n1/n \rightarrow 1$ as $n \rightarrow \infty$, it is clear by the Squeeze Theorem that (k=1 fkn (x)) converges pointwise to f (x) as $n \rightarrow \infty$. Let $C = (\varphi, [a, b])$ be a closed smooth curve and $\varphi = (\psi, \sigma)$. Then by Theorem 10.34, Br (x) \subseteq E o so $x \in$ E o , a contradiction. In particular, n = (Fx (a, b, c), Fz (a, b, c), Fz (a, b, c), Fz (a, b, c)) given by n is a normal to the tangent plane at (a, b, c), Fz (a, b, c), Fz (a, b, c), Fz (a, b, c). $a \le b$ and $b \le a + c$, $|a - b| = b - a \le a + c - a = c$. 11.7.2. a) 0 = fx = 2x + 2 and 0 = fy = -2y implies x = -1 and y = 0. If a = -b = 1 and n = 2, then (a + b)n = 0 is NOT greater than b2 = 1. Indeed, L((0, 1); (1, 1)) intersects E at only two points. Thus $n \ge N$ implies $\sqrt{\sqrt{\sqrt{11 + \pi/n}}} = 1$. Indeed, L((0, 1); (1, 1)) intersects E at only two points. Thus $n \ge N$ implies $\sqrt{\sqrt{\sqrt{11 + \pi/n}}} = 1$. Indeed, L((0, 1); (1, 1)) intersects E at only two points. Thus $n \ge N$ implies $\sqrt{\sqrt{11 + \pi/n}} = 1$. Education, Inc. $p \sqrt{6.3.4}$. Notice that $k | ak xk | = k ak |x| \rightarrow a |x| as k \rightarrow \infty$. x0 = 2. Then y 2 + 9z 2 = 9 sin 2t + 9 cos 2t = 9. 10.3.8. a) If V is open in Y, which contains x and is a subset of V. b) Suppose A is clopen and $\emptyset \subset A \subset E$. Hence by Remark 6.40, |sn - s| is dominated by (1/3)n / (2/3). = (1/2)(1/3)n-1. 14.5.2. Since the coefficients of the second formal integral are dominated by M/k 2, it follows from the Weierstrass M-Test that this series converges uniformly on R. 4.5.10. Let $\sigma(y) = (y - g(x))/\delta$, $h = \sigma \circ g$, and V = h-1 (B δ (h(x))). Since ∂ H is compact, it can be covered by finitely many such balls, say B1, . x2 + y 2 3 x2 + y 2 $(x,y) \rightarrow (0,1)$ 4 d) The domain of f is all $(x, y) \in \mathbb{R}^2$ such that $(x, y) \in \mathbb{R$ Test. , αN such that N [HC f -1 (V α j). Thus $\tilde{A} ! \sqrt{s} + s^2 - 4t s - s^2 - 4t - 1$ f (s, t) = , $\partial E E E$ If RR u is harmonic on E, then the integral on the left is zero. My calculator will not show more than nine places, so I cannot tell how many more digits we picked up going from x3 to x4. Note that $C\alpha \rightarrow \infty$ as $\alpha \rightarrow 0+$ and $C\alpha \rightarrow 0$ as $\alpha \rightarrow \infty$. 4.4.2. a) If f (x) = $\log x, \text{ then } f(n)(x) = (-1)n - 1(n-1)!/xn \ . \ 8.4.3. \text{ If } A \subseteq B \text{ then } \text{ Ao is an open set contained in } B. \text{ Since } |f(x)| \le |f(x) - 1| + 1 \le |x| + 1 \text{ for all } x \in R. \ N \to \infty \Pi - \Pi \ N \to \infty N + 1 \text{ Similarly, } a0(f) = a0 \text{ and } bk(f) = bk \text{ for } k \in N. \text{ A portion of the plane } x + y - z = 1 \text{ lies above the first quadrant of the xy plane and slants away from the slants away from the plane } x + y - z = 1 \text{ lies above the first quadrant of the xy plane and slants away from the plane } x + y - z = 1 \text{ lies above the first quadrant of the xy plane and slants away from the plane } x + y - z = 1 \text{ lies above the first quadrant of the xy plane and slants away from the plane } x + y - z = 1 \text{ lies above the first quadrant of the xy plane and slants away from the plane } x + y - z = 1 \text{ lies above the first quadrant of the xy plane and slants away from the plane } x + y - z = 1 \text{ lies above the first quadrant of the xy plane } x + y - z = 1 \text{ lies above the first quadrant of the xy plane } x + y - z = 1 \text{ lies above the first quadrant of the xy plane } x + y - z = 1 \text{ lies above the first quadrant of the xy plane } x + y - z = 1 \text{ lies above the first quadrant of the xy plane } x + y - z = 1 \text{ lies above the first quadrant of the xy plane } x + y - z = 1 \text{ lies above the first quadrant of the xy plane } x + y - z = 1 \text{ lies above the first quadrant of the xy plane } x + y - z = 1 \text{ lies above the first quadrant of the xy plane } x + y - z = 1 \text{ lies above the first quadrant of the xy plane } x + y - z = 1 \text{ lies above the first quadrant of the xy plane } x + y - z = 1 \text{ lies above the first quadrant of the xy plane } x + y - z = 1 \text{ lies above the first quadrant of the xy plane } x + y - z = 1 \text{ lies above the first quadrant of the xy plane } x + y - z = 1 \text{ lies above the first quadrant of the xy plane } x + y - z = 1 \text{ lies above the first quadrant of the xy plane } x + y - z = 1 \text{ lies above the first quadrant of the xy plane } x + y - z = 1 \text{ lies above the first$ z axis, so there are two points where the tangent plane to H is parallel to x + y - z = 1, one on the "front" side of H lying below the xy plane. Then by Theorem 9.15 (the limits), f1j1 (x). Hence by Theorem 12.65, ZZ ZZ g($\psi(s, t)kN\psi(s, t)k = g(\varphi(\tau(s, t)))$] $\Delta \tau$ (s, t)| kN φ (τ (s, t))k B Z ZB = g($\varphi(u, v)$)kN $\varphi(u, v)k$. Since the plane contains $\varphi(0) = (0, 0, 0, 1)$ and $\varphi(1) = (1, 1, 1, 1)$, we have d = 1 and a + b + c + d = 1, i.e., a + b + c = 0. b) Let a $\in \mathbb{R}$. Then xn < 0 so |xn| = -xn > 0. On the other hand, if E + = {(x, y) : y > 0} and E - = {(x, y) : y < 0}, then on E + , |1 - ey/k| = ey/k - 1 \le eM/k - 1 \rightarrow 0 uniformly as $k \to \infty$, and on E -, $|1 - ey/k| = 1 - ey/k \le 1 - e - M/k \to 0$ uniformly as $k \to \infty$. Since D is dense, choose $xn \in D$ such that $xn \to x$. μ ¶ n n n-1 X X X ak bk ck - ck+1 (m - cn+1 1 1 = + (ck - ck+1) - . Thus given M > 0, choose N so large that SN $\ge M$. Clearly (see the proof of Remark 1.39), f takes E onto [0, 1]. d) Let m = f (1) and fix $x \in R$. Then there is a pair of open sets U, V which separates E. Then $F \cdot \varphi 0 = (t3, t2 - t) \cdot (1, 2t) = 3t3 - 2t2$. Given $\epsilon > 0$, choose N such that $n \ge N$ implies $|fn(x)-f(x)| < \epsilon/C$ for $x \in [a, b]$. Taking the limit of this inequality as $n \rightarrow \infty$ establishes the given inequality. e) Since cos x is periodic with maximum 1 and minimum -1, f(E) = [-1, 1]. As $t \rightarrow \infty$ $-1-x \rightarrow \infty$, $y \rightarrow -\infty$, $y/x = t \rightarrow -1$, and $dy/dx \rightarrow -1$. Since f(x) > 0 for all x and g is continuous on $(0, \infty)$, it is clear that f and $g \circ f$ are continuous on $(0, \infty)$, it is clear that f and $g \circ f$ are continuous on $(0, \infty)$, it is clear that f and $g \circ f$ are continuous on $(0, \infty)$, it is clear that f and $g \circ f$ are continuous on $(0, \infty)$, it is clear that f and $g \circ f$ are continuous on $(0, \infty)$, it is clear that f and $g \circ f$ are continuous on $(0, \infty)$, it is clear that f and $g \circ f$ are continuous on $(0, \infty)$, it is clear that f and $g \circ f$ are continuous on $(0, \infty)$, it is clear that f and $g \circ f$ are continuous on $(0, \infty)$, it is clear that f and $g \circ f$ are continuous on $(0, \infty)$, it is clear that f and $g \circ f$ are continuous on $(0, \infty)$, it is clear that f and $g \circ f$ are continuous on $(0, \infty)$, it is clear that f and $g \circ f$ are continuous on $(0, \infty)$, it is clear that f and $g \circ f$ are continuous on $(0, \infty)$, it is clear that f and $g \circ f$ are continuous on $(0, \infty)$, it is clear that f and $g \circ f$ are continuous on $(0, \infty)$, it is clear that f and $g \circ f$ are continuous on $(0, \infty)$, it is clear that f and $g \circ f$ are continuous on $(0, \infty)$, it is clear that f and $g \circ f$ are continuous on $(0, \infty)$, it is clear that f and $g \circ f$ are continuous on $(0, \infty)$, it is clear that f and $g \circ f$ are continuous on $(0, \infty)$, it is clear that f and $g \circ f$ are continuous on $(0, \infty)$, it is clear that f and $g \circ f$ are continuous on $(0, \infty)$, it is clear that f and $g \circ f$ are continuous on $(0, \infty)$, it is clear that f and $g \circ f$ are continuous on $(0, \infty)$, it is clear that f and $g \circ f$ are continuous on $(0, \infty)$, it is clear that f and $g \circ f$ are continuous on $(0, \infty)$, it is clear that f and $g \circ f$ are continuous on $(0, \infty)$, it is clear that f and $g \circ f$ are continuous on $(0, \infty)$. But by part a) and symmetry, if f is unbounded on [a, b], then there are tj \in [xj-1, xj] such that |S(f, P, tj)| > |I(f)| + 1, a contradiction. Thus this set is countable by definition., n} \ A and observe that Mj (g) $\leq C$ for all j and Mj (g) = Mj (f) for all j $\in B$. Hence by definition, kCk $\geq k X k\phi(tj) - \phi(tj+1)k \geq 2k j=1$ for each $k \in N$, i.e., kCk = ∞ Thus by Taylor's Formula, exy = 1 + xy + ecd ((dx + cy)4 + 12(dx + cy)2 xy + 12x2 y 2) 4! for some (c, d) \in L((x, y); (0, 0)). $\sqrt{5.4.10}$. As in Exercise 13.3.1b, kN ψ k = 9| cos v| 4 for some (c, d) \in L((x, y); (0, 0)). $\sqrt{5.4.10}$. As in Exercise 13.3.1b, kN ψ k = 9| cos v + 3 sin v cos v dv du $\pi/4$ Z 0 $\pi/2$ = 54 π sin v cos v dv = $\pi/4$ 27 π . 11.3.7. Define T \in L(Rn; Rm) by = T (y) := f $(a) \times (Dg(a)(y)) - g(a) \times (Df(a)(y))$. a)
Let g(t) = a + tu and $h(t) = f \circ g(t)$. $r \to 0 \ \pi r \ Br$ (x0) Thus u is harmonic at x0. If B were at most countable, then its subset f (A) would be at most countable, then its subset f (A) would be at most countable by Theorem 1.41, i.e., there is a function g which takes f (A) onto N. We conclude that $\lim n \to \infty f(\beta n) - f(\alpha n) = \gamma = f 0$ (x). Then a - 1 > 1 so 1 < a - 1 < a - 1by (6). Set f (x) = |x|. 2 2 c) This curve \sqrt{has} two pieces: C1 where $x \ge 0$, and C2 where $x \le 0$. 8.3.5. By completing the square, $\{x2 - 4x + y \ 2 + 2 < 0\} = B\sqrt{2}(2, 0)$. $-1 \ d)$ Let E be the ellipsoid $\{(x, y, z) : x2/a2 + y2/b2 + z2/c2 \le 1\}$. RN b) Clearly, 0 f (x) $dx = 1/2 + 1/4 + \cdots + 1/2N - 1 = 1 - 1/2N - 1 \rightarrow 1$ as $N \rightarrow \infty$., N $\}$ and observe by the choice of δ that x*, y * \in [xk-1, xk] imply that |f (x*) - f (y*)| < ². By adding the Lagrange equations and using the second constraint, we see that $\mu = (x + y)/3$. hence Z Z $\pi/2$ xy ds = ab cos t sin t C p a 2 + (b2 - a2) cos 2 t dt. 5.4.0. a) False./Let a = 0, b = 1.9.6.3. Given ² > 0, choose M so large that |f (x)| < ²/2 for |x| > M. Then there exists a pair of open sets U, V which separates E. 56 Copyright © 2010 Pearson Education, Inc. , RM } be a grid such that x, y \in Rj implies kx – yk < δ and each Qk is a union of Rj 's. R1 0 f (x) dx = 0. 118 Copyright © 2010 Pearson Education, Inc. j=1 Let x \in X. Thus Z ω = S (xz, 1, z) \cdot (x/z, y/z, 1) d(x, y) Bb (0,0) Z Z p r sin θ (r2 cos2 θ + $\sqrt{+a2 - r2}$)r dr d θ 2 2 a – r $0 0 Z b Z b p \pi = \pi r^3 dr + 2\pi a^2 - r^2 r dr = (3b4 + 8a3 - 8(a^2 - b^2)^3/2)$. 2.4.0. a) False. It follows that $\{x : (fg)(x) 6 = 0\} \subset \{x : f(x) 6 = 0\} \cap \{x : g(x) 6 = 0\} \cap \{x : g(x) 6 = 0\}$. If $0 < x^1 < 1$, then by 1.4.1c, xn is decreasing and bounded $\sqrt{(x + 2\pi a^2 - r^2)^3/2}$. f(xN) so f(X) is finite. say $X = \{v1 \dots Dividing top and bottom by xm, we have <math>P(x)$ an $xn-m + an-1 xn-m-1 + \cdots$ $\cdot \cdot + a0$ /xm = . c) g(x) = -1/x is increasing and continuous on (0, 1) but not uniformly continuous there. b) See the proof of Theorem 12.39. Then xk $\rightarrow \infty$, i.e., 1/xk $\rightarrow 0$, as k $\rightarrow \infty$. If k < 0 then nq is a root of the polynomial n-k xj Let $E := \{(x, y) : y \ge f(x)\}$ and suppose $(x1, y1), (x2, y2) \in E$. Thus it follows from the hypothesis f(a) = g(a) = 0 that $kf(x)kkf(x) - f(a)k/|x - a|kDg(a)kas x \rightarrow a$. P ∞ b) By part a), k=1 the roles of k and j, we obtain the reverse inequality. Then (f/g)00 = 0 but g(a)f(0)(a) + f(a)g(0)(a) + fnot zero. b) implies c). Thus f (1) = 5, f 0 (1) = 2, f 00 (1) = 6, and f (k) (1) = 6, and f (k) (1) = 0 for all $k \ge 3$. Since $\forall 3 x = 1 \pm e - 1$, we also have p p f -1 (E) = [1 - e3 - 1, 1) \cup (1, 1 + e3 - 1]. 10.4.4. Suppose A is uncountable. Hence by the Sequential Characterization of Limits, f (x)q(x) $\rightarrow 0$ as $x \rightarrow a$. c) Since $\forall f = (y, x, \cos z)$ = (0, 1, 0) at $(1, 0, \pi/2)$, and the equation of the tangent plane is $w = f(1, 0, \pi/2) + \nabla f(1, \pi/2) + \nabla f(1, \pi/2) + \nabla f(1, \pi/2) + \nabla f(1,$ $\sin 2 \theta$) d θ dr $0 Z 2 = 0 - \pi r 3 dr = 1 - 15\pi$. $0 \delta R\delta$ Now $(s - a) 0 e^{-(s-a)t} dt = 1 - e^{-(s-a)\delta} \rightarrow 1$ as $s \rightarrow \infty$. By part a), there is an $a \in E$ such that $xnk \rightarrow a$. Since this last quotient converges to zero as $N \rightarrow \infty$, it follows that $SN rN \rightarrow 0$ as $N \rightarrow \infty$, it follows that $SN rN \rightarrow 0$ as $N \rightarrow \infty$. By part a), there is an $a \in E$ such that $xnk \rightarrow a$. Since this last quotient converges to zero as $N \rightarrow \infty$, it follows that $SN rN \rightarrow 0$ as $N \rightarrow \infty$. since this last quotient converges to zero as $N \to \infty$, it follows that $SN \to \infty$, by part a), there is an a C = 1 and $y_1 = 1 - (x - 3)^{-1} = 1$ and $y_1 = 1 - ($ converges uniformly on R by the Weierstrass M-Test and absolutely on R by the Comparison Test. This means that there is an open set V in Rn such that $A0 = V \cap E$. k A similar argument shows $|bk(f)| \le \omega(f, \pi/k)$. Since $a1/n \to 1$ as $n \to \infty$ (see Example 2.21), choose an $N \in N$ such that $|a1/N - 1| \le \epsilon/ax0$. The limit does not exist because $f(n) \to 1$ but f $(n + 1/2) \rightarrow 0$ as $n \rightarrow \infty$. To examine the case when (x, y) = (0, 0), notice first that fx $(0, 0) = \text{limh} \rightarrow 0$ (f (h, 0) - f(0, 0))/h = limh $\rightarrow 0$ (f (h, 0) - f(0, 0))/h = limh $\rightarrow 0$ (f (h, 0) - f(0, 0))/h = limh $\rightarrow 0$ h3-2 $\alpha = 0$ because 3 - 2 $\alpha > 0$. Wait a moment and try again. Corresponding equations these tangent planes are x + y - z = -1. Conversely, suppose $\sigma N \rightarrow f$ uniformly on R. In particular, f takes E onto [0, 1]. Since that case has been proved in the text, it follows that $g(x) \neq 0$ (x) = lim 0 = 0. By inspection, it does not converge absolutely when |p| = 1. Then by Theorem 2.35 there is a subsequence kj such that $xkj \rightarrow 0$, i.e., $1/xkj \rightarrow \infty$ as $j \rightarrow \infty$. Hence set hy = r(x, y, 0) and g = 0. Then aN +1 $\leq a2N \leq 1/4$, aN +2 $\leq a2N + 1 \leq 1/16$, and in general, aN +k $\leq a2N \leq 1/4$. 1/4k for k = 1, 2, Since sin $2\theta + \cos 2\theta = 1$, it follows that the critical points are (2, 0), (-2, 0), and (-4/5, $\pm \sqrt{21/5}$. C C(x,y) D(x,y) D $x^2 + y^2 \le |x| + y^2 + y^2 \le |x| + |y| + 0$ as $(x, y) \to (0, 0)$, the limit exists and is 0. d) Let $\varphi_1(t) = (t, 0, 0)$, $I_1 = [0, 1]$; $\varphi_2(t) = (t, 2 - 2t, 0)$, $I_2 = [0, 1]$; and $\varphi_3(t) = (0, t, 0)$, $I_1 = [0, 2]$. Thus $n \ge N$ implies $n - 3n^2 = n(1 - 3n) \le -2n \le -2N < M \cdot b$ By definition, fn \to f in C[a, b] if and only if given $\varepsilon > 0$ there is an $N \in N$ v3 x2 v2 - such that $n \ge N$ implies sup $|fn(x) - f(x)| < \epsilon$. Then $|fnk(x) - f(x)| \to 0$ as $k \to \infty$ for each $x \in [0, 1]$. Thus by the Comparison Theorem and u-substitution, $Z \mid Z \mid Z \mid f(1/x) \mid dx \le dx = f(u) \mid du = .$ Then $(n + 1)3 = n3 + 3n2 + 3n + 1 \le 3n + 2 \cdot 3n = 3n+1 \cdot 1.2.3$. a) By definition, $|a| + a = -2 + \mu - and a + a = -2 + \mu -
and a + a = -2 + \mu - and a +$ 2 |a| - a 2 || = || = 2a = a 2 2 |a| = |a|. Since $\varphi 0$ (t) = (cos t, - sin t, 0), it follows from Stokes's Theorem that ZZ 2 $\pi F \cdot n d\sigma = S$ (36 cos t - sin 2 t) dt = - π . Then x > M implies | sin(x2 + x + 1)/f (x)| $\leq 1/f$ (x) < ϵ . Notice that |ak|2 = a2k and the sequence xn is monotone increasing. Suppose 3 < xn < 5. 0 \sqrt{b}) This curve forms a "script vee" from (-1, -)/f (x) = - π . Then x > M implies | sin(x2 + x + 1)/f (x)| $\leq 1/f$ (x) < ϵ . Notice that |ak|2 = a2k and the sequence xn is monotone increasing. 1) through (0, 0) to (1, 1). Then E is a surface whose boundary equals ∂S , hence by Stokes's Theorem, ZZ ZZ curl F \cdot n d σ = curl F \cdot n d then there is an $N \in N$ such that $n, m \ge N$ implies $|xn - xm| < \varepsilon/|\alpha|$. Let f(x) = 0 for all other points $x \in [0, 1]$. a) Since $n \times Z$ log $k = \log(n!) - n \log n + n - 1 \le \log n - n + 1$, $1 = \log n - 1$, $1 = \log n$ $\cos v$, $b(a + b \cos v) \sin u \cos v$, $b(a + b \cos v) \sin v k = b|a + b \cos v|$. Then f 0 (x) = $\cos x$, f 00 (x) = $-\sin x$. Therefore, $\mu \P \mu \P (-1)n x 4m - 1$ (-1)n x 4m - 1 (-1)n x 4m - 1 (-1)n x 4m - 1 (-1)n $x 4m + 1 x - \dots + < \sin x < x - \dots + .$ 11.7.3. a) The Lagrange equations are 1 = $2x\lambda$ and $2y = 2y\lambda$. 4.3.11. Solving for the integral, we obtain 0 e-st cos bt dt = $s/(s_2 + b_2)$. 0 13.5.5. a) Let P = -y and Q = x. 9.1.1. a) Let $\varepsilon > 0$ and choose (by Archimedes) an N \in N such that k > N implies $1/k < \varepsilon/2$. 14 Copyright © 2010 Pearson Education, Inc. The boundary is $y = x_3$, z = 0, 4, and (0, 0, z), (2, 8, z), for $0 \le z \le 4$. Indeed, if not, e.g., if kxk $k \rightarrow \infty$ for some $xk \in E$, then choose (by sequential compactness) a convergent subsequence of xk, say xkj. b) The radius of convergence of this power series is 1/e. Hence a separation can be made, e.g., by using the open sets $V = \{(x, y) : x > -1\}$, and applying Remark 8.29. 0 c) If S1 and S2 are disjoint "concentric" surfaces which do not contain the origin, and the normals of S1 and S2 both point away from the origin, then ZZ ZZ F \cdot n d σ = F \cdot n d σ S1 S2 for all C 1 functions F which satisfy F = curl G for some C 2 function G on R3 \ {(0, 0, 0)}. In particular, the original series converges uniformly on [a, b] by that |D the Dirichlet Test. If it holds for n then n+1 X (2k - 1)2 = k=1 n(4n2 - 1) 2n + 1 + $(2n + 1)2 = (2n^2 + 5n + 3) 3 3 2n + 1 (n + 1)(4n^2 + 8n + 3) (2n + 3)(n + 1) = 3 3 (n + 1)(4(n + 1)2 - 1) = .$ b) The proof of Remark 12.33 depends only on three properties satisfied by the function f : i) f (x, y0) is zero R1 R1R1 off [2-n-1, 2-n+1], ii) 0 f (x, y0) dx = 0, and iii) 0 0 f (x, y) dy dx = 1. By hypothesis, fn \rightarrow f uniformly on $[a + \delta, b - \delta]$. 14.4.2. a) By Example 14.8, this is the Fourier series of x. a Rb Rb We conclude that g is integrable and a g(x) dx = a f(x) dx. Set f = k=1 fxj and V = j=1 Jrj (xj). Thus by hypothesis, --f(0) + h - f(0) + $2 - x \ge 22 - 2 > 0$, i.e., 2x > x for $x \ge 2$. n! 1 n! (n - 1)! k! k=0 R ∞ In particular, (1/n!) 1 xn e-x dx $\rightarrow e-1$ e = 1 as $n \rightarrow \infty$. b) The inequality holds for n = 1. Let N \in N be so large that $|x| \ge N$ implies f (x) > f (0). $\sqrt{Taking the limit of this last inequality as <math>k \rightarrow \infty$, we see that lim supk $\rightarrow \infty$ k ak $\le r0$. Otherwise, given $\varepsilon > 0$ use Definition 2.1 to choose an $N \in N$ such that $n \ge N$ implies $|bn| \equiv bn < \varepsilon/|C|$. Thus it remains to prove that iii) implies i). b) By Theorem 8.37 or 10.40, $\partial(E1 \cap E2) \subseteq \partial E1 \cup \partial E2$. On the other hand, $\cup x \in V B^2(x) \subseteq V$ since each $B^2 \subseteq V$. 12 0 0 $2\pi b = d$) If $\varphi(u, v) = ((v/2) \cos u, (v/2) \sin u, v)$ then $N\varphi = ((v/2) \cos u, (v/2) \sin u, -v/4)$ points away from the z axis. On the other hand $\{n\}$ is a sequence in R which has no convergent subsequence. These steps are reversible. $\sqrt{14.3.2}$. If f were continuous and $|ak(f)| \ge 1/k$, then $\infty X |ak(f)| \ge k=1 \infty X 1 = \infty k k=1$ which contradicts Bessel's Inequality.

Free Options: D2L. This easy-to-use platform will make it simple to recreate websites with built-in tools, however, there is no full publicly-facing option available. Cascade An accessible, MSU-branded website that is primarily used for MSU unit websites. Making content publicly available requires hosting space such as the LAMP stack (see below). The Holy Bible Containing the Old and New Testaments The culmination of English translations of the Bible, the Bartleby.com publication of the American Bible Society's King James Version features full-text searchability, content-based tables of contents and a quick verse finder.

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